Generation Techniques and a Novel On-line Adaptation Strategy for Weight Vectors Within Decomposition-Based MOEAs

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ABSTRACT

The success of Multi-Objective Evolutionary Algorithms based on Decomposition (MOEA/D) has generated great interest in the use of a reference set of weight vectors to promote diversity within non-dominated solutions. However, the quality of the solution set obtained heavily depends on the relation between the weight distribution and the Pareto front's shape.

This study focuses on a performance comparison of classical techniques for weight vector generation, either based on mixture design or low discrepancy sequences, and a novel approach for updating the weight vectors during the evolutionary process. This approach uses information from the non-dominated individuals to generate weights vectors through a repulsion criterion. Preliminary experiments indicate that this dynamic strategy provides significant benefits when compared to the static Simplex Lattice Design (SLD).

CCS CONCEPTS

• **Computing methodologies → Search methodologies**.

KEYWORDS

reference sets based MOEAs, weight vector adaption

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1 INTRODUCTION

Among the diversity of approaches for solving Multi-Objective Optimization Problems (MOPs), those based on a set of reference points have attracted significant attention, since they provide a well-distributed set of non-dominated solutions under certain conditions.

In order to improve the distribution on a wide range of Pareto front geometries, several methods for generating reference points have been proposed in the specialized literature. In most of these techniques, the set of weight vectors does not change during the execution (*static methods*). Usually, these methods are well suited for a specific Pareto front shape. For instance, the Simplex Lattice Design provides a remarkably well distribution in convex and symmetric trade-off surfaces, but it shows a poor distribution in non-convex, disconnected Pareto fronts. In order to deal with a scenario in which the shape of the Pareto front is not known in advance, a promising strategy is to reallocate the weight vectors during the search.

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2 STATIC TECHNIQUES FOR WEIGHT VECTOR GENERATION WITHIN MOEAS

There are two main classes of generation methods for static weight vectors. First, the mixture design theory, which has received a refreshed attention since the publication of MOEA/D [5], combines m different chemical components to meet some properties and the sum of their proportions equals 1. Other strategies aim at producing a set of points over a geometrical object that minimizes a discrepancy function, which measures the non-uniformity of the points. This is the case of Uniform Design and low-discrepancy sequences. For an exhaustive review on uniform point set generation strategies, the interested reader is referred to [2, 3].

Computational experiments are carried out with six static generation methods: simplex lattice (SLD), two layer (2LD) and uniform (UD) designs, and three low-discrepancy sequences (Halton, Hammersley and Sobol). These techniques, embedded within MOEA/D-PBI, are tested on the DTLZ7, Kite[4] and $DTLZ1^{-1}$ functions, which present interesting front characteristics. 50 executions are performed for each technique and for 3, 5 and 8 dimensions, using population sizes respectively equal to 300, 500 and 792 individuals. Performances are evaluated with the hypervolume, Inverted Generational Distance and Δ-Diversity indicators. The results shown in table 1 highlight that even though the mixture-based design are the best options for 3 objectives, low-discrepancy sequences prove to be useful for 5 or more objectives.

3 A NEW METHOD FOR DYNAMIC WEIGHT VECTORS TUNING WITHIN MOEA/D

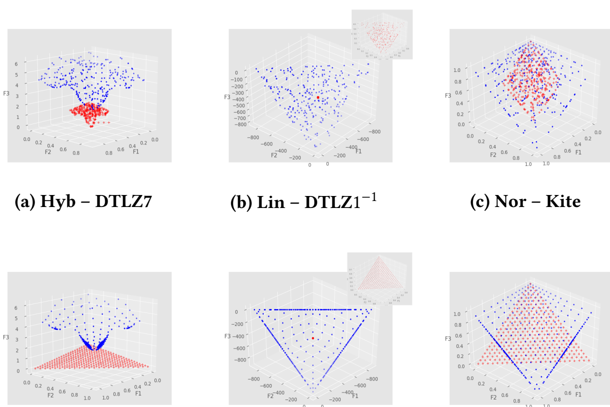
Irregular Pareto front shapes, such as simplex-inverted, disconnected or degenerate fronts, may pose diversity issues for static weight vector generation techniques. An intuitive solution to this problem is to adaptively update the weight vectors during the search. Some interesting attempts have been made along this line (not described here due to space limitations). This work proposes a new strategy embedded within MOEA/D to learn from the search process and modify some weight vectors that are currently useless.

After a normal working stage (convergence phase), an updating step is performed periodically until the end of the run.

Test func.	Ind.	Dim.	SLD	2LD	UD	Halton	Hammersley	Sobol
DTLZ7	HV	3	**990.945 (55.688)**	990.441 (0.037)	986.638 (0.379)	987.842 (0.293)	986.613 (55.027)	986.910 (0.253)
		5	**1.069E+05 (4.850)**	1.061E+05 (5.187)	1.022E+05 (371.819)	1.043E+05 (22.3028)	1.0426E+05 (26.141)	1.034E+05 (24.959)
		8	1.035E+08 (2.468E+06)	7.900E+06 (2.528E+06)	9.990E+07 (2.265E+06)	1.032E+08 (2.466E+06)	**1.071E+08 (1.469E+06)**	1.035E+08 (2.468E+06)
	IGD	3	**0.07394 (0.05019)**	0.09139 (0.00087)	0.09956 (0.00198)	0.15887 (0.00331)	0.15747 (0.03474)	0.11198 (0.00094)
		5	**0.44697 (0.00014)**	0.49108 (0.00109)	0.60650 (0.00234)	0.57903 (0.00147)	0.54387 (0.04625)	0.47385 (0.00168)
		8	5.50966 (0.76683)	**2.06705 (0.03775)**	2.68654 (0.18682)	2.18940 (0.37848)	2.54961(0.15984)	3.31275 (0.09046)
	Δ	3	1.16310 (0.02635)	1.11955 (0.00412)	1.13114 (0.00273)	**1.10188 (0.00362)**	1.13750 (0.00870)	1.14247 (0.00528)
		5	1.13303 (0.00126)	0.99462 (0.00234)	**0.88141 (0.01355)**	0.95872 (0.00482)	0.96102 (0.00682)	0.94166 (0.00273)
		8	0.83376 (0.01582)	**0.77612 (0.00856)**	0.79483 (0.02371)	0.79810 (0.01649)	0.84530 (0.01395)	0.80764 (0.01338)
$DTLZ1^{-1}$	HV	3	**8.748E+07 (5.045E+03)**	8.542E+07 (1.857E+04)	3.014E+07 (1.752E+06)	8.610E+07 (3.194E+04)	8.687E+07 (2.756E+04)	8.500E+07 (2.983E+04)
		5	2.798E+05 (9.857E+04)	3.338E+05 (9.369E+08)	7.477E+10 (5.701E+10)	**17.068E+10 (2.915E+10)**	9.702E+10 (3.245E+10)	3.946E+10 (2.615E+10)
		8	1.384E+10 (1.109E+12)	1.634E+10 (3.123E+12)	1.581E+12 (3.77E+12)	5.706E+11 (1.878E+12)	**2.263E+12 (1.617E+14)**	4.177E+11 (3.97E+12)
	IGD	3	**434.787 (0.003)**	434.900 (0.002)	632.648 (15.161)	443.628 (0.733)	440.675 (0.772)	437.010 (0.287)
		5	**266.636 (0.001)**	329.946 (9.161)	422.731 (31.141)	317.40 (11.325)	314.153 (8.284)	363.138 (17.885)
		8	443.060 (52.621)	359.771 (105.137)	293.426 (47.861)	272.354 (37.366)	**245.807 (27.484)**	256.662 (28.378)
	Δ	3	1.02225 (0.00044)	0.80503 (0.00113)	**0.48451(0.02258)**	0.95142 (0.00290)	0.97185 (0.00250)	0.94629 (0.00297)
		5	1.53902 (0.00128)	1.26958 (0.02335)	0.56430 (0.06038)	**0.42409 (0.01466)**	0.45392 (0.00960)	0.57704 (0.02842)
		8	1.83157 (0.02632)	1.83165 (0.22269)	0.31331 (0.12027)	0.27656 (0.05288)	0.27697 (0.09214)	**0.27210 (0.08816)**
Kite	HV	3	**0.72878 (0.00000)**	0.72504 (0.00000)	0.10624 (0.03259)	0.22778 (0.01447)	0.22778 (0.01447)	0.10624 (0.03259)
		5	0.42707 (0.02603)	0.47951 (0.03029)	0.44070 (0.01066)	0.48874 (0.03209)	0.48878 (0.03146)	**0.48884 (0.02834)**
		8	**0.573399 (0.00001)**	0.573397 (0.00002)	0.573397 (0.00010)	0.573396 (0.00010)	0.573393 (0.00008)	0.573395 (0.00008)
	IGD	3	**0.03519 (0.00000)**	0.04127 (0.00001)	0.54850 (0.03311)	0.39966 (0.03375)	0.39966 (0.03375)	0.34145 (0.02170)
		5	0.22498 (0.05153)	0.18105 (0.04357)	0.27918 (0.02069)	0.17378 (0.06022)	**0.17106 (0.05914)**	0.17425 (0.05245)
		8	1.504657 (0.08738)	**1.504650 (0.08686)**	1.504653 (0.12731)	1.504656 (0.10251)	1.504654 (0.10279)	1.504659 (0.10980)
	Δ	3	0.82997 (0.00007)	**0.34145 (0.02170)**	0.39773 (0.01823)	0.45036 (0.01963)	0.39773 (0.01823)	0.52810 (0.00187)
		5	0.94935 (0.03379)	0.81522 (0.04836)	0.79679 (0.04210)	0.58869 (0.10628)	0.57093 (0.11085)	**0.56877 (0.10286)**
		8	**1.07546 (0.02374)**	1.07547 (0.02818)	11.07546 (0.02407)	1.07548 (0.01684)	1.07549 (0.01739)	1.07552 (0.02153)

Each cell provides the indicator median value (std. deviation).

Table 1: Performance results of the static weight vector generation techniques.



(a) Hyb – DTLZ7 **(b) Lin – $DTLZ1^{-1}$** **(c) Nor – Kite**

(d) SLD – DTLZ7 **(e) SLD – $DTLZ1^{-1}$** **(f) SLD – Kite**

Figure 1: 3D results for dynamic and SLD techniques

Each non-dominated solution is marked as a Tabu point and corresponding weight vectors are re-adjusted over the unitary simplex according to the solution positions. Then, the other weight vectors are recomputed using a sub-population repulsion concept borrowed from multimodal optimization (see [1]). For those solutions close enough (in terms of Mahalanobis distance to avoid the dimensional effect) to any Tabu point, a vector candidate is produced through the combination of two Tabu vectors, subsequently projected on the unitary simplex. Three combination techniques are implemented here: linear (Lin), normal multivariate (Norm) and hybrid (Hyb).

A second set of experiments (with the same parameter settings as before) is carried out to compare the three dynamic methods with their static counterpart, using SLD as the initial weight generator. Figure 1 shows the approximated fronts (blue) and weight vectors (red) obtained with SLD and the best dynamic technique. In all cases, the dynamic techniques transform the weight distribution shape, which finally coincides with the approximate location of the real Pareto front. With static SLD, most solutions lie on the front boundaries, since many points on the simplex surface are oriented towards "blank regions" (without Pareto optimal solutions). Although not provided here, performance indicators confirm the visual observation: dynamic techniques almost always outperform the static one (yet, in some cases, IGD turns out to be better for SLD). Therefore, the weight adaptation strategy allows generating a dense distribution of solutions inside the front boundaries, with some drawbacks though: boundaries and the extreme points may be sparsely described , while the point distribution inside the front shape is dense but no so uniform (explaining the IGD deficiencies).

4 CONCLUSIONS

This study allowed identifying some trends for weight vector generation techniques, when facing MOPs with complex Pareto shapes. Using static methods, mixture based designs work well for low dimensions but low-discrepancy sequences improve their performance with the number of objectives. Further, a novel on-line weight adaptation strategy is introduced, which obtains promising results and provides some guideline for future improvements.

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