Ensemble-Based Constraint Handling in Multiobjective Optimization

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ABSTRACT

Many real-world optimization problems involve both multiple objectives and constraints. Although constraint handling in multiobjective optimization has been considered in the literature, there is still a high demand for more advanced and versatile constraint handling techniques (CHTs) in real-world applications. For this reason, we propose a general approach to combine multiple CHTs into an ensemble-based method, providing a framework to easily construct new CHTs from existing ones. The approach is evaluated on nine test problems from the literature using an ensemble of four widely-used CHTs. The experimental results show that the ensemble is more robust than single CHTs and performs at least as well as the best single CHTs on all the test problems. Moreover, a positive synergistic effect of the ensemble is demonstrated on three problems.

CCS CONCEPTS

• **Applied computing → Multi-criterion optimization and decision-making**; • **Theory of computation → Bio-inspired optimization**;

KEYWORDS

constrained multiobjective optimization, multiobjective evolutionary algorithms, constraint handling techniques, ensemble-based methods, NSGA-II

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1 INTRODUCTION

Real-world optimization problems regularly involve both multiple objectives and constraints. Such problems are called *constrained multiobjective optimization problems* (CMOPs) and are known for being hard to solve. A CMOP can be formulated as

$$\begin{aligned} \text{minimize} \quad & f_m(x), \quad m = 1, \ldots, M \\ \text{subject to} \quad & g_n(x) \leq 0, \quad n = 1, \ldots, N \end{aligned}$$

where $x = (x_1, \ldots, x_D)^{\mathrm{T}}$ is a *decision vector*, $f_m : S \to \mathbb{R}$ are *objective functions*, $g_n : S \to \mathbb{R}$ *constraint functions*, $S \subseteq \mathbb{R}^D$ is a *decision space* of dimension D, and M and N are the numbers of objectives and constraints, respectively. Additionally, $f_m(x)$ is an *objective value* and $\max(g_n(x), 0)$ *constraint violation*.

Among the most widely used approaches to deal with CMOPs are undoubtedly multiobjective evolutionary algorithms (MOEAs) equipped with constraint handling techniques (CHTs), since they are capable of finding a good set of *feasible individuals*—solutions to the given problem—in a single run. However, until recently the development of CHTs for multiobjective optimization problems has received little attention in the evolutionary computation community. According to [9], the main reason for this is that it is generally believed that CHTs developed for single-objective optimization can be easily adapted for multiobjective optimization.

Nevertheless, a comprehensive study on incorporating single-objective CHTs into multiobjective optimization has only recently been presented in [3]. In that study, a generic framework applicable to almost all state-of-the-art MOEAs and all fitness-based and/or rank-based CHTs was proposed. The idea was to use a MOEA-specific approach to assign to each individual a fitness and/or rank, and then consider the given multiobjective problem as a single-objective one. Beside supporting comparative studies, this framework also makes it easy to combine several CHTs into a hybrid CHT, ensemble-based CHT, etc. Similar approaches have already proved effective in single-objective optimization [7] but have not been widley studied on CMOPs.

In this work, we propose a general approach for combining multiple CHTs into an ensemble-based CHT and utilize the proposed framework to incorporate the ensemble into a custom MOEA based on the Nondominated Sorting Genetic Algorithm II (NSGA-II) [1]. In addition, we identify nine relatively difficult CMOPs from the literature and use them to assess the performance of an ensemble of four CHTs.

The rest of this paper is organized as follows. Section 2 introduces the ensemble of CHTs and presents the general framework for incorporating CHTs into MOEAs. Section 3 is dedicated to the experimental setup, while the results are discussed in Section 4. Finally, Section 5 summarizes the study and provides some ideas for future work.

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2 METHODOLOGY

This section presents the methodology used in our study. After introducing the general framework for the ensemble of CHTs, we discuss the inclusion of the ensemble into an MOEA based on NSGA-II.

2.1 Ensemble-Based CHT

We consider only CHTs which are applied in the replacement phase, i.e., survivor selection, of an evolutionary algorithm. For any finite population of individuals, $\mathbb{P} \subseteq \mathbb{R}^D$, a CHT is supposed to provide some sort of a quality measure combining individuals' objective values and constraint violations. For this reason, we define a general CHT by a set of maps, $c : \mathbb{P} \rightarrow [0, 1]$, which assign to each individual x, from a given population $\mathbb{P}$, a quality measure taking values from $[0, 1]$. Here, the value of the quality measure is inversely proportional to the individual quality. In this formulation, the quality measures are normalized to allow for comparison of individuals' quality among various CHTs.

At this point, we can define the ensemble of CHTs as follows. Given a set of CHTs, $\{c_i : \mathbb{P} \rightarrow [0, 1]\}_{i=1}^{L}$, the *ensemble of constraint handling techniques*, $c_e : \mathbb{P} \rightarrow [0, 1]$, is defined as

$$c_e(x) = \sum_{i=1}^{L} w_i c_i(x)$$

where $\sum_i w_i = 1$ and $w_i \geq 0$. The weights, w_i, determine the importance of each CHT in the ensemble. For example, large weights give a high impact to the corresponding CHT during the ensemble-based quality measure assignment. On the other hand, for a weight close to zero the ensemble almost ignores the corresponding CHT.

Since the ensemble is a CHT itself, it can be included in almost any MOEA following the framework discussed in Section 1. A modified version of NSGA-II capable of incorporating ensembles of CHTs is presented in the following subsection.

2.2 Modified NSGA-II

The pseudocode of the modified NSGA-II is shown in Algorithm 1. Until the replacement phase (Lines 11 to 16), the modified version is identical to the original version of NSGA-II [1]. Specifically, binary tournament selection is used for parent selection (Line 6), simulated binary crossover for recombination (Line 7), and polynomial mutation as a mutation procedure (Line 8).

The replacement phase is where the modified algorithm differs from the original. First, the Pareto rank and crowding distance are assigned to each individual using nondominated sorting and crowding-distance assignment (Line 12). Second, for each individual a fitness is calculated as the individual's Pareto rank minus its normalized crowding distance (Line 13). Here, the crowding distance is subtracted in order to consider individuals with larger crowding distance to be fitter than those with smaller crowding distance. In addition, the crowding distance values are normalized to preserve the Pareto ranking of individuals. Next, ranks are assigned to all individuals by sorting them according to their fitness (Line 14). Finally, any fitness-based or rank-based CHT can be used to select among the best individuals (Lines 15 and 16).

However, in order to include a CHT into an ensemble, it has to provide a normalized quality measure for each individual. This is achieved using the CHT-specific approach, usually, by normalizing the fitness and/or ranks generated by the CHT.

Algorithm 1 Modifed version of NSGA-II

Input: population size, CHT, stopping criterion;
Output: population $\mathbb{P}$ of feasible individuals;
1: create the initial population $\mathbb{P}$ of random individuals;
2: evaluate $x \in \mathbb{P}$;
3: **while** stopping criterion not met **do**
4: $\quad \mathbb{P}_{\text{new}} \leftarrow \emptyset$;
5: $\quad$ **for** $i \in 1 : |\mathbb{P}|/2$ **do**
6: $\qquad$ select two parents $x_1, x_2 \in \mathbb{P}$;
7: $\qquad$ performer crossover on x_1, x_2;
8: $\qquad$ mutate x_1, x_2;
9: $\qquad \mathbb{P}_{\text{new}} \leftarrow \mathbb{P}_{\text{new}} \cup \{x_1, x_2\}$;
10: $\quad$ **end for**
11: $\quad \mathbb{P}_{\text{new}} \leftarrow \mathbb{P}_{\text{new}} \cup \mathbb{P}$;
12: $\quad$ assign Pareto rank and crowding distance to $x \in \mathbb{P}_{\text{new}}$;
13: $\quad$ assign fitness to $x \in \mathbb{P}_{\text{new}}$;
14: $\quad$ assign rank to $x \in \mathbb{P}_{\text{new}}$;
15: $\quad$ evaluate $x \in \mathbb{P}_{\text{new}}$ according to the selected CHT;
16: $\quad \mathbb{P} \leftarrow |\mathbb{P}|$ best individuals from $\mathbb{P}_{\text{new}}$;
17: **end while**
18: **return** $\mathbb{P}$;

3 EXPERIMENTAL SETUP

In this section we present the experimental setup of our study. After introducing the CHTs chosen for the experiments, we briefly discuss the test problems and describe the parameter settings.

3.1 Constraint Handling Techniques

In the experiments we used four single CHTs and the ensemble combining these CHTs:

- Nondominated sorting (NDS) [1]: This method selects the new generation of individuals according to the dominance relation not considering constraint violations at all.
- Constrained-domination principle (CDP) [1]: This CHT can be seen as an extension of NDS, where feasible individuals dominate infeasible ones, and infeasible individuals are ranked according to the overall constraint violation.
- Multiple constraint ranking (MCR) [4]: In this approach the individuals are ranked based on the fitness and constraint violations. If there are no feasible individuals, only the rank generated from constraint violations is considered, otherwise a combination of both ranks is taken into account.
- Dynamic penalty function (DPF) [2]: This method augments the fitness of an individual by the addition of a penalty that is proportional to the overall constraint violation. The penalty pressure is increased in each generation.
- Ensemble of CHTs (ENS): Ensemble of NDS, CDP, MCR, DPF as proposed in Section 2.1 with uniform weights, $w_i = 1/4$.

In addition, to provide quality measures used by the ensemble, each CHT was adequately modified using the CHT-specific approach. For MCR and DPF the corresponding ranks and fitness values were normalized only. In contrast, for NDS and CDP we used a similar approach as described in Section 2.2, i.e., assigning fitness from Pareto ranks and crowding distances. The only difference was that the constrained-domination principle was used in the Pareto ranking for CDP instead of the dominance relation.

3.2 Test Problems

From the test CMOPs found in the literature, we chose those that are believed to be difficult to solve or were proposed in [5, 13] as appropriate test problems. The presented ensemble-based CHT was experimentally evaluated and compared with single CHTs on six artificial CMOPs: SRN [12], OSY [11], C3-DTLZ1 [6] and C3-DTLZ4 [6], where the three-objective and four-objective versions of the last two problems were used. In addition, three real-world problems (RWPs) frequently used to evaluate the performance of MOEAs were considered in the experiments as well: the water resource planning problem [10], the car-side impact problem [6] and the vibrating platform problem [8]. Some basic characteristics of the test problems are shown in Table 1.

3.3 Parameter Settings

The experiments were conducted using the algorithm parameter settings that were found suitable in preliminary runs. All the algorithms were run with populations of 200 individuals for 500 generations. Specifically, for MOEA the crossover probability was set to 0.9 and the mutation probability to $1/D$. On each test problem every algorithm was run 30 times.

Table 1: Characteristics of the test CMOPs: number of objectives M, number of constraints N, dimension of the decision space D, feasibility ratio of the decision space experimentally estimated as the proportion of feasible individuals in 10^5 randomly generated individuals.

CMOP	M	N	D	Feasibility ratio
SRN	2	2	2	≈0.16
OSY	2	6	6	≈0.03
C3-DTLZ1	3	3	7	≈1.00
C3-DTLZ4	3	3	7	≈0.01
C3-DTLZ1	4	4	8	≈1.00
C3-DTLZ4	4	4	8	≈0.01
Vibrating platform	2	5	5	≈0.00
Car-side impact	3	10	7	≈0.18
Water resource planning	5	7	3	≈0.92

4 RESULTS

Cumulative hypervolume of Pareto front approximations was used to measure the quality of an algorithm run. In order to present the entire search process, hypervolumes were calculated for each generation on an unbounded external archive storing all the nondominated feasible individuals generated until that generation. Before the hypervolume calculation, the objective values were normalized using the nadir and ideal points. For the artificial problems the nadir and ideal points were known in advance, while for the three RWPs they were estimated using all nondominated and feasible individuals generated by all algorithms during all 30 runs. Given $f_i \in [0, 1]$, reference points for hypervolume calculation were set to $(1.1, \ldots, 1.1)^{\mathrm{T}}$. Note that only feasible individuals dominating the reference point were used to calculate the hypervolume.

The progress of cumulative hypervolume averaged over 30 runs of every algorithm on each test problem is shown in Figure 1. As we can see, the ensemble of CHTs performs at least as well as the best single CHT on all the test problems. Indeed, a deeper inspection of the results shows that the ensemble is much more robust than any other CHT. From this observation we may conclude that the ensemble is able to adopt to the best performing single CHTs on all the observed problems. Moreover, the better ensemble performance on the three RWPs clearly shows that the ensemble is able to extract the information gathered from multiple CHTs and combine them in a synergistic way. Especially on the vibrating platform problem the combination of CHTs played a pivotal role for the MOEA to find a high-quality Pareto front approximation.

After examining the obtained cumulative Pareto front approximations (not shown in the paper due to the lack of space), we observed that the test problems were actually not so hard to solve. For example, all CHTs except NDS were capable of finding good Pareto front approximations for SRN, OSY and C3-DTLZ1. In fact, the vibrating platform was the only problem identified as difficult according to our results.

5 CONCLUSIONS

In this paper, we proposed a general approach to include multiple CHTs into an ensemble-based method for constraint handling in multiobjective optimization. In addition, we presented a modified version of NSGA-II capable of including the ensemble during the replacement phase. To assess the performance of our approach, we experimented with four CHTs and the ensemble of these CHTs on nine test CMOPs from the literature.

The experimental results show that the ensemble of CHTs is far more robust than other CHTs and solves all test problems at least as well as the best performing single CHT. Furthermore, on all three RWPs the ensemble outperformed the single CHTs. Especially on the vibrating platform, which is the hardest test problem according to our results, the synergy of multiple CHTs was crucial for finding a good Pareto front approximation. Nevertheless, we observed that, contrary to general belief, CMOPs regularly used to evaluate the performance of MOEAs can be quite easily solved.

In the future we plan to investigate whether the presented results are statistically significant. We will also analyze the robustness of the proposed ensemble by considering different parameter settings, including non-uniform weights of CHTs in the ensemble, and incorporating additional CHTs. Finally, we will try to identify more difficult test problems or construct new CMOP benchmarks.

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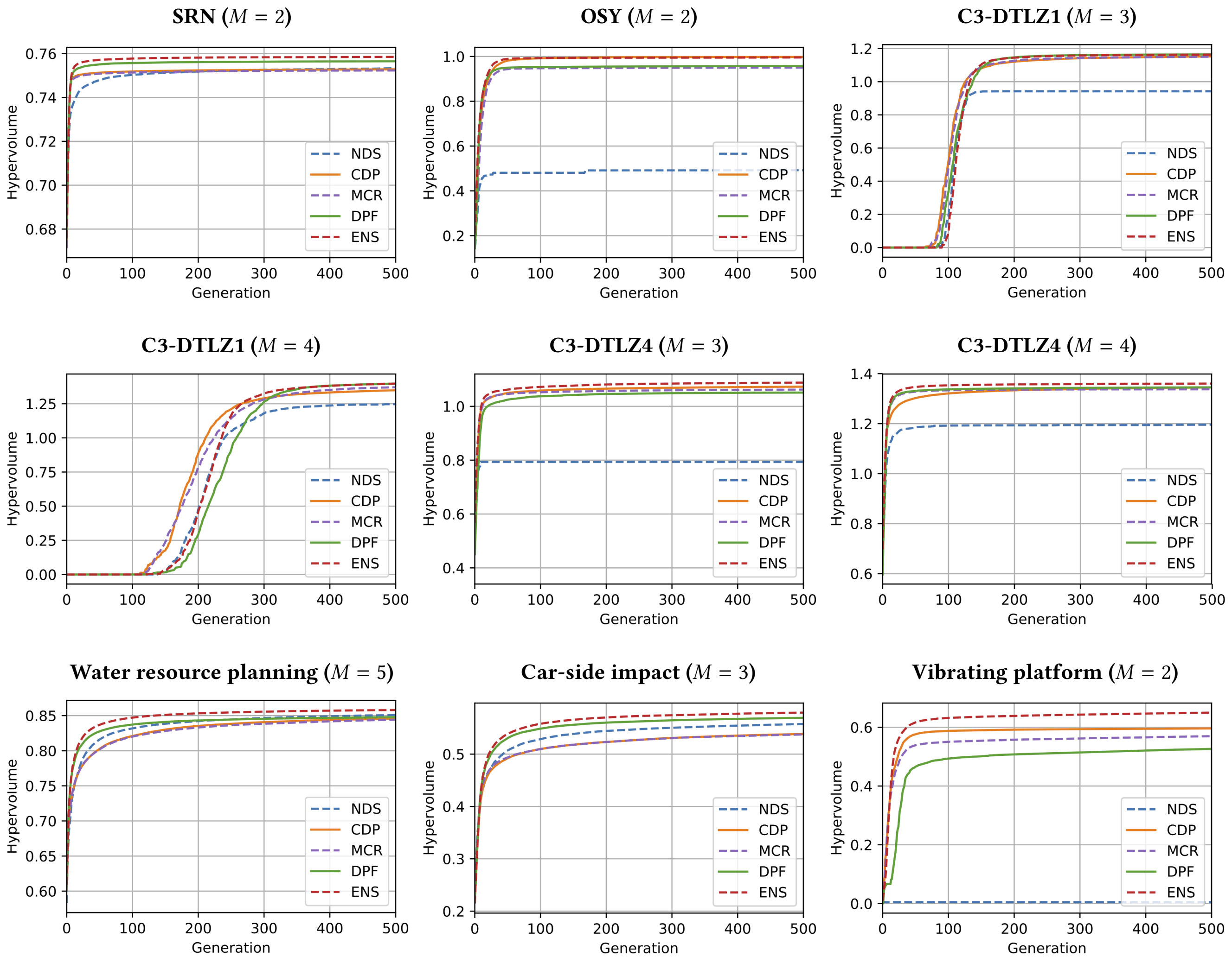

Figure 1: Cumulative hypervolume progresses for all the test problems, where the blue dashed line indicates NDS, the orange solid line CDP, the purple dashed line MCR, the green solid line DPF, and the red dashed line ENS.

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