Exploratory Landscape Analysis

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Instructors

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His current research interests are algorithm selection, as well as *Exploratory Landscape Analysis* for single- and multi-objective optimization problems.

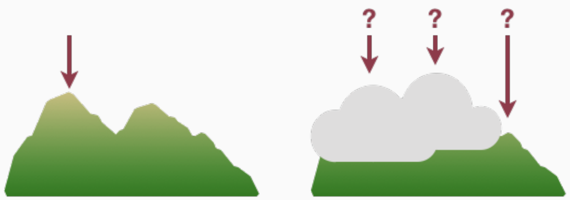
Mike Preuss is Assistant Professor at Leiden University, The Netherlands, and member of the ERCIS network. Previously, he was with the group of *Information Systems and Statistics* at the University of Münster, Germany. In 2013, he received his PhD at TU Dortmund University, Germany.
His research interests focus on the field of evolutionary algorithms for real-valued problems, namely on multimodal and multi-objective optimization. He is also active in computational intelligence methods for computer games.

Introduction

Introduction

Context:

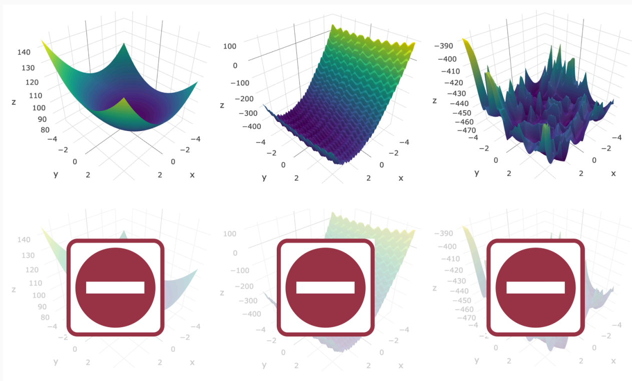
- *Black-Box Optimization*: find the optimum of a given problem without actually knowing its entire landscape



Introduction

- lack of real-world problems

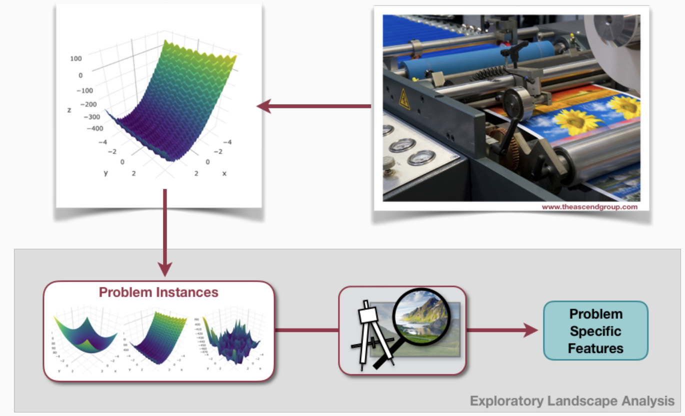
  ⇝ use benchmark problems and handle them as black-box



Introduction

Idea of ELA

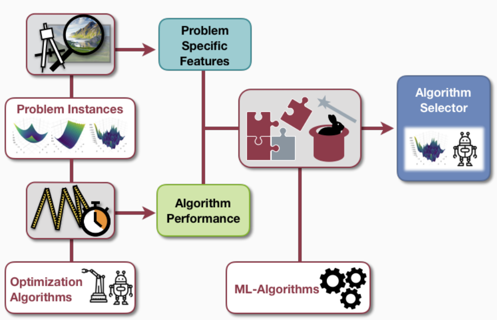
- extract information from (black-box) optimization problems in an automated fashion



Introduction

How is ELA helpful when optimizing a given problem?

- *Algorithm Selection Problem*: find the individually best suited algorithm for an unseen optimization problem



[36] Rice, J. R. (1976). *The Algorithm Selection Problem*. In Advances in Computers (pp. 65-118).

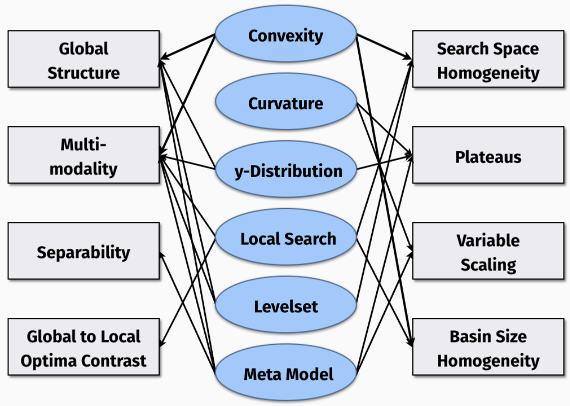
[17] Kerschke, P., Hoos, H. H., Neumann, F. & Trautmann, H. (2019). *Automated Algorithm Selection: Survey and Perspectives*. In Evolutionary Computation, Vol. 27, Number 1 (pp. 3-45).

Introduction

Exploratory Landscape Analysis (ELA):

- we aim at finding the "best" algorithm
- also improve understanding of problems, as well as algorithm/problem dependency
- basic idea (exploratory!): we start with **very simple features without clear purpose**
- match existing high-level features (expert knowledge) with our ELA features
- currently: mostly continuous (black-box) (global) optimization, but also in other domains (e.g., TSP)

Introduction



[29] Mersmann, O., Preuss, M. & Trautmann, H. (2010). *Benchmarking Evolutionary Algorithms: Towards Exploratory Landscape Analysis*. In Proceedings of PPSN XI (pp. 71 - 80).

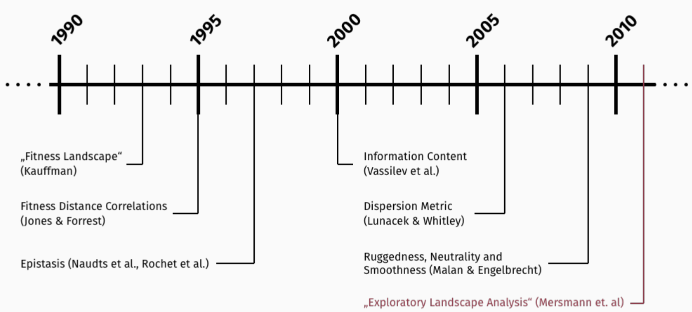
[28] Mersmann, O., Bischl, B., Trautmann, H., Preuss, M., Weihs, C. & Rudolph, G. (2011). *Exploratory Landscape Analysis*. In Proceedings of GECCO 2011 (pp. 829 - 836).

Introduction

- we do not know functional relationships when designing features
- but we can match them to high-level characteristics (multimodality, funnel structure, etc.) of optimization problems
- this enables recognizing important problem properties quickly
- based on initial design of samples $x_{i1}, \ldots, x_{id}$ and their corresponding fitness value y_i, $i = 1, \ldots, n$
- given an evaluated initial design (initial population?), most ELA features are for free
- there are already several different feature sets

Introduction

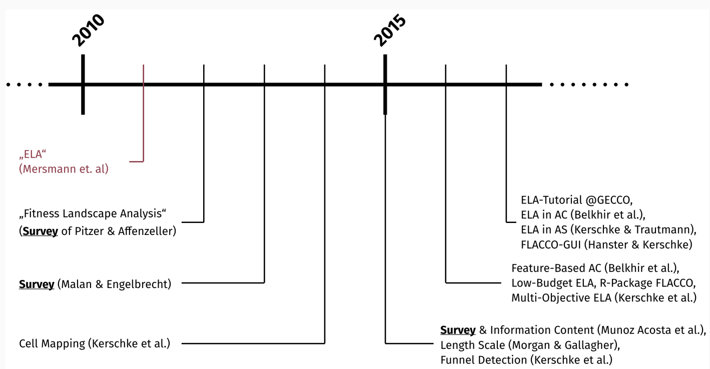
History of Landscape Analysis – Before ELA:



Further details are given in [13, 12, 33, 37, 40, 25, 26, 28]

Introduction

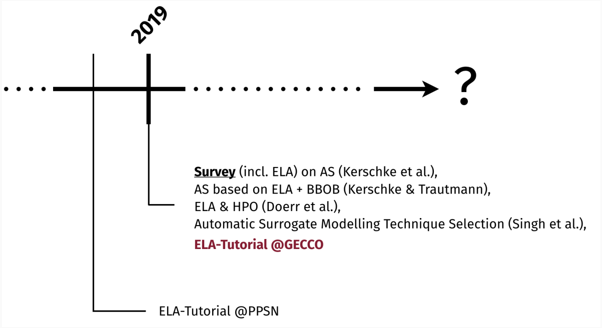
History of Landscape Analysis – Since ELA:



Further details are given in [28, 34, 27, 19, 31, 32, 30, 20, 2, 21, 22, 15, 18, 10, 23, 3]

Introduction

History of Landscape Analysis – Most Recent Developments:



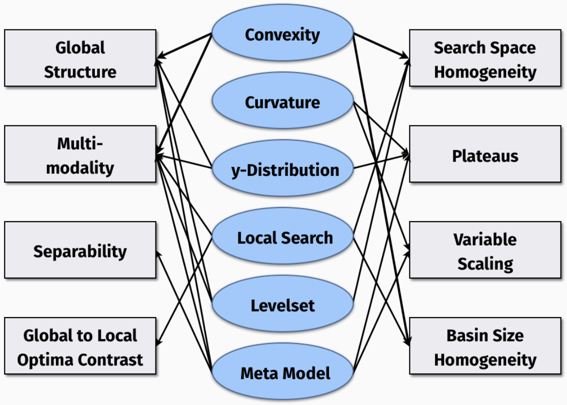
Further details are given in [23, 17, 7, 39]

Single-Objective ELA Features

Single-Objective ELA Features

Classical ELA Features

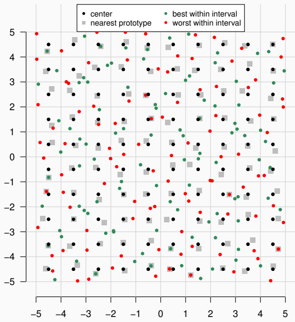
[29] Mersmann, O., Preuss, M. & Trautmann, H. (2010). *Benchmarking Evolutionary Algorithms: Towards Exploratory Landscape Analysis.* In Proceedings of PPSN XI (pp. 71 - 80).

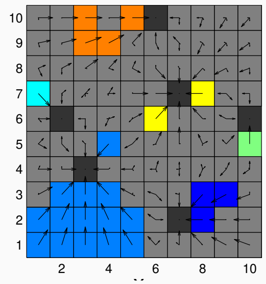


Single-Objective ELA Features

General Cell Mapping Features

[19] Kerschke, P., Preuss, M., Hernández, C., Schütze, O., Sun, J.-Q., Grimme, C., Rudolph, G., Bischl, B. & Trautmann, H. (2014). *Cell Mapping Techniques for Exploratory Landscape Analysis.* In Proceedings of EVOLVE 2014 (pp. 115 - 131).



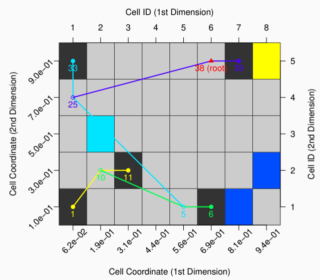


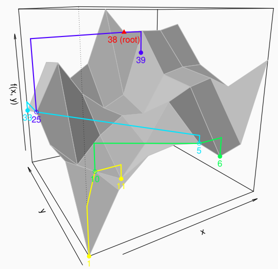
Single-Objective ELA Features

Barrier Tree Features

[11] Hernández, C., Schütze, O., Emmerich, M. T. M., & Xiong, F. R. (2014). *Barrier Tree for Continuous Landscapes by Means of Generalized Cell Mapping.* In Proceedings of EVOLVE 2014.

[8] Flamm, C., Hofacker, I. L., Stadler, P. F. & Wolfinger, M. T. (2002). *Barrier Trees of Degenerate Landscapes.* In International Journal of Research in Physical Chemistry and Chemical Physics (pp. 155 - 173).

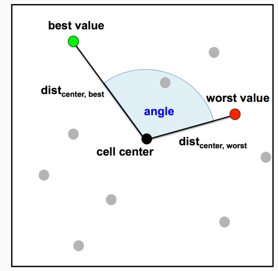


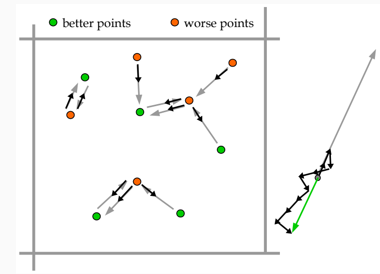


Single-Objective ELA Features

Cell Mapping Features

[19] Kerschke, P., Preuss, M., Hernández, C., Schütze, O., Sun, J.-Q., Grimme, C., Rudolph, G., Bischl, B. & Trautmann, H. (2014). *Cell Mapping Techniques for Exploratory Landscape Analysis.* In Proceedings of EVOLVE 2014 (pp. 115 - 131).



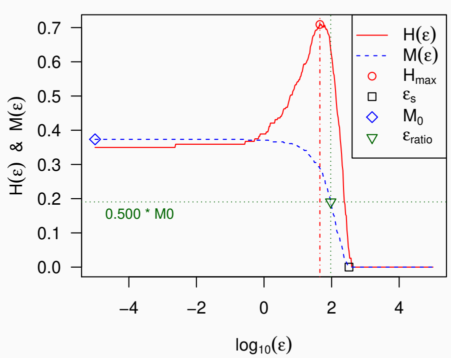


Single-Objective ELA Features

Information Content Features

[31] Muñoz, M. A., Kirley, M., Halgamuge, S. K. (2015). *Exploratory Landscape Analysis of Continuous Space Optimization Problems using Information Content.* In IEEE Transactions on Evolutionary Computation (pp. 74 - 87).

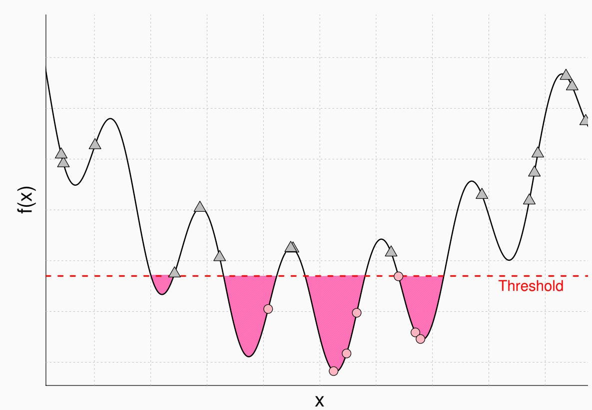
[40] Vassilev, V. K., Fogarty, T. C. & Miller, J. F. (2000). *Information Characteristics and the Structure of Landscapes.* In Evolutionary Computation, Vol. 8, Number 1 (pp. 31 - 60).



Single-Objective ELA Features

Dispersion Features

[25] Lunacek, M. & Whitley, D. (2006). *The Dispersion Metric and the CMA Evolution Strategy.* In Proceedings of GECCO 2006 (pp. 477 - 484).



Single-Objective ELA Features

Hill Climbing Features

[1] Abell, T., Malitsky, Y. & Tierney, K. (2013). *Features for Exploiting Black-Box Optimization Problem Structure.* In Proceedings of LION 2013 (pp. 30 - 36).

Ruggedness Features

[27] Malan, K. M. & Engelbrecht, A. P. (2013). *Ruggedness, Funnels and Gradients in Fitness Landscapes and the Effect on PSO Performance.* In Proceedings of CEC 2013 (pp. 963 - 970).

Nearest Better Clustering Features

[20] Kerschke, P., Preuss, M., Wessing, S. & Trautmann H. (2015). *Detecting Funnel Structures by Means of Exploratory Landscape Analysis.* In Proceedings of GECCO 2015 (pp. 265 - 272).

Length Scale Features

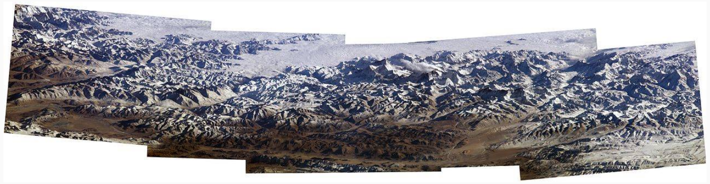
[30] Morgan, R. & Gallagher M. (2015). *Analyzing and Characterising Optimization Problems Using Length Scale.* In Soft Computing (pp. 1 - 18).

Bag of Local Landscape Features

[38] Shirakawa, S. & Nagao, T. (2016). *Bag of Local Landscape Features for Fitness Landscape Analysis.* In Soft Computing, 20(10) (pp. 3787 – 3802).

(ELA for Single-Objective) Multimodal Optimization

Multimodal Optimization



Multimodal Optimization

- algorithmic ancestry goes back to the 1980s
- yearly competitions and publications steadily produce new methods
- two main algorithmic approaches:
  - parallel, large populations
  - sequential, coordinated restarts
- several components that may be used: archives, clustering methods, methods for obtaining well distributed samples
- ELA could be helpful for selecting components/methods

Multimodal Optimization

- different aims possible
- currently most important (competitions): multiglobal
  = find all search space points that are globally optimal
- other options: all optima, well distributed good optima, multiglobal over time, etc.
- interesting: feature values do not change because they do not depend on actual measure, we can reuse feature data
- but we need different algorithms for different goals
- algorithm performances and one sample on the problems enable algorithm selection per criterion

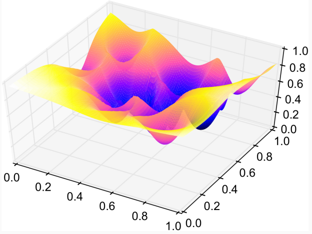
Exemplary Use Cases of ELA

Exemplary Use Cases of ELA: Funnel Detection

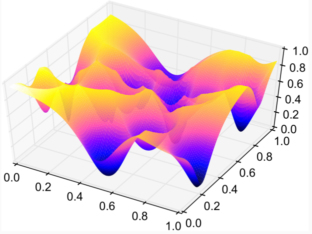
Example 1: **Funnel Detection**

Exemplary Use Cases of ELA: Funnel Detection

- funnel: local optima are located near to each other and pile up to an "upside-down mountain"
- knowledge about underlying global structure, i.e. funnels, helps selecting the right algorithm



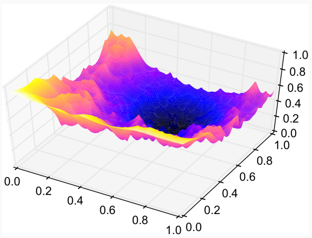
(a) funnel



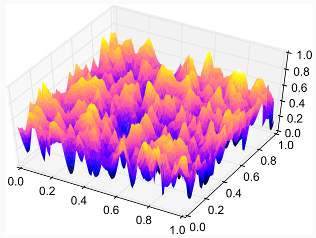
(b) non-funnel ("random")

Exemplary Use Cases of ELA: Funnel Detection

- different algorithm candidates for either category
- but there is a wide variety within classes funnel and non-funnel



(a) funnel



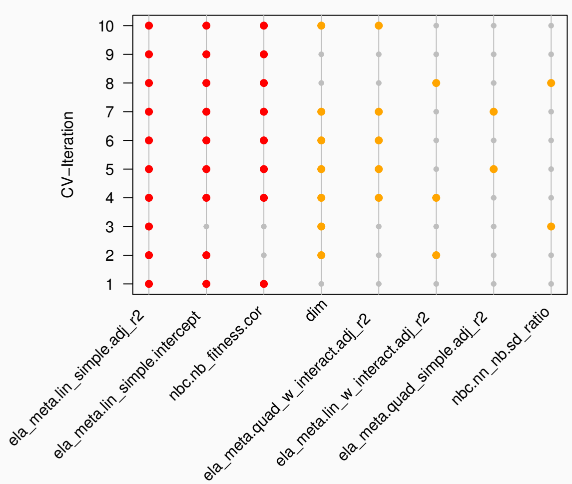
(b) non-funnel (“random”)

Exemplary Use Cases of ELA: Funnel Detection

- detailed results in our GECCO paper[1]
- used MPM2[2] to generate a set of 4,000 training instances
- initial designs of size $50 \times d$ observations (small!)
- trained four classifiers (random forest, rpart, kknn and ksvm)
- experimentally driven reduction of the full feature set (300+ features) to 8 features
- validated results on BBOB and subset of problems from CEC-2013 niching competition

1. [21] Kerschke, P., Preuss, M., Wessing, S. & Trautmann H. (2016). Low-Budget Exploratory Landscape Analysis on Multiple Peaks Models. In Proceedings of GECCO 2016 (pp. 229-236)
2. [43, 4] multiple peaks model 2 generator, available in python (`optproblems0.9`, Wessing, S.) and R (`smoof`, Bossek, J.)

Exemplary Use Cases of ELA: Funnel Detection

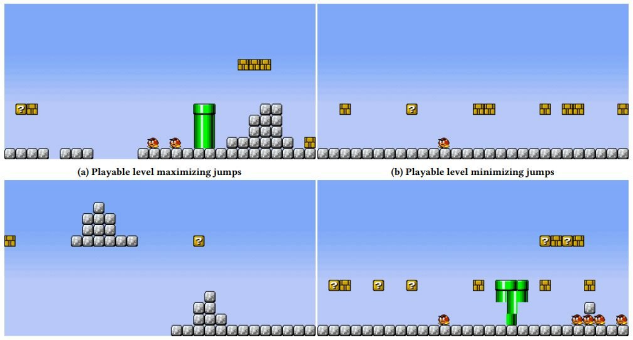


Exemplary Use Cases of ELA: Benchmark Comparison

Example 2: **Benchmark Comparison (BBOB vs. Mario GAN)**

Exemplary Use Cases of ELA: Benchmark Comparison

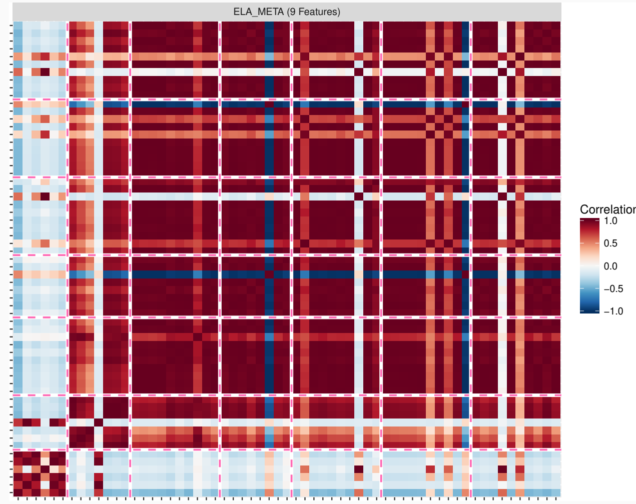
- using GANs (Generative Adversarial Networks) for generating Super Mario levels
- comparison of underlying optimization problems against BBOB



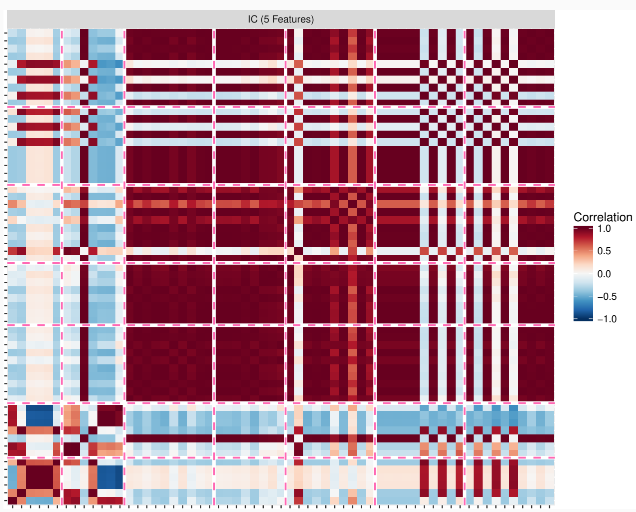
[3]

3. [42] Volz, V., Schrum, J., Liu, J., Lucas, S. M., Smith, A., & Risi, S. (2018). *Evolving Mario Levels in the Latent Space of a Deep Convolutional Generative Adversarial Network*. In Proceedings of GECCO 2018 (pp. 221 - 228).

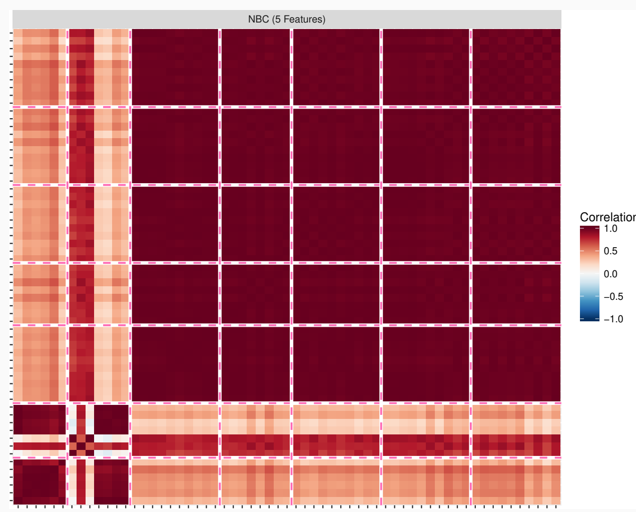
Exemplary Use Cases of ELA: Benchmark Comparison



Exemplary Use Cases of ELA: Benchmark Comparison



Exemplary Use Cases of ELA: Benchmark Comparison

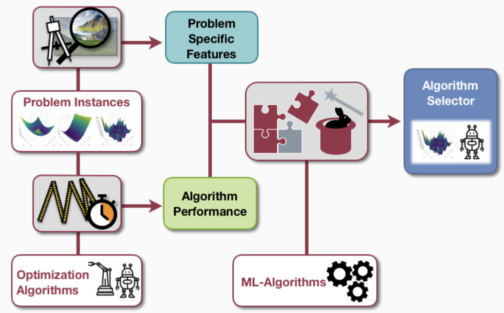


Exemplary Use Cases of ELA: ELA for Algorithm Selection

Example 3: **Using ELA for Algorithm Selection**

Exemplary Use Cases of ELA: ELA for Algorithm Selection

- *Algorithm Selection Problem*: find the individually best suited algorithm for an unseen optimization problem



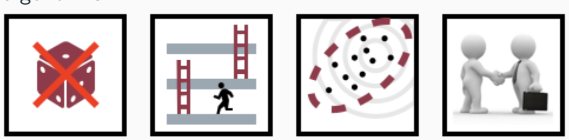
[36] Rice, J. R. (1976). *The Algorithm Selection Problem.* In Advances in Computers (pp. 65-118).

[17] Kerschke, P., Hoos, H. H., Neumann, F. & Trautmann, H. (2019). *Automated Algorithm Selection: Survey and Perspectives.* In Evolutionary Computation, Vol. 27, Number 1 (pp. 3-45).

Exemplary Use Cases of ELA: ELA for Algorithm Selection

Experimental setup - Part 1:

- COCO[4]: platform storing the performances of (129) optimization algorithms



$\rightsquigarrow$ considered 12 solvers from COCO

- 2x deterministic: BSrr, BSqi
- 5x multi-level approaches: MLSL, fmincon, fminunc, HMLSL, MCS
- 4x CMA-ES variants: CMA-CSA, IPOP400D, HCMA, SMAC-BBOB
- 1x commercial solver: OQNLP

4. [9] Hansen, N., Auger, A., Mersmann, O., Tušar, T. & Brockhoff, D. (2016). *COCO: A Platform for Comparing Continuous Optimizers in a Black-Box Setting.* ArXiv e-print arXiv:1603.08785. Link to COCO: `http://coco.gforge.inria.fr/`

Exemplary Use Cases of ELA: ELA for Algorithm Selection

Experimental setup - Part 2:

- all 24 BBOB problems
- problem dimensionality: $d \in \{2, 3, 5, 10\}$
- accuracy threshold: $\tau = 10^{-2}$
- performance measure: relative ERT (per problem)
- computed ca. 100 ELA features per problem based on initial designs of $50 \times d$ observations
- performed automated feature selection
- tried different machine learning algorithms

Exemplary Use Cases of ELA: ELA for Algorithm Selection

Results - Part 1:

- Single-Best Solver from Portfolio: HCMA (relERT $\approx$ 30.4)

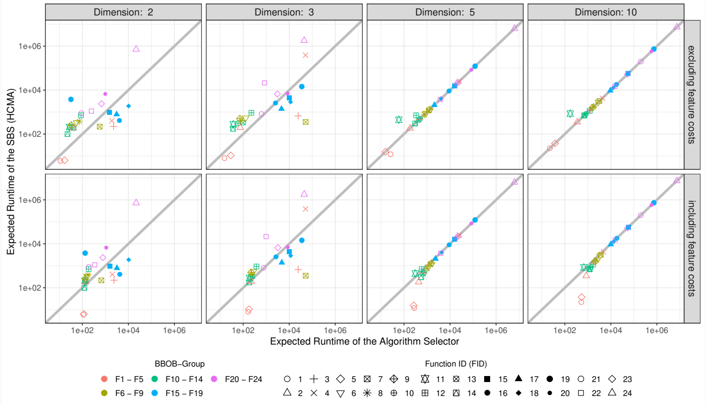
  $\rightsquigarrow$ on average 30x number of function evaluations (in relation to best possible solver per problem)
- Best Algorithm Selector: classification-based SVM (relERT $\approx$ 14.2)

  $\rightsquigarrow$ less than half of the number of function evaluations of HCMA
- always predicts either fmincon, HCMA, HMLSL or MLSL
- nine features employed by selector:
  - 1 (cell mapping) angle,
  - 1 levelset,
  - 1 y-distribution,
  - 2 meta-model and
  - 4 NBC features

Results - Part 2:

- detailed results can be found here:

  [23] Kerschke, P. & Trautmann, H. (2018). *Automated Algorithm Selection on Continuous Black-Box Problems By Combining Exploratory Landscape Analysis and Machine Learning.* In Evolutionary Computation, Vol. 27, Number 1 (pp. 99 - 127)



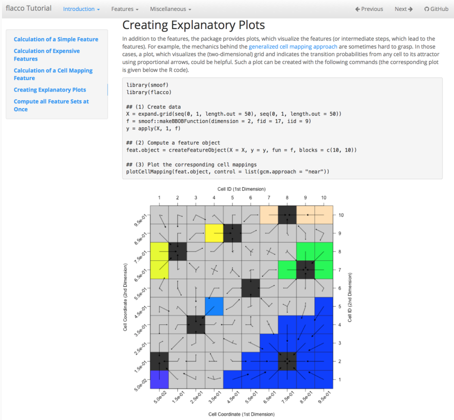
FLACCO + GUI

FLACCO + GUI

- `flacco`: **F**eature-Based **L**andscape **A**nalysis of **C**ontinuous and **C**onstraint **O**ptimization Problems
- unified interface for multiple (single-objective) sets of configurable features
- stable release on CRAN / developers version on GitHub
- multiple vizualisation techniques (partially shown on these slides)
- tracks # of function evaluations <u>and</u> run time - per feature set
- a comprehensive description of FLACCO can be found here[5,6]

5. [22] Kerschke, P. & Trautmann, H. (2016). *The R-Package FLACCO for Exploratory Landscape Analysis with Applications to Multi-Objective Optimization Problems.* In Proceedings of CEC 2016.
6. [14] Kerschke, P. (2017). *Comprehensive Feature-Based Landscape Analysis of Continuous and Constrained Optimization Problems Using the R-Package flacco.* In arXiv 1708.05258 URL: `https://arxiv.org/abs/1708.05258`.

FLACCO + GUI



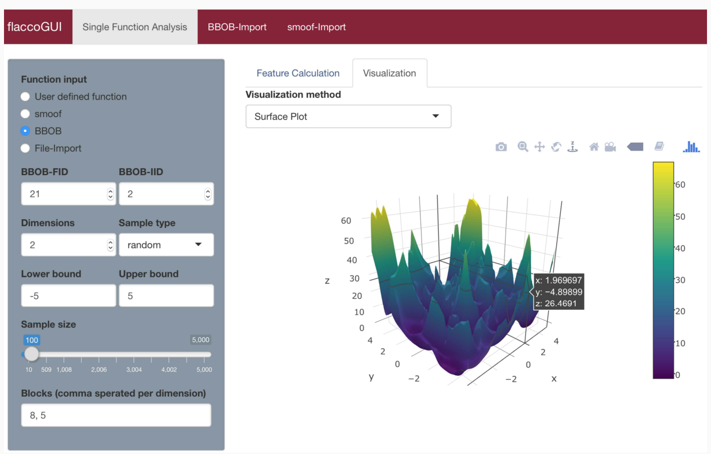
7. Tutorial: `http://kerschke.github.io/flacco-tutorial/site/`

FLACCO + GUI

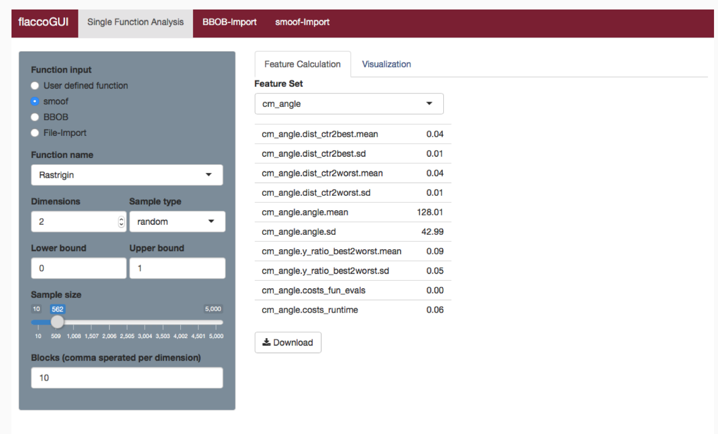
- drawback of `flacco`:
  - it is an R-package ⇝ only people, who are familiar with (programming in) R benefit of it
- solution:
  - user-friendly GUI[8] (graphical user interface)
  - platform independent (web-)application: `https://flacco.shinyapps.io/flacco/`
  - the GUI helps people, who
    - (a) are familiar with R, but don't want to bother with the coding
    - (b) are not familiar with R (and just want to perform ELA)
    - (c) don't have access to a computer
    - (d) have access to a computer, but don't have the rights to install R

8. [10] Hanster, C. & Kerschke, P. (2017). *flaccogui: Exploratory Landscape Analysis for Everyone.* In Proceedings of GECCO 2017 Companion (pp. 1215 – 1222).

FLACCO + GUI

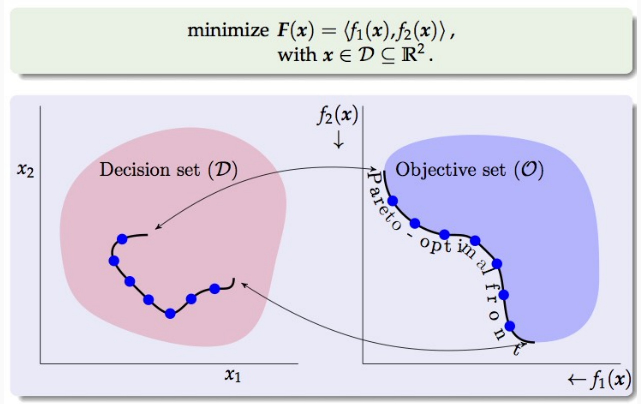


FLACCO + GUI



(ELA for) Multi-Objective Optimization

Multi-Objective Optimization



source: `lmarti.github.io`

Multi-Objective Optimization

- in single-objective optimization, ELA has shown to be useful for describing the problem landscape based on a small initial design
- currently, there exist almost no landscape features for continuous multi-objective optimization problems
- first approaches for using ELA in the multi-objective setting

Multi-Objective Optimization

- `flacco` originally intended to deal with single-objective optimization problems
- features can also be used to characterize multi-objective problems
- we used DTLZ-[9] and ZDT-problems[10] (using the R-package `smoof`[11])

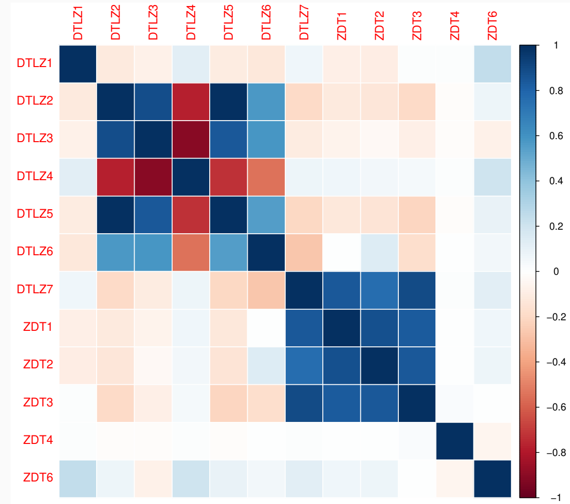
9. [6] Deb, K., Thiele, L., Laumanns, M. & Zitzler, E. (2005). Scalable Test Problems for Evolutionary Multiobjective Optimization. In Evolutionary Multiobjective Optimization (pp. 105 - 145)
10. [44] Zitzler, E., Deb, K. & Thiele (2000). Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. In Evolutionary Computation, Vol. 8, Number 2 (pp. 173 - 195)
11. [4] Bossek, J. (2017). smoof: Single- and Multi-Objective Optimization Test Functions. In The R Journal. `https://CRAN.R-project.org/package=smoof`

Multi-Objective Optimization

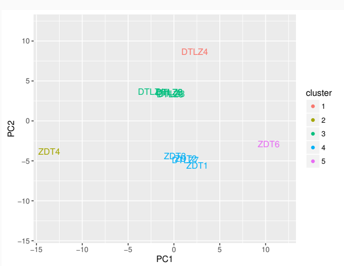
Experimental Setup:

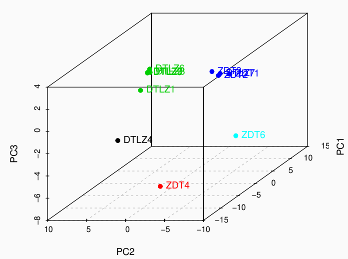
- DTLZ1 to DTLZ7 and ZDT1 to ZDT6 (without ZDT5)
  $\rightsquigarrow$ 120 instances (12 functions with 10 replicates each)
- initial designs: $100 \times d$ samples with $d = 3$
- considered 131 artificially designed "interaction-features":
  - all 15 feature sets except for GCM and Barrier Trees
    $\rightsquigarrow$ 682 features (341 per objective)
  - aggregated by feature-ratio (objective 1 / objective 2)
  - removed runtimes, as well as all features that contained infinite or non-defined values

Multi-Objective Optimization



Multi-Objective Optimization





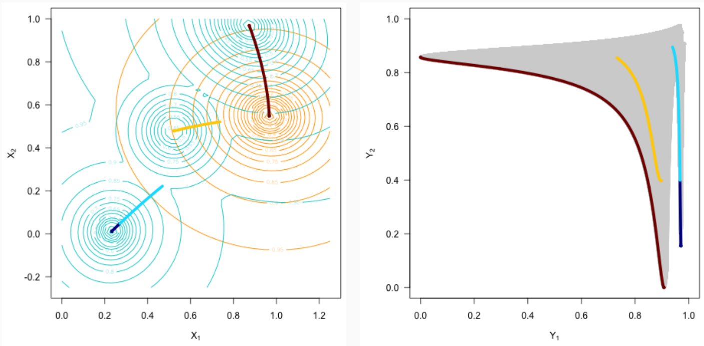
Multi-Objective Optimization

- in single-objective optimization:
  people visualize interaction effects of decision and objective space simultaneously $\rightsquigarrow$ line plots, heatmaps, 3D plots, etc.
- in multi-objective optimization:
  visualizing $d \geq 2$ decision variables and $p \geq 2$ objective values (within a single image) is much more complicated
- effect: researchers mainly only focus on the objective space, but neglect the decision space[12]
- conflict: optimization algorithms usually "act" in the decision space (e.g., mutation / recombination within an EA)

12. Exception: Cost landscapes based on Pareto-ranking as defined by Carlos Fonseca.
[5] Fonseca, C. M. M. (1995). *Multi Objective Genetic Algorithms with Application to Control Engineering Problems.* PhD Thesis at the Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, UK.

Multi-Objective Optimization

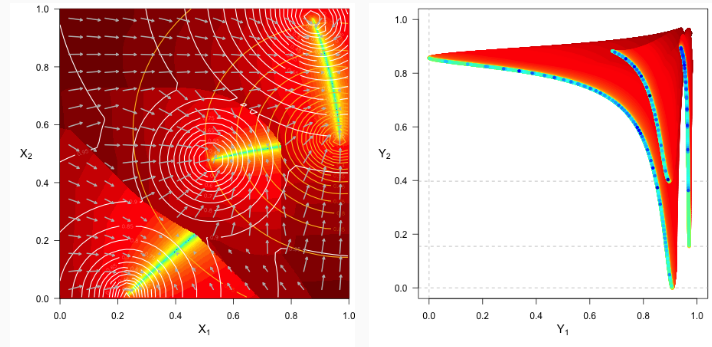
- idea introduced within our paper[13] from the previous PPSN: visualize efficient sets, i.e., the set of points from the decision space whose images are multi-objective local or global optima



13. [24] Kerschke, P., Wang, H., Preuss, M., Grimme, C., Deutz, A., Trautmann, H. & Emmerich, M. (2016). *Towards Analyzing Multimodality of Multiobjective Landscapes.* In Proceedings of PPSN XIV (pp. 962 – 972).

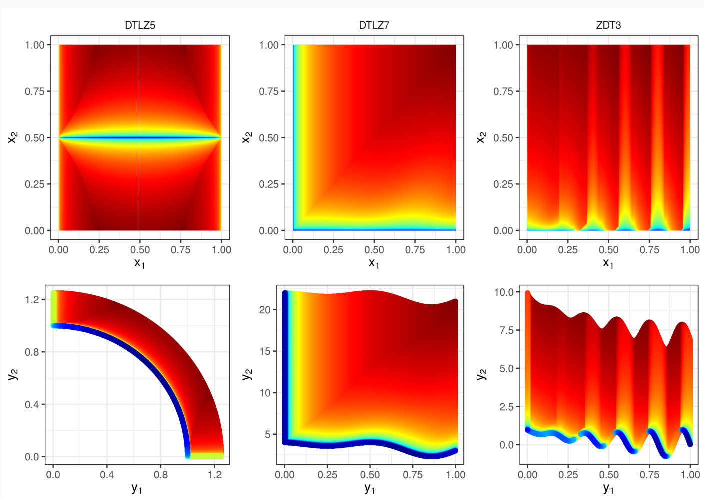
Multi-Objective Optimization

- extended to a visualization of the multi-objective basins of attraction[14]

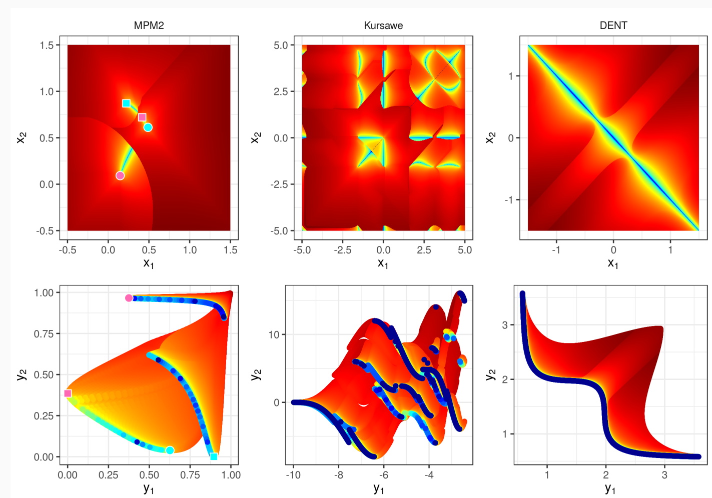


14. [16] Kerschke, P. & Grimme, C. (2017). *An Expedition to Multimodal Multi-Objective Optimization Landscapes.* In Proceedings of EMO 2017 (pp. 329 – 343).

Multi-Objective Optimization



Multi-Objective Optimization



Multi-Objective Optimization

Next Steps w.r.t. ELA for Multi-Objective Optimization:

- analyze numerous multi-objective benchmark problems visually and detect meaningful properties of the landscapes
- develop (simple and) automatically computable features, which *might* capture these properties
- conduct experimental studies to test their applicability
- employ the insights gained from the joined visualization of decision and objective space to either
  (a) construct better performing algorithms, or
  (b) use the derived features for training a suitable algorithm selector

Open Issues

Open Issues

- how can we characterize multimodal and/or multi-objective landscapes? ⇒ develop new landscape features
- enhance `flacco` with more ELA features
- how can we find the smallest most informative feature set?
- by how much can we still reduce the size of the initial designs without losing (too much) information?

Open Issues

- (how) can we transfer features from / to different domains? (e.g., funnels also exist in discrete optimization)
- use features to learn more about the algorithms and problems
  1. train well-performing algorithm selection or configuration models based on ELA features
  2. interpret the model's behavior based on the employed features (black-box to white-box)

Open Issues

- how should we extend the existing / established benchmarks?
- GECCO 2019 offers numerous related workshops:
  - Understanding Machine Learning Optimization Problems (UMLOP)
  - Black Box Optimization Benchmarking (BBOB)
  - Game-Benchmark for Evolutionary Algorithms (GBEA)
  - Black Box Discrete Optimization Benchmarking (BB-DOB)
- directions for possible extensions:
  - machine learning problems [35]
  - landscapes of (hyper-)parameter optimization problems [7]
  - problems from domains such as computational games [41]

Thank you!

Comments, Questions and/or Suggestions?

References

[1] Abell, T., Malitsky, Y., and Tierney, K. Features for Exploiting Black-Box Optimization Problem Structure. In *Learning and Intelligent Optimization* (2013), Springer, pp. 30–36.

[2] Belkhir, N., Dréo, J., Savéant, P., and Schoenauer, M. Feature Based Algorithm Configuration: A Case Study with Differential Evolution. In *Proceedings of the 14th International Conference on Parallel Problem Solving from Nature (PPSN XIV)* (September 2016), J. Handl, E. Hart, P. R. Lewis, M. López-Ibáñez, G. Ochoa, and B. Paechter, Eds., vol. 9921 of *Lecture Notes in Computer Science (LNCS)*, Springer, pp. 156 – 166.

[3] Belkhir, N., Dréo, J., Savéant, P., and Schoenauer, M. Per Instance Algorithm Configuration of CMA-ES with Limited Budget. In *Proceedings of the 19th Annual Conference on Genetic and Evolutionary Computation (GECCO)* (July 2017), ACM, pp. 681 – 688.

[4] Bossek, J. smoof: Single- and Multi-Objective Optimization Test Functions. *The R Journal* (2017).

[5] da Fonseca, C. M. M. *Multiobjective Genetic Algorithms with Application to Control Engineering Problems.* PhD Thesis, Department of Automatic Control and Systems Engineering, University of Sheffield, September 1995.

[6] Deb, K., Thiele, L., Laumanns, M., and Zitzler, E. Scalable Test Problems for Evolutionary Multiobjective Optimization. In *Evolutionary Multiobjective Optimization*, A. Abraham, L. Jain, and R. Goldberg, Eds., Advanced Information and Knowledge Processing (AI & KP). Springer, 2005, pp. 105 – 145.

[7] Doerr, C., Dreo, J., and Kerschke, P. Making a Case for (Hyper-)Parameter Tuning as Benchmark Problems. In *Proceedings of the 21st Annual Conference on Genetic and Evolutionary Computation (GECCO) Companion* (2019), ACM. to appear.

[8] Flamm, C., Hofacker, I. L., Stadler, P. F., and Wolfinger, M. T. Barrier Trees of Degenerate Landscapes. *Zeitschrift für Physikalische Chemie. International Journal of Research in Physical Chemistry and Chemical Physics 216*, 2/2002 (2002), 155 – 173.

[9] Hansen, N., Auger, A., Mersmann, O., Tušar, T., and Brockhoff, D. COCO: A Platform for Comparing Continuous Optimizers in a Black-Box Setting. *arXiv preprint abs/1603.08785v3* (August 2016).

[10] Hanster, C., and Kerschke, P. flaccogui: Exploratory Landscape Analysis for Everyone. In *Proceedings of the 19th Annual Conference on Genetic and Evolutionary Computation (GECCO) Companion* (July 2017), ACM, pp. 1215 – 1222.

[11] Hernández, C., Schütze, O., Emmerich, M., Xiong, F.-R., and Sun, J.-Q. Barrier Tree forContinuous Landscapes by Means of Generalized Cell Mapping. In *EVOLVE - A Bridge between Probability, Set Oriented Numerics, and Evolutionary Computation V*. Springer, 2014.

[12] Jones, T., and Forrest, S. Fitness Distance Correlation as a Measure of Problem Difficulty for Genetic Algorithms. In *Proceedings of the 6th International Conference on Genetic Algorithms (ICGA)* (1995), Morgan Kaufmann Publishers Inc., pp. 184 – 192.

[13] Kauffman, S. A. *The Origins of Order: Self-Organization and Selection in Evolution*. Oxford University Press, 1993.

[14] Kerschke, P. Comprehensive Feature-Based Landscape Analysis of Continuous and Constrained Optimization Problems Using the R-Package flacco. *arXiv preprint abs/1708.05258* (August 2017).

[15] Kerschke, P. *flacco: Feature-Based Landscape Analysis of Continuous and Constrained Optimization Problems*, 2017. R-package version 1.7.

[16] Kerschke, P., and Grimme, C. An Expedition to Multimodal Multi-Objective Optimization Landscapes. In *Proceedings of the 9th International Conference on Evolutionary Multi-Criterion Optimization (EMO)* (March 2017), H. Trautmann, G. Rudolph, K. Kathrin, O. Schütze, M. Wiecek, Y. Jin, and C. Grimme, Eds., Springer, pp. 329 – 343.

[17] Kerschke, P., Hoos, H. H., Neumann, F., and Trautmann, H. Automated Algorithm Selection: Survey and Perspectives. *Evolutionary Computation 27*, 1 (2019), 3 – 45.

[18] Kerschke, P., and Preuss, M. Exploratory Landscape Analysis: Advanced Tutorial at GECCO 2017. In *Proceedings of the 19th Annual Conference on Genetic and Evolutionary Computation (GECCO) Companion* (2017), pp. 762 – 781.

[19] Kerschke, P., Preuss, M., Hernández Castellanos, C. I., Schütze, O., Sun, J.-Q., Grimme, C., Rudolph, G., Bischl, B., and Trautmann, H. Cell Mapping Techniques for Exploratory Landscape Analysis. In *EVOLVE – A Bridge between Probability, Set Oriented Numerics, and Evolutionary Computation V*. Springer, July 2014, pp. 115 – 131.

[20] Kerschke, P., Preuss, M., Wessing, S., and Trautmann, H. Detecting Funnel Structures by Means of Exploratory Landscape Analysis. In *Proceedings of the 17th Annual Conference on Genetic and Evolutionary Computation (GECCO)* (July 2015), ACM, pp. 265 – 272.

[21] Kerschke, P., Preuss, M., Wessing, S., and Trautmann, H. Low-Budget Exploratory Landscape Analysis on Multiple Peaks Models. In *Proceedings of the 18th Annual Conference on Genetic and Evolutionary Computation (GECCO)* (July 2016), ACM, pp. 229 – 236.

[22] Kerschke, P., and Trautmann, H. The R-Package FLACCO for Exploratory Landscape Analysis with Applications to Multi-Objective Optimization Problems. In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC)* (July 2016), IEEE, pp. 5262 – 5269.

[23] Kerschke, P., and Trautmann, H. Automated Algorithm Selection on Continuous Black-Box Problems By Combining Exploratory Landscape Analysis and Machine Learning. *Evolutionary Computation 27*, 1 (2019), 99 – 127.

[24] Kerschke, P., Wang, H., Preuss, M., Grimme, C., Deutz, A. H., Trautmann, H., and Emmerich, M. T. M. Towards Analyzing Multimodality of Multiobjective Landscapes. In *Proceedings of the 14th International Conference on Parallel Problem Solving from Nature (PPSN XIV)* (September 2016), J. Handl, E. Hart, P. R. Lewis, M. López-Ibáñez, G. Ochoa, and B. Paechter, Eds., vol. 9921 of *Lecture Notes in Computer Science (LNCS)*, Springer, pp. 962 – 972.

[25] Lunacek, M., and Whitley, L. D. The Dispersion Metric and the CMA Evolution Strategy. In *Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation (GECCO)* (2006), ACM, pp. 477 – 484.

[26] Malan, K. M., and Engelbrecht, A. P. Quantifying Ruggedness of Continuous Landscapes Using Entropy. In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC)* (2009), IEEE, pp. 1440 – 1447.

[27] Malan, K. M., and Engelbrecht, A. P. A Survey of Techniques for Characterising Fitness Landscapes and Some Possible Ways Forward. *Information Sciences (JIS) 241* (2013), 148 – 163.

[28] Mersmann, O., Bischl, B., Trautmann, H., Preuss, M., Weihs, C., and Rudolph, G. Exploratory Landscape Analysis. In *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation (GECCO)* (2011), ACM, pp. 829 – 836.

[29] Mersmann, O., Preuss, M., and Trautmann, H. Benchmarking Evolutionary Algorithms: Towards Exploratory Landscape Analysis. In *PPSN XI: Proceedings of the 11th International Conference on Parallel Problem Solving from Nature* (2010), Springer, pp. 71 – 80.

[30] Morgan, R., and Gallagher, M. Analysing and Characterising Optimization Problems Using Length Scale. *Soft Computing* (2015), 1 – 18.

[31] Muñoz Acosta, M. A., Kirley, M., and Halgamuge, S. K. Exploratory Landscape Analysis of Continuous Space Optimization Problems Using Information Content. *IEEE Transactions on Evolutionary Computation (TEVC) 19*, 1 (2015), 74 – 87.

[32] Muñoz Acosta, M. A., Sun, Y., Kirley, M., and Halgamuge, S. K. Algorithm Selection for Black-Box Continuous Optimization Problems: A Survey on Methods and Challenges. *Information Sciences (JIS) 317* (October 2015), 224 – 245.

[33] Naudts, B., Suys, D., and Verschoren, A. Epistasis as a Basic Concept in Formal Landscape Analysis. In *Proceedings of the 7th International Conference on Genetic Algorithms (ICGA)* (July 1997), T. H. W. Bäck, Ed., Citeseer, pp. 65 – 72.

[34] Pitzer, E., and Affenzeller, M. A Comprehensive Survey on Fitness Landscape Analysis. In *Recent Advances in Intelligent Engineering Systems*, J. Fodor, R. Klempous, and C. P. Suárez Araujo, Eds., Studies in Computational Intelligence. Springer, 2012, pp. 161 – 191.

[35] Rapin, J., Gallagher, M., Kerschke, P., Preuss, M., and Teytaud, O. Exploring the MLDA Benchmark on the Nevergrad Platform. In *Proceedings of the 21st Annual Conference on Genetic and Evolutionary Computation (GECCO'19) Companion* (2019), ACM. to appear.

[36] Rice, J. R. The Algorithm Selection Problem. *Advances in Computers 15* (1976), 65 – 118.

[37] Rochet, S., Venturini, G., Slimane, M., and El Kharoubi, E. A Critical and Empirical Study of Epistasis Measures for Predicting GA Performances: A Summary. In *European Conference on Artificial Evolution (AE)* (1997), vol. 1363 of *Lecture Notes in Computer Science (LNCS)*, Springer, pp. 275 – 285.

[38] Shirakawa, S., and Nagao, T. Bag of local landscape features for fitness landscape analysis. *Soft Computing 20*, 10 (2016), 3787–3802.

[39] Singh Saini, B., López-Ibáñez, M., and Miettinen, K. Automatic Surrogate Modelling Technique Selection based on Features of Optimization Problems. In *Proceedings of the 21st Annual Conference on Genetic and Evolutionary Computation (GECCO) Companion* (2019), ACM. to appear.

[40] Vassilev, V. K., Fogarty, T. C., and Miller, J. F. Information Characteristics and the Structure of Landscapes. *Evolutionary Computation (ECJ) 8*, 1 (March 2000), 31 – 60.

[41] Volz, V., Naujoks, B., Kerschke, P., and Tušar, T. Single- and Multi-Objective Game-Benchmark for Evolutionary Algorithms. In *Proceedings of the 21st Annual Conference on Genetic and Evolutionary Computation (GECCO)* (2019), ACM. to appear.

[42] Volz, V., Schrum, J., Liu, J., Lucas, S. M., Smith, A., and Risi, S. Evolving Mario Levels in the Latent Space of a Deep Convolutional Generative Adversarial Network. In *Proceedings of the 20th Annual Conference on Genetic and Evolutionary Computation (GECCO)* (2018), ACM, pp. 221 – 228.

[43] Wessing, S. *optproblems: Infrastructure to define optimization problems and some test problems for black-box optimization*, 2016. `Python`-package version 0.6.

[44] Zitzler, E., Deb, K., and Thiele, L. Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. *Evolutionary Computation (ECJ)*, 2 (June 2000), 173 – 195.