Improved Binary Additive Epsilon Indicator for Obtaining Uniformly Distributed Solutions in Multi-Objective Optimization

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ABSTRACT

Several studies report that the solutions obtained by indicator-based evolutionary algorithm (IBEA) with the binary additive epsilon indicator are biased to specific regions in the objective space. This paper reveals the reason of the bias is due to the asymmetricity of the epsilon indicator without considering the dominance relationship. This paper also proposes a modified epsilon indicator to obtain uniformly distributed solutions in the objective space. The proposed indicator uses indicator value based on Chebyshev distance when the dominance relationship does not exist between two solutions. The results of the experiments show that the diversity of IBEA is enhanced by the proposed indicator on WFG5 problem.

CCS CONCEPTS

• **Mathematics of computing → Evolutionary algorithms;**

KEYWORDS

Evolutionary Computations, Multi-objective Optimization

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1 INTRODUCTION

Indicator-Based Evolutionary Algorithm (IBEA) is an MOEA of using an indicator for the mating and environmental selection [1]. Zitzler and Künzli proposed two types of IBEA. IBEA$_{\epsilon+}$, which employs the binary additive epsilon indicator ($I_{\epsilon+}$), is one of IBEAs proposed in [1]. In the previous benchmarking study [2], IBEA$_{\epsilon+}$ shows the highest performance among the 21 MOEAs including state-of-the-art algorithms. On the other hand, several studies report that the solutions obtained by IBEA$_{\epsilon+}$ are biased to specific regions on the objective space [3]. The paper by Tanabe and Oyama shown that IBEA$_{\epsilon+}$ can generate well-distributed solutions but cannot maintain them in the next population [4]. However, it is not clear why IBEA$_{\epsilon+}$ cannot maintain them in the next population. Several studies have modified IBEA$_{\epsilon+}$ to obtain uniformly distributed solutions [3, 5]. However, the reason why the solutions obtained by IBEA$_{\epsilon+}$ are biased to specific regions in the objective space has not been clarified in these papers.

Objective of this paper is to reveal the reason why IBEA$_{\epsilon+}$ cannot obtain uniform solutions. In particular, we focus on $I_{\epsilon+}$. Then, we propose an improved $I_{\epsilon+}$ and investigate the effects of proposed indicator on WFG5 problem.

2 WHY IBEA$_{\epsilon+}$ CANNOT MAINTAIN DIVERSITY?

IBEA$_{\epsilon+}$ uses fitness in enviromental and mating selections. The fitness value for a solution $\boldsymbol{x}$ is caclulated using the following function:

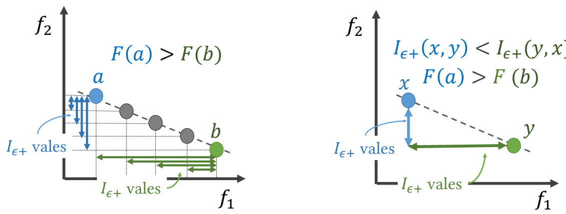
$$F(\boldsymbol{x}) = \sum_{\boldsymbol{y} \in \boldsymbol{P} \setminus \{x\}} e^{-\frac{I_{\epsilon+}(\boldsymbol{x},\boldsymbol{y})}{c \cdot k}} \tag{1}$$

where k is a scaling factor and c is maximum absolute $I_{\epsilon+}$ value. Generally, k is set to 0.05. $I_{\epsilon+}$ is calculated as follows:

$$I_{\epsilon+}(\boldsymbol{x}, \boldsymbol{y}) = \max_{i \in \{1,\ldots,M\}} \{f_i^{'}(\boldsymbol{y}) - f_i^{'}(\boldsymbol{x})\} \tag{2}$$

where $f'(\boldsymbol{x})$ is objective function value normalized to [0,1].

It is generally agreed today that $I_{\epsilon+}$ can assess the diversity of the solution. However, we find that the assessment of the diversity of the solutions by $\text{IBEA}_{\epsilon+}$ does not work well in some situations. Figure 1 (a) shows an example where two solutions are located on a linear Pareto front. As shown in Fig. 1 (a) , $I_{\epsilon+}(\boldsymbol{x},\boldsymbol{y})$ is smaller than $I_{\epsilon+}(\boldsymbol{y},\boldsymbol{x})$; thus solution $\boldsymbol{x}$ is estimated inferior. It is assumed that such a property of the asymmetricity[1] of $I_{\epsilon+}$ for a pair of solutions without the dominance relationship has a negative impact to estimate the diversity of the solution. Figure 1 (b) shows an example where five solutions are located uniformly on a linear Pareto front. As shown in Fig. 1 (b), solution $\boldsymbol{a}$ inferior to solution $\boldsymbol{b}$.



(a) Two solutions on a linear Pareto front

(b) Five solutions on a linear Pareto front

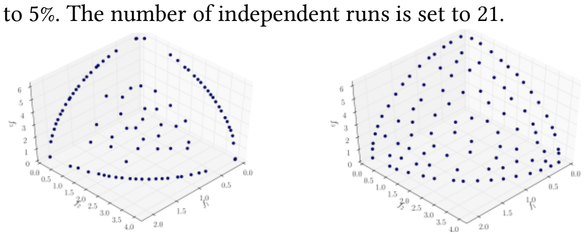
Figure 1: Examples of $I_{\epsilon+}$ calculation

Now we propose a new indicator $I_{\epsilon++}$, that fixes the asymmetricity problem of $I_{\epsilon+}$.

$$I_{\epsilon++}(\boldsymbol{x},\boldsymbol{y}) = \begin{cases} \max_{i\in\{1,\dots,M\}}\{f_i'(\boldsymbol{y}) - f_i'(\boldsymbol{x})\} & , if\ \boldsymbol{x} \prec \boldsymbol{y}\ or\ \boldsymbol{x} \succ \boldsymbol{y} \\ \max_{i\in\{1,\dots,M\}}\{|f_i'(\boldsymbol{y}) - f_i'(\boldsymbol{x})|\} & , otherwise \end{cases} \quad (3)$$

3 EXPERIMENTAL RESULTS

We compare $\text{IBEA}_{\epsilon+}$ and $\text{IBEA}_{\epsilon++}$ on the WFG5 problem. Hypervolume (HV) indicator and the diversity indicator based on reference vectors (DIR) [6] are used to evaluate the quality of the set of the obtained nondominated solutions. The Wilcoxon rank-sum test is applied to make sure statistically differences in HV and DIR. In the Wilcoxon rank-sum test, the significance level is set to 5%. The number of independent runs is set to 21.



$\text{IBEA}_{\epsilon+}$ $\qquad$ $\text{IBEA}_{\epsilon++}$

Figure 2: Population obtained in the median DIR run on 3 objective WFG5 (200 generations, population size of 100).

Figure 2 shows the median DIR run on 3 objective WFG5. As shown in Fig. 2, $\text{IBEA}_{\epsilon++}$ obtains the uniformly distributed solutions in the objective space, while $\text{IBEA}_{\epsilon+}$ cannot maintain diversity of the population. The average HV and DIR values are shown in Table 1. Significance in the difference is shown by the Wilcoxon rank-sum test for all cases. This table show that $\text{IBEA}_{\epsilon++}$ significantly improves diversity in the obtained Pareto-optimal solutions though $\text{IBEA}_{\epsilon++}$ is slightly inferior to $\text{IBEA}_{\epsilon+}$ in terms of hypervolume.

Table 1: Average and standard deviation (in parentheses) of indicator values on WFG5. Higher average is highlighted with gray background.

Indicator	M	$\text{IBEA}_{\epsilon+}$	$\text{IBEA}_{\epsilon++}$
DIR	2	4.6035e-02 (8.3493e-03)	3.2433e-04 (1.9298e-04)*
	3	3.8461e-02 (1.8879e-02)	1.6268e-03 (1.0664e-03)*
	6	1.5532e-03 (7.6825e-04)	3.4697e-04 (3.2909e-04)*
HV	2	3.7536e-01 (1.0613e-03)*	3.7191e-01 (1.5447e-03)
	3	6.8986e-01 (3.4109e-03)*	6.5797e-01 (5.4459e-03)
	6	1.3242e+00 (7.4282e-03)*	1.1651e+00 (2.7632e-02)

4 CONCLUSIONS

In this study, we revealed the reason why the solutions obtained by $\text{IBEA}_{\epsilon+}$ are biased to a specific regions in the objective space. Then, we proposed a new indicator $I_{\epsilon++}$ to obtain uniformly distributed solutions. From our results, it is expected that $\text{IBEA}_{\epsilon++}$ improves the bias to the specific regions in the objective space on WFG5.

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[1] That means "$I_{\epsilon+}(\boldsymbol{x},\boldsymbol{y}) \neq I_{\epsilon+}(\boldsymbol{y},\boldsymbol{x})$, if two solutions without dominance relationship".