Recent Advances in Fitness Landscape Analysis

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http://gecco-2019.sigevo.org/

GECCO '19 Companion, July 13–17, 2019, Prague, Czech Republic

ACM ISBN 978-1-4503-6748-6/19/07.
https://doi.org/10.1145/3319619.3323383

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Instructors

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Outline

- Fundamental Concepts of Fitness Landscapes
  - Motivation for analysing fitness landscapes
  - Basics of fitness landscapes
  - Features of fitness & violation landscapes
- Local Optima Networks (LONs)
  - Definition of Nodes & Edges
  - Detecting Funnels
  - Visualisation & Metrics
- Case Studies
  - LONs applied to feature selection for classification
  - Landscape-aware algorithm selection for constraint-handling
- Closing

Demo
- Sampling
- Constructing LONs
- Visualisation
- Metrics

Download Materials
lonmaps.com

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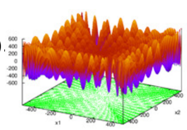
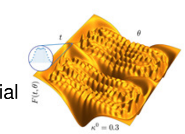
Complex Optimisation

- When classical techniques are not feasible & metaheuristics are needed:
  - Problem complexity is too large (not of the required structure for classical techniques, too many variables).
  - When there is no objective function in mathematical form.
  - Objective function exhibits noise or uncertainty.



- "Massive optimisation"
  - Large scale optimisation (many dimensions)
  - Any-objective optimisation (single-, multi- many-objective)
  - Cross-domain optimisation (continuous / combinatorial / mixed)
  - Expensive optimisation (costly / simulation-based black-box evaluations)



- Many many metaheuristic approaches…

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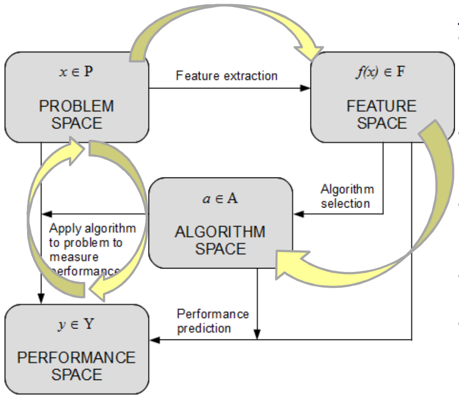
General Algorithm Selection Problem (Rice, 1976)



Trial-and-error approach to finding the best algorithm

Research areas:

- Feature extraction (characterising problems)
- Analysis and design of problems
- Understanding algorithm behaviour
- Performance prediction
- Algorithm selection

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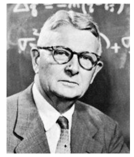
Wright's fitness landscape

- ❖ Surface of selective values (Wright, 1932).
- ❖ No axes, units or labels.
- ❖ Commentary 56 years later:
  - *"useless for mathematical purposes"*
  - Aim: provide an intuitive picture of evolutionary processes taking place in higher dimensional space.



1889 - 1988

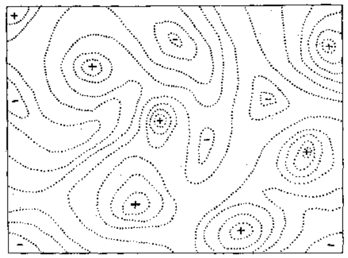


FIGURE 2—Diagrammatic representation of the field of gene combinations in two dimensions instead of many thousands. Dotted lines represent contours with respect to adaptiveness.

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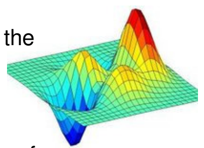
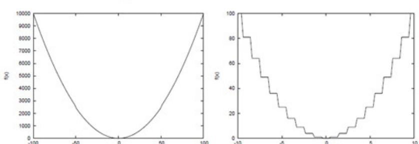
Fitness landscapes today

- ❖ We now have (useful) formalised mathematical models.
- ❖ Essential elements: search space, fitness function, notion of neighbourhood or accessibility.
- ❖ Intuitively, a fitness landscape is a visualisation of the terrain capturing how fitness changes between neighbouring solutions.
- ❖ Idea of "valleys", "peaks", "ridges", "plateaus", etc.
- ❖ One fitness function, many fitness landscapes (even for real-valued spaces).
- ❖ Example: Step benchmark function (same function, two different landscapes).



$$f(\mathbf{x}) = \sum_{i=1}^{D} (\lfloor x_i + 0.5 \rfloor)^2$$



(a) Larger domain (b) Smaller domain

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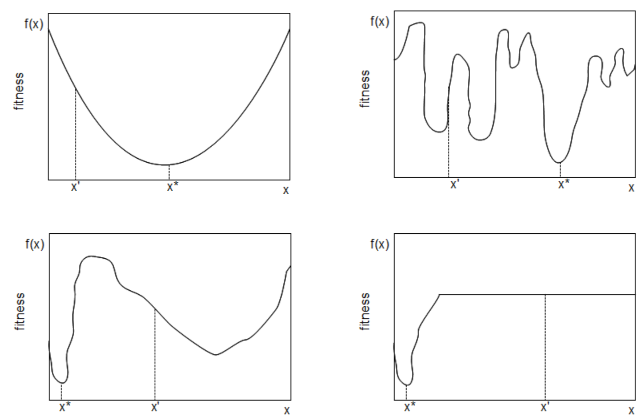
Features of fitness landscapes (1)



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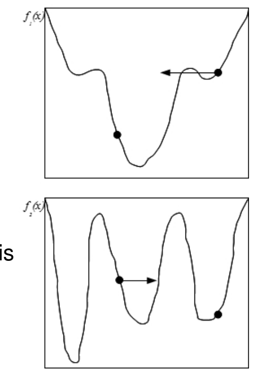
Features of fitness landscapes (2)

- Modality (number of optima) is frequently referred to as affecting difficulty, but too simplistic.
- Example landscapes both with three optima.
- Top landscape: global basin is wider and deeper than local basins.
- Bottom landscape: global basin narrow and local basins deep.
- Consider simple PSO with 2 particles: top landscape not deceptive, bottom landscape is deceptive.
- Distribution & relative sizes of basins of attraction more important than modality.
- Funnels vs ruggedness.



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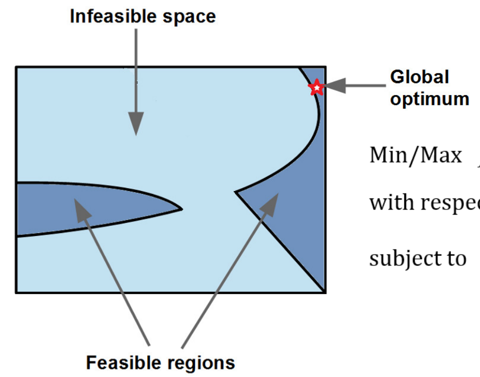
Constrained optimisation spaces



Min/Max $f(x_1, x_2, \dots, x_n)$

with respect to $x_1, x_2, \dots, x_n$

subject to $g_k(x_1, x_2, \dots, x_n) \begin{Bmatrix} \le \\ = \end{Bmatrix} b_k$

$k = 1, \dots, m$

- How do constraints impact on the landscape?
- Can view the landscape i.t.o fitness or level of violation.

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Fitness and Violation Landscapes

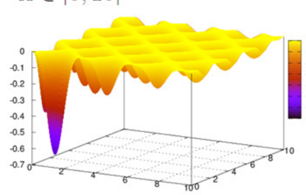
- CEC 2010 Benchmark suite, problem C01:

$$\text{Min } f(\mathbf{x}) = -\left|\frac{\sum_{i=1}^{D}\cos^4(z_i) - 2\prod_{i=1}^{D}\cos^2(z_i)}{\sqrt{\sum_{i=1}^{D} i z_i^2}}\right|, \ \mathbf{z} = \mathbf{x} - \mathbf{o}$$

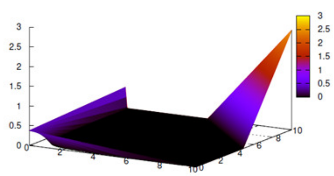
$$g_1(\mathbf{x}) = 0.75 - \prod_{i=1}^{D} z_i \le 0$$

$$g_2(\mathbf{x}) = \sum_{i=1}^{D} z_i - 7.5D \le 0$$

$$\mathbf{x} \in [0, 10]^D$$



Fitness landscape



Violation landscape

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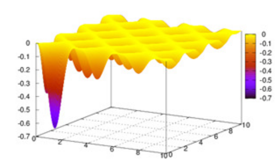
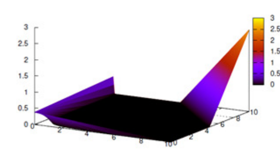
Approaches to constraint handling

- Metaheuristics do not naturally handle constraints, so a constraint-handling technique has to be added on.
- Many different approaches to handling constraints:
  - Avoid infeasible regions (repair / prevention).
  - Use penalties: adapt fitness function to guide search away from infeasible regions.
  - Feasibility ranking: rules of preference using objectives and constraints.
  - Multi-objective optimisation (constraints treated as objective to be minimised).
- What landscape does each approach explore?





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Which constraint handling technique is better?

- Six approaches: no constraint handling, death penalty, weighted penalty, feasibility ranking, epsilon feasibility ranking, bi-objective.
- Above constraint handling techniques used with evolutionary algorithms performed <u>both the worst and the best</u> on a large set of problems: 142 problem instances (70 combinatorial, 72 continuous).

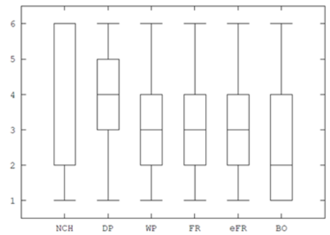
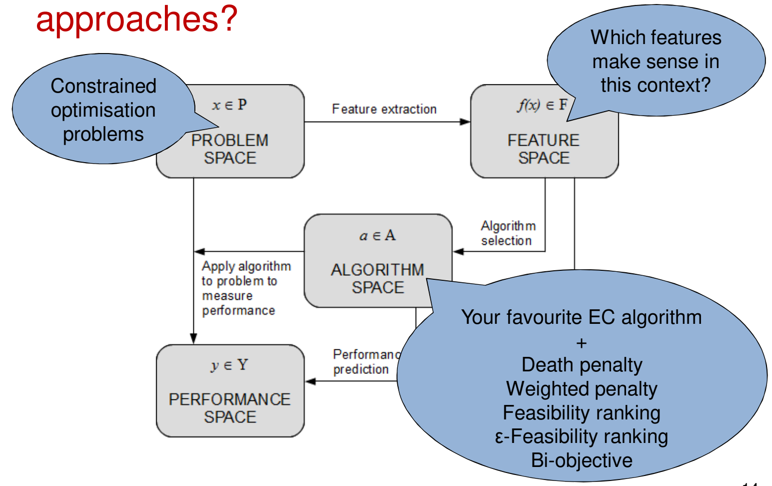


Figure 2: Distribution of algorithm ranks over all problem instances

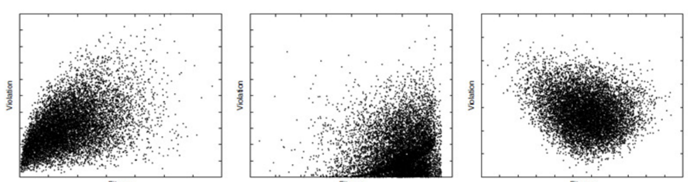
- K.M. Malan and I. Moser. Constraint handling guided by landscape analysis in combinatorial and continuous search spaces. *Evolutionary Computation*, 27(2), 2019.

Algorithm selection for constraint handling approaches?



Features of constrained landscapes

- What proportion of the space is feasible?
- How disjoint are the feasible areas?
- How correlated are the fitness and violation landscapes? Do they “pull” in the same direction?
- What proportion of the solutions are both feasible and fit?



(d) C03 problem in 10D (e) C04 problem in 10D (f) C13 problem in 10D

Sampling to characterise features

Image: pixabay

- Some features require just a random sample, without neighbourhood (e.g. ratio of feasible solutions).
- To characterise how disjoint the feasible areas are, we use a “walk” through the search space.
- Can be random or directed in some way, depending on the purpose of the study.
- Multiple hill climbing walks:
  - More representative of the search process than random walks.
  - Ensures that the sample contains good quality solutions (especially important when some problems have an over-representation of poor solutions).
- Initial solutions must be distributed across the space:
  - Use Latin-hypercube sampling in continuous spaces to prevent over-sampling of the centre of the search space.

Metrics for constrained landscapes

- Feasibility ratio (FsR) estimates the size of the feasible space in relation to the overall search space.
- Given a sample of n solutions, if there are n_f feasible solutions, then

$$\mathrm{FsR} = \frac{n_f}{n}$$

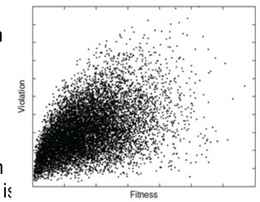
- Ratio of feasible boundary crossings (RFBx) quantifies how disjoint the feasible areas are.
- Given a sequence of n solutions, $x_1, x_2, \ldots, x_n$ obtained by a walk through the search space, a binary string $\boldsymbol{b} = b_1, b_2, \ldots, b_n$ is defined such that $b_i = 0$ if x_i is feasible and $b_i = 1$ if x_i is infeasible, then

$$\mathrm{RFB\times} = \frac{\sum_{i=1}^{n-1} cross(i)}{n-1} \quad \text{where} \quad cross(i) = \begin{cases} 0 \text{ if } b_i = b_{i+1} \\ 1 \text{ otherwise.} \end{cases}$$

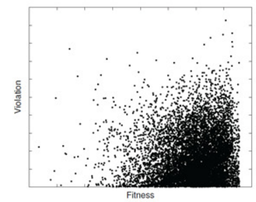
K.M. Malan, J.F. Oberholzer, and A.P. Engelbrecht, A.P. Characterising Constrained Continuous Optimisation Problems. IEEE CEC 2015.

Metrics for constrained landscapes (2)

- Fitness violation correlation (FVC):
  - Based on a sample of solutions resulting in fitness-violation pairs, the FVC is the Spearman's rank correlation coefficient between the fitness and violation values.



- Ideal zone (IZ) metrics:
  - Based on the scatterplot of fitness-violation pairs of a sample of solutions: "ideal zone" is the bottom left corner for a minimisation problem.
  - A small proportion of solutions in the ideal zone could indicate narrow basins of attraction in a penalised landscape.
  - 25_IZ: proportion of points below the 50% percentile for both fitness and violation.
  - 4_IZ: proportion of points below the 20% percentile for both fitness and violation.



Can features predict success / failure?

- 142 problem instances (half combinatorial, half continuous):
  - Samples generated using multiple hill climbing walks from random starting positions.
  - Size: 1% of computational budget used for solving.
  - Samples used as the basis for five landscape metrics.
  - Spearman's correlation coefficient between metrics and success/failure.

	FsR	RFBx	FVC	4_IZ	25_IZ
NCH	0.13	0.26	0.40	0.26	0.23
DP	0.41	0.40	-0.12	-0.33	-0.33
WP	-0.19	-0.10	0.31	0.21	0.32
FR	-0.09	-0.09	0.08	0.01	0.02
ϵFR	-0.04	-0.06	0.10	0.05	0.03
BO	0.17	0.10	-0.10	0.13	-0.15

- DP failure associated with low feasibility
- NCH does well when fitness and violation are correlated.
- Others have weak correlations.

- K.M. Malan and I. Moser. Constraint handling guided by landscape analysis in combinatorial and continuous search spaces. *Evolutionary Computation*, 27(2), 2019.

Dig deeper – split dataset

Table 4: Spearman's correlation coefficients between algorithm performances (success/failure) and landscape metrics for the split datasets.

(a) Correlations for dataset DS_NoFeas.

	FVC	4_IZ	25_IZ
NCH	0.43	0.49	0.40
DP	-0.31	-0.23	-0.31
WP	0.02	0.00	0.02
FR	-0.06	-0.10	-0.06
ϵFR	-0.01	-0.04	0.01
BO	0.35	0.57	0.30

(b) Correlations for dataset DS_Feas.

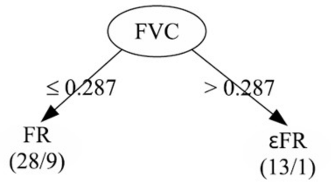
	FsR	RFBx	FVC	4_IZ	25_IZ
NCH	-0.16	0.14	0.51	0.36	0.37
DP	0.31	0.30	0.01	-0.28	-0.22
WP	-0.19	-0.07	0.43	0.24	0.36
FR	0.14	0.13	0.09	-0.06	-0.08
ϵFR	0.08	0.02	0.11	0.03	-0.05
BO	-0.11	-0.30	-0.28	0.15	-0.14

When there is no measurable feasibility (FsR = 0):
- BO correlates positively with 4_IZ metric

When there is measurable feasibility (FsR > 0):
- WP correlates positively with FVC
- BO correlates negatively with RFBx

Decision tree induction sometimes useful

- Predicting when one technique more suited than another technique.



Testing accuracy:
71% using 10-fold cross validation

Prediction in fuzzy terms: *When FsR is zero, if FVC is high, then εFR will probably perform better than FR.*

If there are very few feasible solutions, it is not helpful to ignore the constraint violation early on in the search process (unless the fitness and violation are correlated)

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The next steps

- Your initial investigation shows some links between problem features and algorithm performance.
- Next step: design a landscape-aware approach that exploits this knowledge ….

Image: pixabay

- See the last case study of the tutorial.

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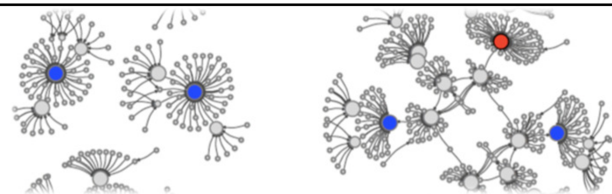

- Overview
- Definition of nodes
- Definition of Edges: basin, escape, monotonic, crossover
- Visualisation & Metrics

LOCAL OPTIMA NETWORKS

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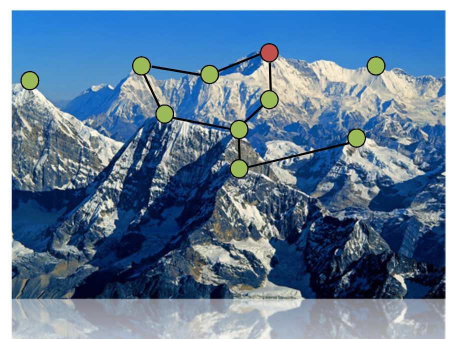
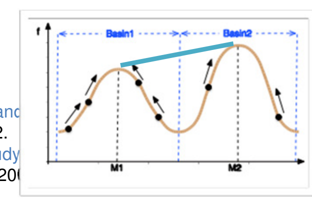
Local Optima Networks (LONs)



- Nodes: local optima according to a hill-climbing heuristic
- Edges: possible transitions between optima



- P. K. Doye. The network topology of a potential energy land a static scale-free network. *Physical Review Letter*, 2002.
- G. Ochoa, M. Tomassini, S. Verel, and C. Darabos. A study landscapes' basins and local optima networks. *GECCO* 20

LON original model

❖ Space S, Neigborhood $N(s)$, fitness $f(s)$

❖ $h(s)$ stochastic operator that associates each solution s to its local optimum (Alg. 1)

❖ The basin of attraction of a local optimum $l_i \in L$ is the set $B_i = \{s \in S \mid h(s) = l_i\}$

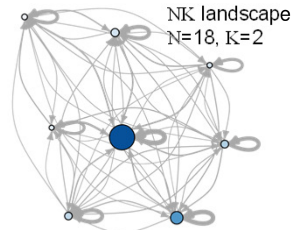
❖ Nodes (L). A local optima is a solution l such that $\forall\ s \in N(s), f(s) \leq f(l)$

❖ Basin Edges (E). Two local optima are connected if their basins of attraction intersect. At least one solution within a basin has a neighbour within the other basin.

❖ LON Model. Directed graph LON = (L, E)

Algorithm 1: Best-improvement local search

```
Choose initial solution s ∈ S
repeat
  choose s' ∈ N(s), f(s') = max_{x∈N(s)} f(x)
  if f(s) ≤ f(s') then
    s ← s'
  end if
until s is a Local optimum
```



w_{ij} proportion of transitions from solutions $s \in Bi$ to solutions $s' \in Bj$

Escape edges

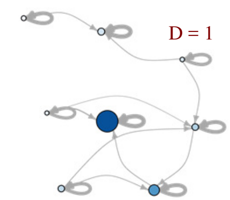
❖ Account for the chances of escaping a local optimum after a controlled mutation (e.g. 1 or 2 bit-flips in binary space) followed by hill-climbing

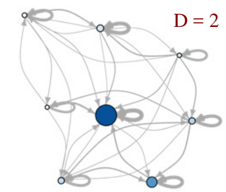
❖ Given a distance function d and integer value D, there is and edge e_{ij} between l_i and l_j if a solution s exists such that $d(s,l_i) \leq D$ and $h(s) = l_j$

❖ w_{ij} cardinality of $\{s \in S \mid d(s,l_i) \leq D$ and $h(s) = l_j\}$

❖ Sampled networks. There is an edge e_{ij} between l_i and l_j if l_j can be obtained after applying a *perturbation* to l_i followed by hill-climbing. Weights are estimated by the sampling process.

NK landscape
N=18, K=2



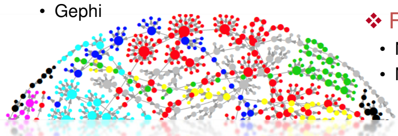


Complex network tools

Visualisation

❖ Force directed layout
- Position nodes in 2D
- Edges of similar length
- Minimise crossings
- Exhibit symmetries
- Example algorithms
  - Fruchterman & Reingold
  - Kamada & Kawai

❖ Software packages
- R igraph
- Gephi

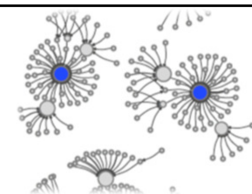


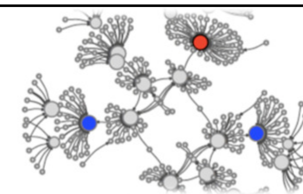
Metrics

❖ Network metrics
- Number of nodes and edges
- Number of global optima
- Weight of self-loops
- Avg. fitness of local optima
- Number of connected components
- Avg. path length to a global optimum
- Centrality (PageRank) of global optima
- Clustering coefficient

❖ Fractal metrics

❖ Funnel metrics
- Number of funnels
- Normalised size of global funnel



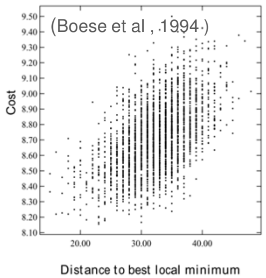


- The Big-Valley Hypothesis
- What is a Funnel?
- Characterization of Funnels with LONs
- Number Partitioning and the Phase transition

FUNNELS

The *big-valley* structure in combinatorial optimisation

- Several studies in the 90s. TSP (Boese et al, 1994), NK landscapes (Kauffman, 1993), graph bipartitioning (Merz & Freisleben, 1998) flowshop scheduling (Reeves, 1999)
- Distribution of local optima is not uniform. Clustered in a big-valley (*globally convex)* structure
- Many local optima, but easy to escape. Gradient at the coarse level leads to the global optimum.



100 cities TSP instance, 2,500 2-opt local optima

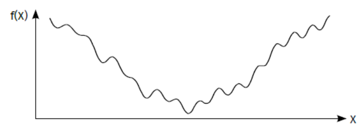


Fig. 1: Depiction of the 'big-valley' structure.

TSP: big-valley. Local optima confined to a small region

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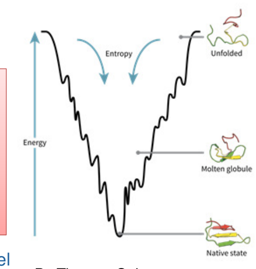
What is a Funnel?

"A key concept that has arisen within the protein folding community is that of a *funnel* consisting of a set of downhill pathways that converge on a single low-energy minimum."

Doye, J. P. K., Miller, M. A., & Wales, D. J . The double-funnel energy landscape of the 38-atom Lennard-Jones cluster. *Journal of Chemical Physics*, 1999



By Thomas Splettstoesser (link)(www.scistyle.com) - Own work

Funnels in continuous optimisation
- Multilevel global structure (Locatelli, 2005)
- *Dispersion* metric (Lunacek &Whitley, 2006, 2008)
- Feature-based detection of (single) funnel structure (Kerschke et al., 2015)

Funnels in combinatorial optimisation
- Related to the big-valley (central-massif) hypothesis (previous slide)
- The big-valley re-visited (Hains, Whitley & Howe, 2011)
- Characterisation of funnels with Local Optima Networks (our contribution)

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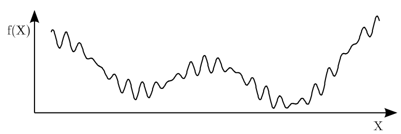
Characterisation of funnels



- Funnels can be loosely defined as groups of local optima, which are close in configuration space within a group, but well-separated between groups.
- A funnel conforms a coarse-grained gradient towards a low cost optimum.
- How to characterise funnels more rigorously using LONs?
  - Connected components. Funnels are sub-graphs, connected components within LONs. (EvoCOP, 2016)
  - Communities. Funnels are *communities* within LONs. (GECCO, 2016, 2017)
  - Monotonic sequences. Concept from energy landscapes. Conceptually sound characterisation, incorporating both grouping and coarse-grained gradient. (EvoCOP 2017, 2018; JoH 2017)

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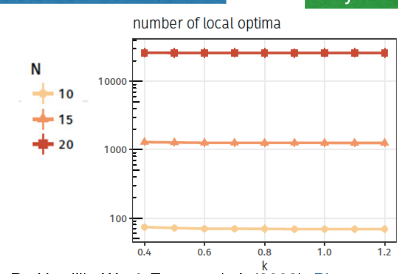
NPP fitness landscape

What features of the fitness landscape are responsible for the widely different behaviours?

Most fitness landscape metrics are insensitive/oblivious to the easy/hard phase transition!



- Stadler, P., Hordijk, W., & Fontanari, J. (2003). Phase transition and landscape statistics of the number partitioning problem. *Physical Review E*
- K. Alyahya, J. Rowe (2014). Phase Transition and Landscape Properties of the Number Partitioning Problem. *EvoCOP*.

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2

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Characterisation of funnels with LONs

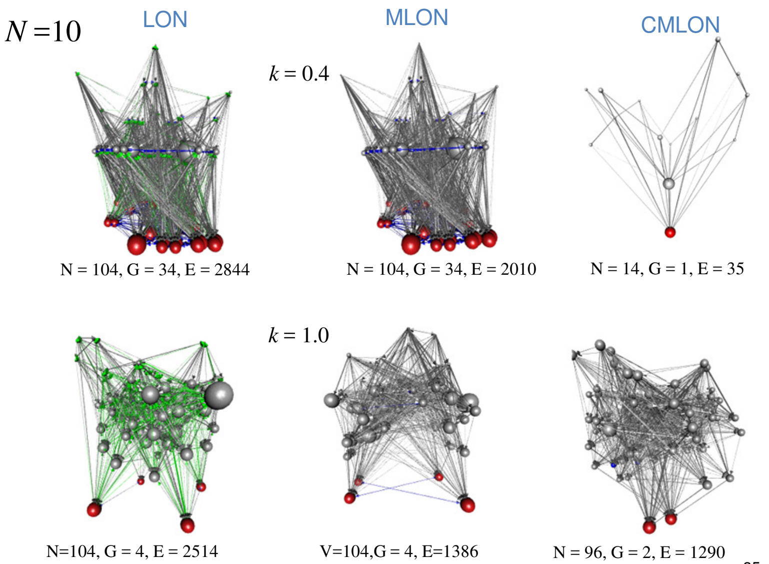
- Monotonic edges. Keep only non-deteriorating edges
  $l_s \rightarrow l_e$, if $f(l_e) \leq f(l_s)$
- Monotonic sequence. Path of connected local optima
  $l_1 \rightarrow l_2 \rightarrow l_3 \ldots \rightarrow l_s$ $f(l_i) \leq f(l_{i-1})$
- Sink. Natural end of the sequence, when there is no adjacent improving local optima
- Definition of Funnel
  - Aggregation of all monotonic sequences ending at the same point (sink).
  - Basin of attraction level of local optima

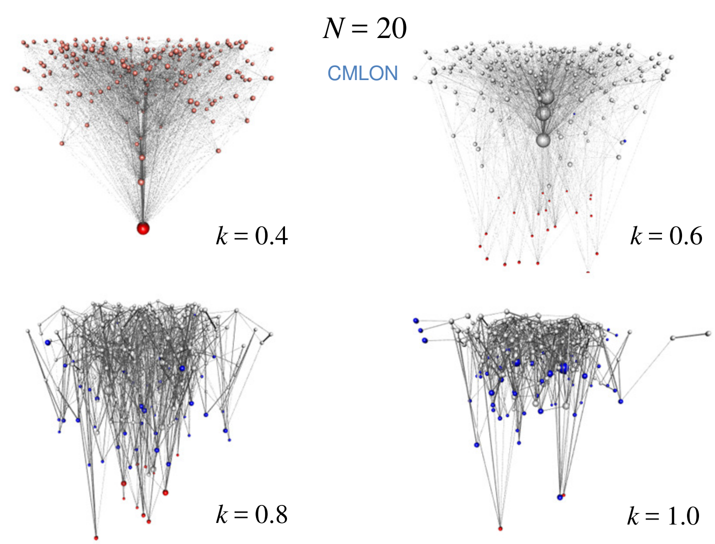
Sink. Node without outgoing edges

$i \leftarrow 0$ $\qquad$ S set of sinks
for $s \in S$ **do**
$\quad fbasin[i] \leftarrow$ breadthFirstSearch(LON, s)
$\quad fbsize[i] \leftarrow$ length($fbasin[i]$)
$\quad i \leftarrow i+1$
end

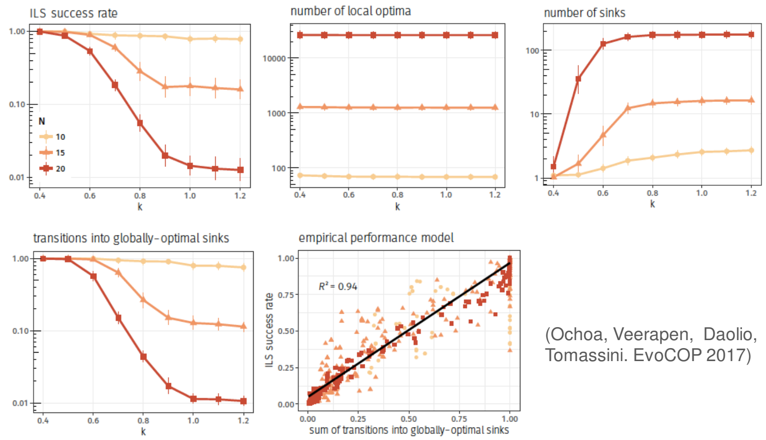
Methodology

- Full enumeration and extraction of LONs
- $N = \{10, 15, 20\}$, k in $[0.4, 1.2]$ step 0.1
- 30 instances for each N and k
- LON. 1-flip local search, 2-flip perturbation $(D = 2)$
- MLON. Monotonic LON, worsening edges pruned
- CMLON. compressed MLON, LON plateaus contracted in a single node
- Empirical search performance: ILS success rate

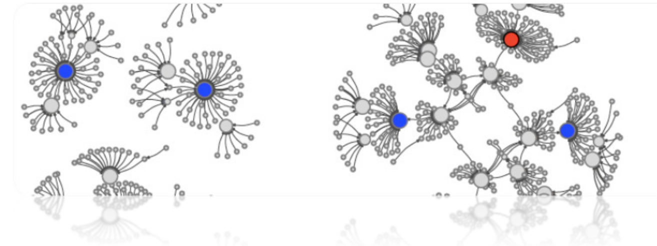




LON metrics & Search Performance



(Ochoa, Veerapen, Daolio, Tomassini. EvoCOP 2017)



- Travelling Salesperson Problem (TSP) and multiple funnels
- Exploiting knowledge of the global structure
  - Increasing perturbation strength
  - Models including recombination
- Feature Selection for Classification

CASE STUDIES

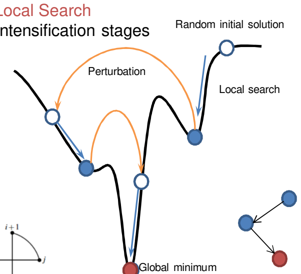
Travelling Salesperson Problem

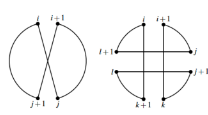
Sampling and constructing LONs with escape edges

Data: I, TSP instance
Result: L, set of local optima,
E, set of escape edges
$L \leftarrow \{\}; E \leftarrow \{\}$
for $i \leftarrow 1$ **to** 1000 **do**
$s_{start} \leftarrow$ initialSolution()
$s_{start} \leftarrow$ LK(s_{start})
$L \leftarrow L \cup \{s_{start}\}$
while $j < 10000$ **do**
$s_{end} \leftarrow$ applyKick(s_{start})
$s_{end} \leftarrow$ LK(s_{end})
$j \leftarrow j + 1$
if $Objective(s_{end}) \leq Objective(s_{start})$ **then**
$L \leftarrow L \cup \{s_{end}\}$
$E \leftarrow E \cup \{(s_{start}, s_{end})\}$
$s_{start} \leftarrow s_{end}$
$j \leftarrow 0$
end
end
end

TSP heuristic, Chained Lin-Kernighan
(Martin, Otto, Felten, 1992)

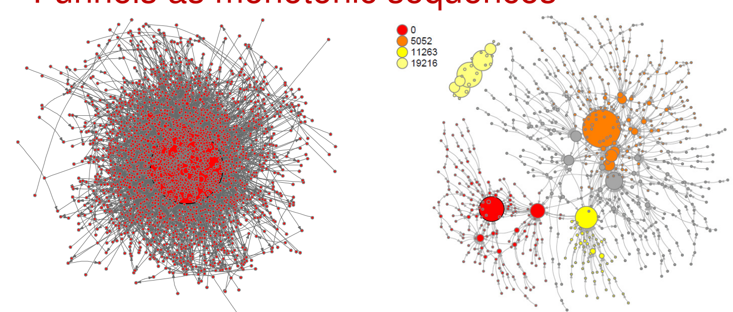
- Form of Iterated Local Search
- Diversification & Intensification stages





- Nodes. Lin-Kernighan
- Edges. Double-bridge

TSP Synthetic Instances
Funnels as monotonic sequences



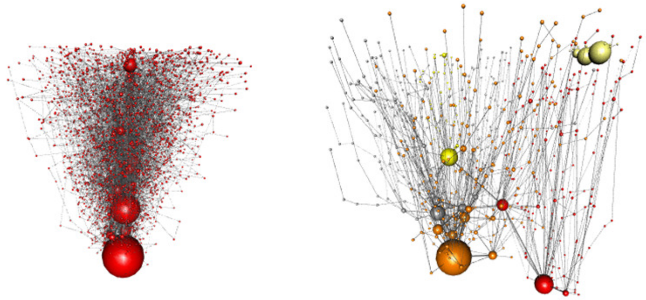
C755 Clustered Cities
Funnels: 1, Success: 100%

E755 Uniform Cities
Funnels: 4, Success: 13%

DIMACS random instances

(Ochoa & Veerapen, JoH 2018)

TSP Synthetic Instances



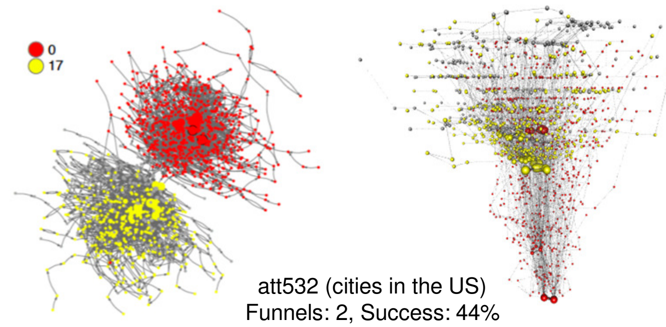
C755 Clustered Cities
Funnels: 1, Success: 100%

E755 Uniform Cities
Funnels: 4, Success: 13%

DIMACS random instances

Same layout, 3D projection where z coordinate is fitness

TSPLIB City Instance att532



att532 (cities in the US)
Funnels: 2, Success: 44%

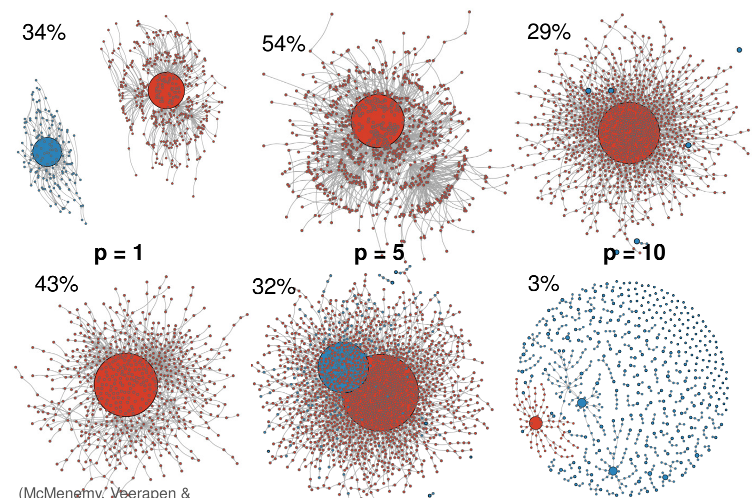
2D layout and 3D projection where z coordinate is fitness

Exploiting knowledge of the global structure

- Instances of several combinatorial optimisation problems have a multi-funnel structure
- Sub-optimal funnels act as traps to the search process
- Can we devise mechanisms for escaping sub-optimal funnels?
  - Restarts
  - Stronger perturbation in ILS implementations
  - Crossover

Increasing perturbation strength

TSP Uniform E755



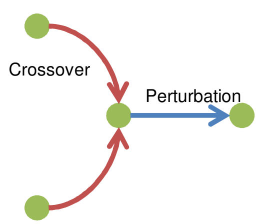
(McMenemy, Veerapen & Ochoa. EvoCOP 2018)

TSP Clustered C755

LONs for Hybrid EAs

- Hydrid EAs which incorporate a local search component to generate local optima.
- Two types of edges
  - Crossover (followed by local search)
  - Perturbation (followed by loca

Grey-box Optimisation
DRILS Deterministic Recombination (PX) + ILS



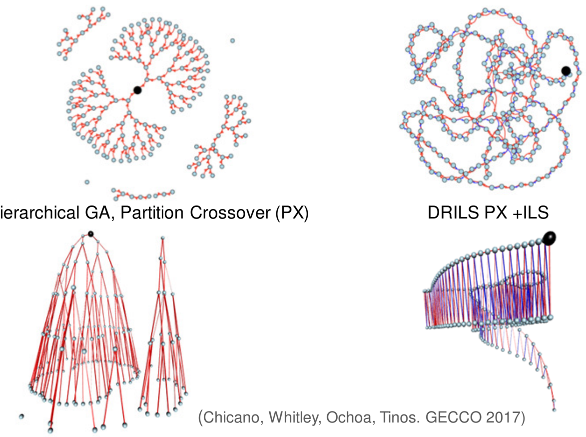
Algorithm 1 DRILS

```
current ← hillClimber(random());
while not stopping condition do
    next ← hillClimber (perturb(current));
    child ← PX(current, next);
    if child = current or child = next then
        current ← next;
    else
        current ← hillClimber(child);
    end if
end while
```

(Chicano, Whitley, Ochoa, Tinos. GECCO 2017)

Contrasting LONs from two solving methods

- Grey-box hybrid EA, 1 million variables NK



(Chicano, Whitley, Ochoa, Tinos. GECCO 2017)

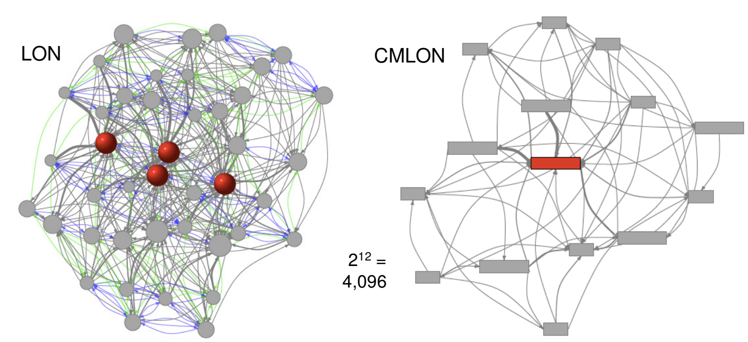
Binary feature selection for classification

- 7 real-world datasets from UCL ML & Open ML, with up to 16 features
- Search space: binary strings, subset of features, 2^n
- Fitness function: classification accuracy
  - *k*-Nearest Neighbour (*k*-NN) algorithm
  - *Cohen's Kappa* statistic
- Fully enumerated LONs (1-bitflip, Escape $D = 2$)
- To test problem difficulty, 3 features selection methods: filter, *sequential forward selection* (SFS) and simple GA wrapper

(Mostert, Malan, Ochoa, Engelbrecht EvoCOP 2019)

Feature selection: vowel recognition dataset

12 features (numeric - extracted from speech) , 11 classes (vowels of British English)



noptima: 42, *nglobals*: 4
SFS_{pr}: 1.00, GA_{pr}:0.98, GA_{sr}: 0.7

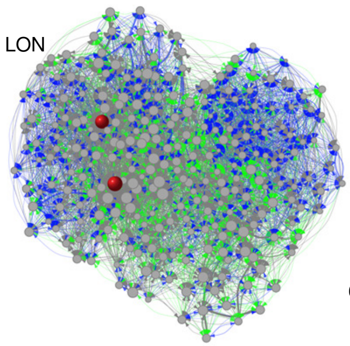
ncoptima: 15, *ncglobal*: 1
neutral: 0.64, *str_global*: 0.50

(Mostert, Malan, Ochoa, Engelbretch EvoCOP 2019)

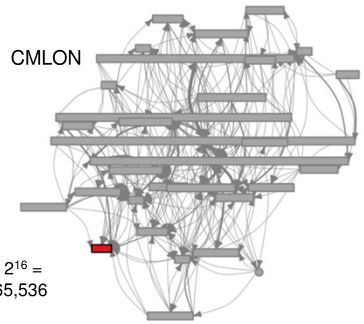
Feature selection: zoo dataset

16 features (B: feathers, aquatic, etc.; I: no.legs), 7 classes (type of animal: mammal, bird, etc.)

LON



CMLON



2^{16} = 65,536

noptima: 4,938, *nglobals*: 2
SFS$_{pr}$: 0.83, GA$_{pr}$:0.99, GA$_{sr}$: 0.13

ncoptima: 1,342, *ncglobal*: 1
neutral: 0.73, *str_global*: 0.001

(Mostert, Malan, Ochoa, Engelbrecht EvoCOP 2019)

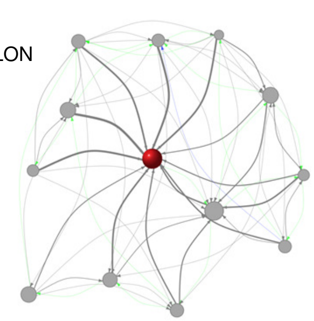
Feature selection: irrelevant features

- Four global optima vowel dataset
  - 0011111100100
  - 0111111100100
  - 1011111100100
  - 1111111100100
- The first two features seem irrelevant
- Checking the dataset confirms that!

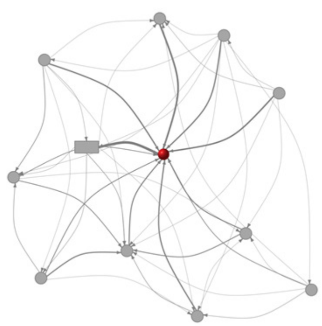
Speaker	Sex	Feature_0	Feature_1	Feature_2	Feature_3	Feature_4	Feature_5	Feature_6	Feature_7	Feature_8	Feature_9	Class
Andrew	Male	-3.639	0.418	-0.67	1.779	-0.168	1.627	-0.388	0.529	-0.874	-0.814	**hid**
Andrew	Male	-3.327	0.496	-0.694	1.365	-0.265	1.933	-0.363	0.51	-0.621	-0.488	**hId**
Andrew	Male	-2.12	0.894	-1.576	0.147	-0.707	1.559	-0.579	0.676	-0.809	-0.049	**hEd**
Andrew	Male	-2.287	1.809	-1.498	1.012	-1.053	1.06	-0.567	0.235	-0.091	-0.795	**hAd**
Andrew	Male	-2.598	1.938	-0.846	1.062	-1.633	0.764	0.394	-0.15	0.277	-0.396	**hYd**
Andrew	Male	-2.852	1.914	-0.755	0.825	-1.588	0.855	0.217	-0.246	0.238	-0.365	**had**

Feature selection: vowel recognition dataset after removing irrelevant features

LON



CMLON



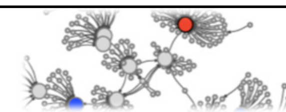
2^{10} = 1,024

noptima: 13, *nglobals*: 1
SFS$_{pr}$: 1.00, GA$_{pr}$:0.98, GA$_{sr}$: 0.8

ncoptima: 12, *ncoptima*: 1
neutral: 0.08, *str_global*: 0.59

(Mostert, Malan, Ochoa, Engelbrecht EvoCOP 2019)

LONs



Conclusions

- More accessible (visual) approach to heuristic understanding
- Global structure impacts search
- Real-world problems are neutral and have multiple-funnels

Contributions

- Sampling & Visualisation
- Characterisation of funnels
- New LON metrics can improve performance prediction.
- Using knowledge to select/configure algorithms

lonmaps.com - Website with LON resources

Image: pixabay

CASE STUDY 2: LANDSCAPE-AWARE CONSTRAINT HANDLING

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Landscape aware constraint handling

Overall approach:

- ❖ Benchmark suite of training instances: characterise instances using landscape analysis based on sampling.
- ❖ Solve the problem instances using different constraint handling techniques (CHTs) with the same base algorithm and rank the performances of CHTs on training instances.
- ❖ Perform data mining on training set to model the relationship between problem features and winning algorithm approaches.
- ❖ Formulate high-level rules for selecting appropriate CHTs.
- ❖ Take a different set of problem instances as the test set.
- ❖ Solve them by switching to the CHT that is predicted to be the best, given the landscape characteristics experienced during search (online landscape analysis).
- ❖ Compare the landscape-aware approach to the individual CHTs.

- K.M. Malan, Landscape-aware Constraint Handling Applied to Differential Evolution. TPNC 2018, LNCS 11324, pp. 176-187.

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Benchmark suite

- ❖ Limited to real-valued minimisation problems:

$$\text{Minimise } f(\mathbf{x}), \quad \mathbf{x} = (x_0, x_1, \ldots, x_{n-1}) \in \mathbb{R}^n \qquad (1)$$

$$\text{subject to } \begin{array}{ll} g_i(\mathbf{x}) \leq 0, & i = 1, \ldots, p \\ h_j(\mathbf{x}) = 0, & j = p+1, \ldots, m \end{array} \qquad (2)$$

- ❖ Equality constraints re-expressed as inequality constraints ($\varepsilon = 10^{-4}$):

$$|h_j(\mathbf{x})| - \epsilon \leq 0, \quad j = p+1, \ldots, m \qquad (3)$$

- ❖ CEC 2010 Competition on Constrained Real-Parameter Optimization:
  - 18 problems, scalable to any dimension.
  - Training data set: nine odd numbered functions in 5D, 10D, 15D, 20D, 25D and 30D (54 instances).
  - Testing data set: nine even numbered functions in 5D, 10D, 15D, 20D, 25D and 30D (54 instances).

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Base algorithm: Differential Evolution

- ❖ Differential evolution:
  - Established, popular population-based metaheuristic for real-valued optimisation.
  - Performed well in CEC Constrained Real-Parameter Optimisation competitions: CEC 2006, CEC2010.

1st	**ε_DE**
2nd	**DMS-PSO**
3rd	**SaDE, MDE**
5th	**PCX**
6th	**MPDE**
7th	**DE**
8th	**jDE-2**
9th	**GDE, PESO+**

Rank	Algorithm
1st	εDEg
2nd	ECHT
3rd	jDEsoco
4th	DCDE
5th	Co-CLPSO
6th	IEMA
7th	DE-VPS
8th	CDEb6e6rl
9th	RGA
10th	E-ABC
11th	MTS
12th	sp-MODE

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Feature extraction of training set

- 54 training instances characterised based on samples.
- Approach to sampling:
  - Sample size: 200 x D (1% of computational budget for solving the problem) generated for each instance using multiple hill climbing walks.
  - From a random initial position, neighbours sampled (from a Gaussian distribution with mean = current position and std deviation = 5% of the range of domain of search space).
  - Walk terminated if no better neighbour found after sampling 100 random neighbours.
- Sample used as the basis for calculating five landscape metrics: FsR, RFBx, FVC, 25_IZ, 4_IZ.

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Constraint handling techniques

Base algorithm: *DE/rand/1*, with uniform crossover, a population size of 100, a scale factor of 0.5, and a crossover rate of 0.5.

1. Weighted Penalty:
   - Combine constraint violation as a penalty in the objective function (50% penalty, 50% objective value).
2. Feasibility Ranking (Deb, 2000):
   - Two feasible solutions compared on objective value
   - Feasible solution always preferred to an infeasible solution
   - Two infeasible solutions compared by level of constraint violation.
3. ε-Feasibility Ranking (Takahama & Sakai, 2006):
   - Like Deb's rules, but with a tolerance (ε) to constraint violations that reduces over time.
4. Bi-objective:
   - Constraint violation treated as a 2^{nd} objective.
   - Non-dominated sorting of NSGA II (Deb et al., 2002).

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Measuring performance on training set

- Classic DE run on training instances with each CHT 30 times.
- Computational budget: 20 000 x D.
- A run of an algorithm regarded as feasible if a feasible solution found within the budget of function evaluations.
- Two algorithms compared using CEC2010 competition rules:
  - If two algorithms have different success rates, the algorithm with the higher success rate wins.
  - If two algorithms have the same success rate > 0, the algorithm with the superior mean fitness value of feasible runs wins.
  - If two algorithms have success rate = 0, the algorithm with the lowest mean violation wins.

Image: pixabay

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Example performance on training set

CEC 2010 problem C01 in 15 dimensions				
	Success rate	Mean fitness (feasible runs)	Mean violation	Algorithm rank
WP	0.233	-0.7824	0.2125	3
FR	1	-0.7815	0	2
ϵFR	1	-0.7820	0	1
BO	0	n/a	0.3750	4
CEC 2010 problem C09 in 5 dimensions				
	Success rate	Mean fitness (feasible runs)	Mean violation	Algorithm rank
WP	1	0.2561	0	2
FR	0	n/a	0.4117	4
ϵFR	0	n/a	0.2637	3
BO	1	0.0000	0	1

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Deriving CHT selection rules

- Training dataset for a classification problem:
  - 54 problem instances with 5 features (landscape metrics based on sampling).
  - Binary performance class for each algorithm: the best or not.
- Use data mining to predict when each CHT will be the best:
  - C4.5 algorithm used to induce decision trees using the full training dataset.
  - Rules extracted for predicting when each algorithm is expected to be the best.

1. WP is predicted to be the best when (4_IZ > 0.006) AND ((FsR > 0) OR (FsR $= 0$ AND FVC ≤ 0.06)).
2. FR is predicted to be the best when (25_IZ ≤ 0.259) AND (RFBx ≤ 0.083).
3. ϵFR is predicted to be the best when FsR > 0.28.
4. BO is predicted to be the best when (FVC > 0.28) AND (FsR $= 0$).

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Online landscape analysis

- Landscape information collected during search:
  - Path of each individual in the population is treated as a "walk": position, objective value & constraint values stored in a queue.
  - With each new generation, if the new position differed from previous solution, the new child solution appended to the walk of that individual.
  - *OLA_limit*: parameter for queue length.
- Switching constraint handling using general rules (derived from data mining):
  - After a set number of iterations (*SW_freq* parameter), the landscape metrics are calculated.
  - Based on the landscape profile, the CHT is switched to the predicted best strategy (using the rules derived previously through data mining).

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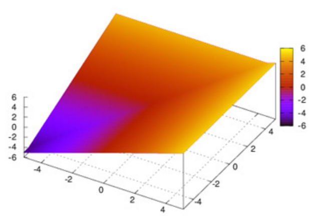
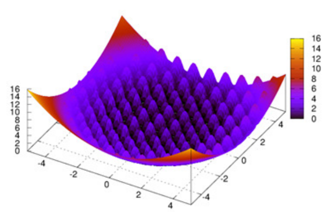
Sample run

Table 3. Data from a sample run of the LA approach on problem C02 in 5D

Iteration	Online Landscape Metrics					Selected Strategy
	FsR	RFBx	FVC	25_IZ	4_IZ	
50	0	0	0.553	0.335	0.017	BO
100	0	0	-0.810	0.063	0	FR
600	0.001	0.001	-0.872	0.050	0	FR



Fitness landscape



Violation landscape

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Results

- Performance of six constraint handling approaches on 54 test problem instances (rank: 1 – 6):
  - Four base constraint handling techniques: WP, FR, εFR, BO.
  - RS: Random switching between above 4 techniques.
  - LA: Landscape-aware switching based on online landscape features.
- Parameters: *SW_freq* = 10 x D, *OLA_limit* = 10 x D.

Strategy	Mean Rank	Best Performing		Worst Performing	
WP	3.44	**16 instances**	**(30%)**	14 instances	(26%)
FR	3.69	7 instances	(13%)	8 instances	(15%)
ϵFR	3.59	3 instances	(6%)	**0 instances**	**(0%)**
BO	4.54	9 instances	(17%)	32 instances	(59%)
RS	3.19	9 instances	(17%)	**0 instances**	**(0%)**
LA	**2.46**	15 instances	(28%)	**0 instances**	**(0%)**

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Case study conclusion

- There is value in utilising a range of constraint-handling techniques.
- Proposed switching technique:
  - Pre-processing landscape analysis step to derive rules for predicting when each constraint handling technique will perform the best.
  - Rules applied during search using features extracted from the search path (no additional sampling or fitness evaluations needed).
- Results show that the proposed landscape-aware approach performed better than the constituent approaches when used in isolation.
- Similar approach to landscape-aware search can be used in other contexts.

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Tutorial conclusion

- Fitness landscape analysis has come a long way in the last 10 years
  - Different perspectives of landscapes: local scale (e.g. ruggedness), global scale (e.g. funnels)
  - Different landscapes (e.g. fitness and violation landscapes).
- We showed how local optima networks can be used
  - To visualise global structure
  - To characterise funnels
- Case studies demonstrated
  - Using LONs to analyse funnels of TSP
  - Exploiting knowledge of global structure to configure search algorithms.
  - Using LONs to gain insight into the feature selection problem
  - How Rice's general algorithm selection framework can be used to implement landscape-aware search in the context of constraint handling techniques for evolutionary algorithms

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