A New Constrained Multi-objective Optimization Problems Algorithm Based on Group-sorting

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ABSTRACT

Constrained multi-objective optimization problems(CMOPs) have wide applications in many areas. One of the difficulties in solving CMOPs is to handle constraints and optimize objective values simultaneously. In this paper, three improvements are proposed for CMOPs. Firstly, a new strategy of adaptive grouping is proposed to ensure the diversity of the algorithm. Secondly, each sub-population evolves according to the designed crossover operator, which improves the searchability of the algorithm, thus accelerates the convergence process of the algorithm. Finally, a new sorting method based on the information of representative solutions is designed to measure the quality of individuals. The experimental results show that the proposed algorithm performs better in convergence and diversity.

CCS CONCEPTS

• **Applied computing → Multi-criterion optimization and decision-making;**

KEYWORDS

constrained multi-objective optimization, adaptive grouping, the mixed crossover operator

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1 INTRODUCTION

Compared with MOPs, the existence of constraints in CMOPs makes the topology of the search space more complicated. Firstly, when the number of constraints of the problems is large, the feasible space will become very small, thereby a good balance between global search and local search is needed. Secondly, when there are many non-connected feasible spaces in the problem, there will be massive local optimal solutions which require the algorithm to take diversity and convergence into account [1, 2]. Therefore, the research of CMOEAs mainly about constraint processing and optimization method. For example, NSGA-II-CDP [1] uses CDP to handle constraints. The series of CMOEA [2] algorithms add various constraint handling methods to MOEA/D. However, they are difficult to ensure convergence and diversity simultaneously. In order to balance the diversity and convergence performance of the algorithm, a new constrained multi-objective optimization problems algorithm based on group-sorting called CMOPs-GS is proposed in this paper.

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2 PROPOSED ALGORITHM

In the proposed algorithm, population P is randomly generated firstly. Secondly, the uniformly adaptive grouping method is used to divide P into k sub-populations, after which the individuals update themselves with a new crossover operator in each sub-population. Thirdly, the improved non-dominated sorting [3] is used for ranking, some good individuals are selected from the sorted population for the next generation based on representative solutions and constraint violations. The framework of CMOPs-GS is shown as algorithm 1.

Algorithm 1: Framework of CMOPs-GS

```
Input: P(population), N(size of population)
Output: P(population)
  P = Initialize (N)
  while stopping condition not reached do
    P' = (P'_1, P'_2, ..., P'_k) ⟵ Adaptive grouping(P)
    for i = 1 to k do
      [x_con, x_dis] ⟵ Find representative points in each
      sub-population P'_i (i = 1, 2, ..., k)
      P'_i ⟵ Updation(P'_i, x_con, x_dis)
    end
    F ⟵ Sorting(P ∪ P')
    P ⟵ Environmental selection(F)
  end while
```

The steps of the proposed grouping method are described in algorithm 2. Firstly, evenly divide the target space and uniformly generate r weight vectors. Each weight vector $V_k(k = 1, \dots, r)$ will select a corresponding value from the set $\{\frac{0}{H}, \frac{1}{H}, \dots, \frac{H}{H}\}$. The number r of weight vectors can be calculated by the formula (1):

$$r = C_{H+m-1}^{m-1} \quad (1)$$

where m is the number of objective functions, the positive integer H is a parameter. Secondly, we sort population by improved non-dominated sorting [3], and then obtain f-layer population, each layer is represented as $F_i(i = 1,2,\ldots,f)$. Thirdly, grouping individuals in each layer F_i. Assign each individual F_i^j $(j = 1,2,\ldots,n)$ of layer F_i to the group $P_j^{i,K}$. n is the number of individuals in F_i. V_k is the weight vector with the minimum Euclidean distance $D_{k,i}^j$ to F_i^j. After grouping individuals in F_i, group individuals in the next F_{i+1} until all individuals are assigned. Then $M = f * r$ groups are obtained.

Algorithm 2: Adaptive Grouping

Input: P(population), V(weight vectors)
Output: grouped population $P' = (P_1', P_2', \ldots, P_k')$

Generate a set of uniformly distributed weight vectors $V = \{V_1, V_2, \ldots, V_r\}$, $r = C_{H+m-1}^{m-1}$
$\{F_1, F_2, \ldots, F_f\} \longleftarrow$ non-dominated sorting(P)
for i = 1 to f **do**
for j = 1 to n **do**
Calculate $D_{k,i}^j$
$K \in \{1,2,\ldots,i\} = \{k | min(D_{k,i}^j), k = 1,2,\ldots,r\}$
assign F_i^j to $P_j^{i,K}$
end
end

After adaptive grouping, the first step is to traverse the individuals in units of sub-population and select two representative solutions in each sub-population. One is the individual with the smallest Euclidean distance to PF, called x_{dis}[4], the PF is defined by points which are the maximum value of each objective function, the other is the individual with the smallest constraint violation, called x_{con}. The second step is to build a mixed crossover operator. For individuals except for representative points, update them with formula (2) according to probability p_c.

$$x_i' = \alpha(x_{dis} - x_i) + (1 - \alpha)(x_{con} - x_i) + x_i \quad (2)$$

where α is an adaptive parameter. The crossover population is mutated according to the polynomial variation method.

After updating, the hierarchical sorting of $P \cup P'$ is performed by the improved non-dominated sorting. Then we do the elite selection process based on the representative individuals which go directly to the next generation, and the insufficient individuals are selected from the remaining individuals by sorting.

3 EXPERIMENTAL RESULTS

In order to examine the performance of CMOPs-GS, we adopt the IGD and 22 common widely tested CMOPs including CF, CTP, BNH, OSY, SRN and TNK problems. Five commonly used CMOEAs are adopted to compare with CMOPs-GS: ATM, CMOEA/D, IDEA, MOEA/D-CDP and NSGA-II-CDP [5]. Due to the limitation of the length of this paper, we only show the result of CMOPs-GS compared with others on CF1~CF10.

Parameter H for weight vectors takes 2, and the parameter PU for mutation is 2.0. For the cross operator, we set α variate from 0.6 to 0.5 and p_c from 0.8 to 1. Population size N =1000, maximum evolution iteration number G_{max} =100. In the same operating environment, all the compared algorithms run times $1*10^5$ function evaluations and the independent experiments run times is 30.

Table 1: IGD test results of compared algorithms on 10 test functions

Problem	CMOPs-GS	ATM	CMOEA/D	IDEA	MOEA/D-CDP	NSGA-II-CDP
CF1 mean	**8.279E-03**	5.786E-02	1.449E-02	2.035E-02	1.440E-02	5.964E-02
CF2 mean	8.271E-02	1.372E-01	1.534E-01	5.229E-02	**1.184E-01**	1.605E-01
CF3 mean	**1.581E-01**	6.360E-01	2.610E-01	2.043E-01	3.255E-01	5.414E-01
CF4 mean	**5.792E-02**	1.478E-01	1.772E-01	8.301E-02	1.930E-01	1.558E-01
CF5 mean	**2.353E-01**	3.885E-01	3.653E-01	2.645E-01	3.892E-01	3.779E-01
CF6 mean	**4.200E-02**	1.221E-01	1.242E-01	6.602E-02	1.217E-01	1.188E-01
CF7 mean	3.626E-01	3.755E-01	4.681E-01	**2.848E-01**	3.862E-01	4.150E-01
CF8 mean	**1.933E-01**	2.574E-01	Inf	2.000E-01	Inf	1.781E+00
CF9 mean	7.901E-01	2.030E-01	**1.060E-01**	1.248E-01	1.169E-01	1.900E-01
CF10 mean	**1.803E-01**	Inf	Inf	Inf	Inf	Inf

From Table 1, we can see that CMOPs-GS performs the minimum IGD value on 7/10 test functions. And the result will be better with a smaller IGD value, the mean IGD obtained on the test functions CF1 ~ 10 are largely improved. Overall, IGD mean value has been reduced significantly, which indicates that CMOPs-GS has the best distribution and convergence of pareto solution set among several algorithms, so CMOPs-GS has advantages over other algorithms in diversity maintenance and convergence performance obviously.

4 CONCLUSIONS

This paper mainly studies the CMOPs and proposes a new algorithm based on group sorting. Three improvements are proposed: new adaptive grouping strategy, sub-population evolves according to the designed crossover operator and a new method based on the information of representative solutions to measure the quality of individuals. From our analysis on the experimental results, we can see CMOPs-GS performs well in balancing the feasibility, diversity and convergence of the CMOPs.

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