The Quadratic Assignment Problem: Metaheuristic Optimization Using HC12 Algorithm

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ABSTRACT

The Quadratic Assignment Problem (QAP) is a classical NP-hard combinatorial optimization problem. In the paper will be presented suitable metaheuristic algorithm HC12. The algorithm is population based and uses a massive parallel search of the binary space which represents the solution space of the QAP. The presented implementation of the metaheuristic HC12 utilizes the latest GPU CUDA platform. The results are presented on standard test problems from the QAP library.*

CCS CONCEPTS

• **Design and analysis of algorithms** → Approximation algorithms analysis, Parallel algorithms • **Mathematical optimization** → Optimization with randomized heuristics

KEYWORDS

Quadratic assignment problem, Massively parallel algorithm

ACM Reference format:

R. Matousek, L. Dobrovsky, and J. Kudela. 2019. The Quadratic Assignment Problem: Metaheuristic Optimization Using HC12 Algorithm. In *Companion Proceedings of ACM GECCO conference, Prague, Czech Republic, July 2019 (GECCO'19),* 2 pages.
DOI: 10.1145/nnnnnnn.nnnnnnn

1 INTRODUCTION

The NP-hard quadratic assignment problem (QAP), in its Koopmans and Beckmann form [1], can be described as follows: The problem is structured on a complete directed graph with n nodes and n^2 arcs, together with a set of n facilities, that have to be assigned to the nodes. The indices i, j correspond to the nodes, the indices f, g correspond to the facilities, $b_{i,j} \geq 0$ is a given (directed) distance from node i to node j, $a_{f,g} \geq 0$ is a flow from facility f to facility g, and $c_{i,j}$ is a cost of assigning facility f to node i. By using binary variables $x_{i,f} = 1$ if facility f is assigned to node i, and 0 otherwise, the QAP can be stated as the following 0-1 optimization problem:

$$min \quad \sum_i \sum_f \sum_j \sum_g a_{f,g}\, b_{i,j} x_{i,f} x_{j,g} + \sum_i \sum_f c_{i,f}\, x_{i,f} \qquad (1)$$

$$s.t. \quad \sum_i x_{i,f} = 1, \ \sum_f x_{i,f} = 1, \qquad \forall f, \ \forall i \qquad (2)$$

$$x_{i,f} \in \{0,1\}, \qquad \forall i, \forall f. \qquad (3)$$

Several directions for enriching the QAP formulation have been proposed – among the most notable of these are the multi-objective formulation [2] and stochastic formulation [3].

2 ALGORITHM HC12

The binary HC12 algorithm [4] is a stochastic heuristic searching algorithm which belongs to the class of pseudo global search methods. The basic step of the algorithm is a generation of a neighborhood of the current solution, which serves as a base for the new population. The method of generating the neighborhood is the pivotal characteristic of HC12. The paradigm of the algorithm is the search of the optimal solution in the binary (Hamming) space, that encodes the solution. In this context, it is a parallel approach to genetic algorithms, where the solution is encoded as a binary vector. The best individual of the $i-$th generation (or iteration) is chosen as the base for the following $(i+1)$ generation. The approach is depicted in Fig. 1.

3 RESULTS AND DISCUSSION

The HC12 algorithm is extremely suitable for parallel implementation. In the presented experiments, it was implemented for HPC computations on NVIDIA RTX 2080 (8GB). Even the larger memory requirements of the QAP problems, not more than 6GB were used. The implementation searches for the best solution in multiple runs (restarts of the algorithm). The effectivity of the algorithm (in regard to the number of found optimal solutions) can be determined as a success rate (the ration of runs that ended in an optimal solution).

GECCO'19, July 13-17, 2019, Prague, Czech Republic

ACM ISBN 978-1-4503-6748-6/19/07…$15.00
https://doi.org/10.1145/3319619.3322088

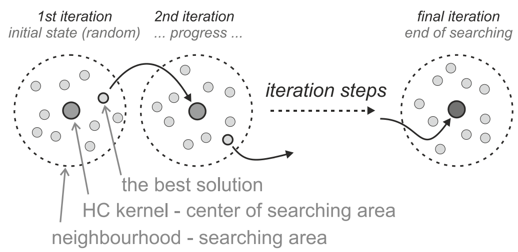

Figure 1: The scheme of HC12 iterations.

A rather interesting insight is provided by the dependence of the number of used swaps on the number of found optimal solutions. More swaps also result in a higher computation time. There appear to be "optimal" number of swaps for the given problem (the number of swaps that results in the most successful runs).

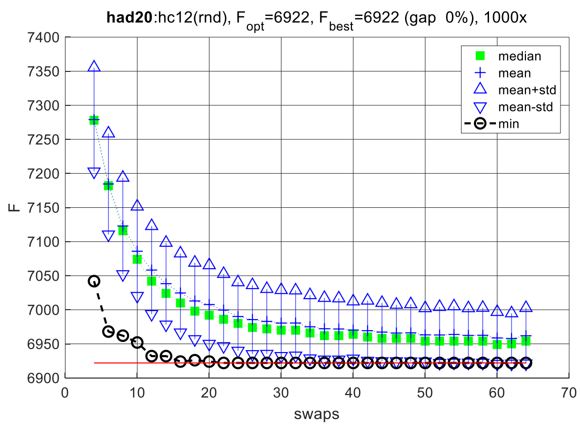

Figure 2: An influence of swap operator to convergence features of "had20" test problem.

The computational comparison of HC12 with other state-of-the-art metaheuristics is done on the standard test problems from the QAPLIB library [5].

The selected metaheuristics are the hybrid teaching-learning optimization implemented on a cluster [6], the parallel implementation of hybrid algorithms [7,8], and the bee algorithm implemented on a CUDA platform [9]. The results of the computation and the comparison are reported in Table 1.

Although the running times of the HC12 algorithm are extremely fast (compare to the other heuristics), the robustness of the resulting solutions is still rather low and requires additional research and tuning.

ACKNOWLEDGMENTS

This work was also supported by The Ministry of Education, Youth and Sports of the Czech Republic, INTER-COST project LTC18053 and European COST Action CA15140 and by IGA BUT: No. FSI-S-17-4785.

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Table 1: Comparison with other results

problem instance	optimal solution	HC12 (GPU implementation)			[6]		[7]		[8]		[9]
		swaps	success*	time [s]	APD	time	APD	time	APD	time	APD
esc16a	68	62	1	0.0014	0	6	0	151.8	0	702	0
esc32a	130	52	0.001	5.9462	0	72	-	-	-	-	10.77
had16	3720	64	0.348	0.0288	0	6	0	149.4	0	594	0
had18	5358	64	0.123	0.1476	0	12	0	183.6	0	618	-
had20	6922	62	0.067	0.3072	0	18	0	223.8	0	600	-
rou12	235528	60	0.116	0.0338	0	6	0	87.6	0	90	-
rou15	354210	50	0.02	0.2378	0	6	0	133.8	0	600	-
rou20	725522	44	0.001	8.4109	0	18	-	-	-	-	0.25

* success: The effectivity of the algorithm (in regard to the number of found optimal solutions) is determined as ratio of number of optimal solutions to number of algorithms runs.