Operational Decomposition for Large Scale Multi-objective Optimization Problems

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ABSTRACT

Most multi-objective evolutionary algorithms (MOEAs) of the state of the art treat the decision variables of a multi-objective optimization problem (MOP) as a whole. However, when dealing with MOPs with a large number of decision variables (more than 100) their efficacy decreases as the number of decision variables of the MOP increases. Problem decomposition, in terms of decision variables, has been found to be extremely efficient and effective for solving large scale optimization problems. In this work, we study the effect of what we call "operational decomposition", which is a novel framework based on coevolutionary concepts to apply MOEAs's crossover operator without adding any extra cost. We investigate the improvements that NSGA-III can achieve when combined with operational decomposition. This new scheme is capable of improving efficiency of a MOEA when dealing with large scale MOPs having from 200 up to 1200 decision variables.

CCS CONCEPTS

• **Theory of computation → Bio-inspired optimization**; • **Computing methodologies → Continuous space search**;

KEYWORDS

large scale multi-objective optimization, decomposition, multi-objective optimization

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1 INTRODUCTION

The current practice of MOEAs is to assess their performance using benchmark problems test suites, normally adopted with a relatively low number of decision variables, usually, no more than 30 decision variables. However, in real-world applications, many MOPs have hundreds of decision variables and the effect of parameter scalability in modern MOEAs has not been properly analyzed. Besides, there exists empirical evidence that indicates that most of the currently available MOEAs significantly decrease their efficacy as the number of decision variables of a MOP increases [3, 4]. In this paper, we propose a new scheme to apply a MOEAs' crossover operators which improves the performance over large scale MOPs. We study here the effect of parameter scalability and investigate the improvements that a MOEA can achieve when adopting this scheme. For this purpose, we propose to combine the NSGA-III [2] with Cooperative Coevolutionary techniques, giving rise to a novel MOEA based on what we call *operational decomposition*.

2 PREVIOUS RELATED WORK

Regarding studies on parameter scalability in MOEAs, the most significant ones that we are aware of are those reported by Durillo et al. [3, 4], in which the behavior and effect of parameter scalability over eight state-of-the-art multi-objective metaheuristics is analyzed. Another work in this direction is a small study presented in [7], where ZDT1 is solved with up to 100 decision variables using MOEA/D. Later on, an algorithm based on interdependence variable analysis and control variable analysis designed to deal with large scale MOPs was presented in [6]. This work was then improved in [8], where the decomposition is based on a decision variable clustering method.

3 OUR PROPOSED APPROACH

We propose here the so-called *operational decomposition* (OD) approach, which is a coevolutionary step added to a MOEA, where we make use of the divide-and-conquer technique that splits the MOP to be solved (in *decision variables* space) when applying crossover. We use decision variable decomposition to perform crossover operations which allows us to handle in a better way the *curse of dimensionality* present in MOEAs. So, individuals will still be representing a whole solution, but operators will be applied based on the corresponding species, and not based on the individuals. This makes the crossover operator to be more effective, since decomposition of the operations causes a bigger effect than when adopting the usual scheme in which most MOEAs are implemented. The algorithm of our proposed operational decomposition scheme, when incorporated to a MOEA, works as follows:

Input:
- S: Species for decision variables decomposition
- T: Neighborhood size for coevolutionary collaboration

Output:
- PS: the final solutions found during the search

Step 1) Initialization:
Step 1.1) Set the population of final solutions $PS = \emptyset$.
Step 1.2) Generate an initial population $X = x^1, \ldots x^N$. Set $FV^i = f(x^i)$.
Step 1.3) Divide the problem into S subcomponents $c^1, \ldots, c^S$ each one of dimension m, such that, for each $j = 1, \ldots, N$, $x^j = [c^1_j, \ldots, c^S_j]$.

Step 2) Update:
Step 2.1) Find the T closest decision variables vectors to each solution $x_i \in X$. For each x_i, set $B(i) = \{i_1, \ldots, i_T\}$, where $x^{i_1}, \ldots, x^{i_T}$ are the T closest solutions to x^i .
For $i = 1, \ldots, N$ do
Step 2.2) OD Operation and Mutation:
For $j = 1, \ldots, S$ do
Step 2.2.1) Randomly select two indexes p, q from $B(i)$, and then generate a new solution y^j_c from c^j_p and c^j_q using crossover.
Step 2.3) Assemble y' from $[y'^1_c, \ldots, y'^S_c]$.
Step 2.4) Apply mutation operator and evaluate solution y'.
Step 2.5) Remove from the external population PS all the vectors dominated by $f(y')$. Add $f(y')$ to PS if no vectors in PS dominate it.

Step 3) Stopping Criterion: Stop if the termination criterion is satisfied. Otherwise, go to Step 2.

Since c^j_p and c^j_q in Step 2.2.1 are subcomponent (in decision variables space) neighbor solutions and their dimensionality is lower than that of the original vector of decision variables x, their offspring y'^j_c (later improved by mutation) should be a good contribution to the complete assemble of the new final solution y'. The use of neighbors allow the new solution to have a more controlled modification and the absence of it makes the approach to have a very poor performance, causing a poor convergence of the solutions.

3.1 Experimental Results

In order to validate our approach we adopted NSGA-III [2] and incorporated operational decomposition to it, giving rise to a new MOEA called OD-NSGA. For the purposes of this study, we adopted the Deb-Thiele-Laumanns-Zitzler (DTLZ) test suite and the Walking Fish Group test suite [5] with instances of three objectives with a number of decision variables that ranges from 200 to 1200. In order to assess the performance of each approach, we selected the hypervolume indicator [4], since this measure can differentiate between degrees of complete outperformance of two sets.

Function	No. Vars	NSGAIII HV	ODNSGA HV	ODNSGA - NSGAIII Improvement	P(H)
WFG4	200	274	317	43	0.000000 (1)
	400	245	311	66	0.000000 (1)
	600	228	308	80	0.000000 (1)
	800	219	305	87	0.000000 (1)
	1000	208	303	95	0.000000 (1)
	1200	210	301	91	0.000000 (1)

Table 1: Average of the hypervolume indicator for the WFG4 test problem.

For both OD-NSGA and NSGA-III, we adopted Simulated-binary crossover (SBX) and polynomial-based mutation [1]. The mutation probability was set to $p_m = 1/l$, where l is the number of decision variables; the distribution indexes were set as: $\eta_c = 20$ and $\eta_m = 20$. For the case of OD-NSGA, the numbers of species were set such that 2 decision variables per species are used. The maximum number of iterations adopted was set to 1000. Finally, the population size in all problem instances was set to 120 and the number of supplied reference points was set to 12. In our experiments, we obtained the hypervolume value over the 25 independents runs performed. Table 1 show the average hypervolume value of the two MOEAs being compared for WFG4 test problem adopted, as well as the results of the statistical analysis that we made to validate our experiments, for which we adopted Wilcoxon's rank sum. The cells containing the best hypervolume value for each problem have a grey colored background. Based on the results of Wilcoxon's test, we can confirm that the null hypothesis can be rejected, so OD-NSGA yields the best overall results.

4 CONCLUSIONS AND FUTURE WORK

Our approach was able to deal with all the difficulties presented in the DTLZ and WFG test suites, even in high dimensionality. The results confirmed that our proposed approach is very effective and efficient in tackling large scale MOPs. As part of our future work, we intend to study other decomposition techniques for decision variable space. Also, we want to test our approach in many-objective MOPs. We are also interested in studying the incorporation of operational decomposition in other state-of-the-art MOEAs.

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