Solving the Parameterless Firefighter Problem using Multiobjective Evolutionary Algorithms

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ABSTRACT

The Firefighter Problem (FFP) is a graph-based optimization problem that is an abstraction of real-life problems such as epidemics control, economic crises prevention, etc. In the FFP spreading of fire is simulated on a graph in discrete time steps. In the original formulation of the problem a fixed number of graph nodes N_f can be defended in each time step. In this paper the problem is reformulated, and three different solution representations are studied. In one of the representations (N+P), the N_f parameter is a decision variable and in the other two (P using permutations and T using integer vectors) it is determined when the solution is decoded. Because higher N_f values mean more resources used for defense it is desirable to minimize this value, but on the other hand we want to minimize the number of graph nodes consumed by fire. Therefore the Parameterless FFP is tackled using two well-known multiobjective evolutionary algorithms: the MOEA/D and the NSGA-II as a multiobjective optimization problem with two and three objectives. The results presented in the paper show that for the Parameterless FFP the best solution representation is N+P.

CCS CONCEPTS

• **Mathematics of computing → Evolutionary algorithms**; **Combinatorial optimization**;

KEYWORDS

Graph-based optimization, Solution representation, Combinatorial optimization, MOEA/D, NSGA-II

ACM Reference Format:
Krzysztof Michalak. 2019. Solving the Parameterless Firefighter Problem using Multiobjective Evolutionary Algorithms. In *Genetic and Evolutionary Computation Conference Companion (GECCO '19 Companion), July 13–17, 2019, Prague, Czech Republic.* ACM, New York, NY, USA, 8 pages. https://doi.org/10.1145/3319619.3326812

1 INTRODUCTION

Many real-life problems can be described using a scenario in which a threat spreads among entities connected by relationships, forming a networked system. Epidemics, bankruptcies, computer viruses and fake news can be described using a similar formalism in which some entities are affected by a threat (e.g. a pathogen) and this threat subsequently spreads to other entities via certain contacts (e.g. meeting an infected person). Depending on the problem, the dynamic of the spreading of the threat can, of course, be different and different countermeasures can be used to prevent a particular threat from spreading.

The Firefighter Problem (FFP) is an optimization problem in which the spreading of fire is simulated on an undirected graph G with N_v nodes. The nodes of the graph can be in one of the states from the set $L = \{$ 'B', 'D', 'U' $\}$ which indicate, respectively: 'B' - burning, 'D' - defended and 'U' - untouched (neither burning nor defended). Each FFP instance is a triple $\langle G, S_0, N_f \rangle$, where $S_0 \in L^{N_v}$ is the initial graph state and N_f is the number of nodes that can be defended in each time step. The most common approach is to set the initial state S_0 in such a way that some nodes are initially burning ('B') and all the remaining nodes are untouched ('U'). Starting from S_0 the spreading of fire is simulated in discrete time steps. In each time step N_f untouched nodes become defended, so their state is changed from 'U' to 'D'. After the nodes to defend are selected, fire spreads from burning nodes to untouched ones along the edges of the graph. Nodes in the 'D' state are immune, that is fire cannot spread to, nor through, them, and they remain defended until the end of the simulation. Edges of the graph define connections between nodes and are not themselves subject to burning nor defense.

The definition of an optimization problem must include the definition of the search space and one or more objective functions. Most commonly, solutions of the FFP are represented as permutations and therefore the search space is $\Omega = \Pi_{N_v}$ - the set of all permutations of N_v elements. A solution $\pi \in \Pi_{N_v}$ is evaluated by simulating the spreading of fire, starting from the initial state S_0 and using the permutation π to determine the order in which the nodes should be protected. In each time step the first N_f elements of π are taken for which the corresponding nodes in the graph are in the 'U' state and these nodes are defended. The evaluation of the solution depends on the nodes that were protected from fire, both the defended ones ('D') and nodes which were not reached by fire ('U'). In the original version of the problem [15] the evaluation of the solution is equal to the number of nodes protected from fire. In some other papers variable node costs were used in a single-objective [26] and multiobjective [24] versions of the problem. Naturally, both the maximization version of the problem can be studied (maximize the number or total value of the saved nodes) as well as the minimization version (minimize the number or total value of the burnt nodes).

GECCO '19 Companion, July 13–17, 2019, Prague, Czech Republic

ACM ISBN 978-1-4503-6748-6/19/07…$15.00
https://doi.org/10.1145/3319619.3326812

The FFP has been to date a subject of numerous theoretical works focused on such aspects as the conditions under which it is possible to save the graph [11]. Numerous works focus on special cases which lend themselves to theoretical investigations, such as multidimensional grids [29], trees [8], digraphs [20], graphs with a limited degree [12], planar graphs [19] or cubic graphs [18]. Recently, numerous metaheuristic approaches to the FFP have been studied. The classical version of the FFP was solved using Ant Colony Optimization (ACO) method by Blum *et al.* [4] and using the Variable Neighbourhood Search (VNS) method by Hu [16] and coauthors. Also, Estimation of Distribution Algorithms (EDAs) with probabilistic models dedicated to the FFP have been studied in the literature [26]. Various metaheuristic approaches have been applied to the multiobjective FFP to date, such as multipopulation evolutionary algorithms [24] and dedicated local search methods [25]. Also, an extended variant of the FFP has been proposed in which two different threats spread simultaneously in the same graph [22]. Apart from the most often studied deterministic variant of the problem a non-deterministic version has been studied [28], to which a simheuristic approach [17] has been applied combining metaheuristic optimization with extensive simulations.

In all the works discussed above the number of nodes that can be defended in each time step N_f is treated as a parameter of a problem instance, which is set to a fixed value before the algorithm starts solving this problem instance. In this paper a different approach is considered which is to allow the algorithm to find solutions for many different values of N_f in the same run. The contributions of this paper are threefold:

(1) The Parameterless Firefighter Problem is formulated in which the number of firefighters N_f assigned per a time step is not predetermined. In this problem formulation the optimization algorithm has to find good trade-offs between the value of the N_f parameter and the number of finally burnt nodes.
(2) Three different solution representations are proposed in which the parameter N_f is encoded either explicitly as a decision variable or implicitly (in which case its value is determined when the solution is decoded).
(3) The effectiveness of various genetic operators is tested for the proposed solution representations.

The rest of this paper is organized as follows. Section 2 describes the parameterless version of the FFP and solution representations proposed in this paper. Section 3 presents the experiments in which the performance of multiobjective evolutionary algorithms using different solution representations and genetic operators was compared. Section 4 concludes the paper.

2 THE PARAMETERLESS FIREFIGHTER PROBLEM

In the original Firefighter Problem formulation [15] the number of graph nodes that can be protected in each time step is set to a constant number N_f. This parameter represents a limitation of available resources that can be used for protecting the graph. The value of N_f can have a very strong impact on the number of nodes that can be saved from fire. For example, if the limit N_f is high enough to cut off the nodes burning in the initial state S_0 from the rest of the graph, it is easy to save many nodes in the graph. Conversely, for a fixed N_f, the difficulty of problem instances strongly depends on the graph structure. Figure 1 shows the number of test instances for which a given final number of burnt nodes was obtained depending on the number of firefighters N_f for optimized solutions of the FFP with $N_v = 1000$ nodes.

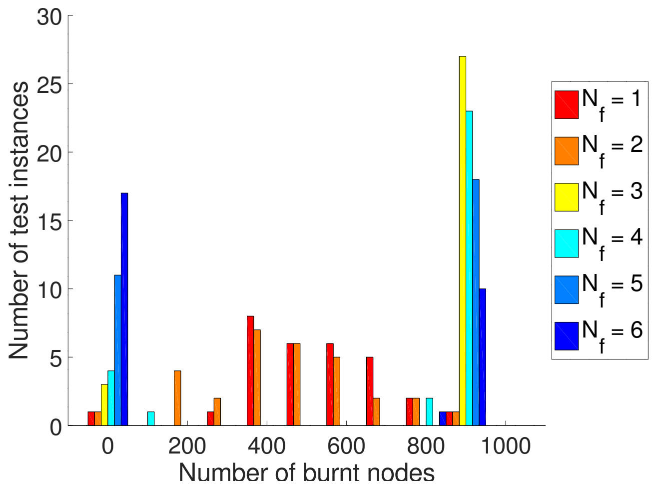

Figure 1: The number of test instances for which a given final number of burnt nodes was obtained depending on the number of firefighters N_f.

For $N_f \geq 3$ it is clearly visible that for some instances it is very easy to contain fire (very few nodes are burnt) and for some it is impossible (almost all nodes are burnt) with no instances for which a medium result was obtained. Because of this effect when generating tests instances for the FFP it is difficult to obtain instances that allow testing the algorithms adequately. Some of the instances are very easy and they do not pose a real challenge for the optimizers and some are very hard and all the algorithms perform equally poorly.

2.1 Parameterless FFP as a multiobjective optimization problem

As noted above, the number of graph nodes that can be protected in each time step N_f can have a very strong impact on the difficulty of problem instances. Conversely, problem instances with the same number of graph nodes N_v and the N_f parameter value can be of very different difficulty depending on the graph structure (cf. Figure 1). From this observation a motivation arises to study a parameterless version of the FFP for which optimization algorithms have to find good trade-offs between the number of firefighters N_f and the number of finally burnt nodes. The Parameterless FFP can thus be formulated as a multiobjective optimization problem. The objective f_1 is the number of burnt nodes and is to be minimized. The objective f_2 is the maximum number of firefighters N_f that were assigned to nodes in one time step (thereby changing the state of these nodes from 'U' to 'D'). The f_2 objective is the equivalent of the N_f parameter in the classical Firefighter Problem, in that it represents the value of N_f for which a given solution would attain the objective f_1 in the classical FFP. The f_3 objective is the total number of defended nodes in the entire simulation. Both objectives f_2 and f_3 represent different kinds of resources needed to prevent the threat from spreading. The f_2 objective is "the number of firefighters" and can indeed represent the number of people (or teams) involved in threat containment. Apart from firefighters these can be medical personnel necessary for vaccinations or brigades that reinforce floodbanks to prevent a flood. After the work is done in one location they move to another place, and hence the resource limitation is expressed as the maximum number of graph nodes that become (and later stay) defended per a time step. The f_3 objective represents the total cost of countermeasures applied in the entire simulation. This can be the cost of vaccine doses or the cost of material used to reinforce floodbanks. Obviously, we want to minimize both f_2 and f_3. In this paper, two multiobjective variants of the Parameterless FFP are studied: a bi-objective one with objectives f_1 and f_2 and a three-objective one with objectives f_1, f_2 and f_3.

2.2 Solution representations

In this paper three different solution representations are studied.

Permutation (P) in which solutions are permutations from the set Π_{N_v} of all permutations of N_v elements. This representation takes advantage of the fact that permutation-based representation is redundant when used for the classical FFP. For example if S_0 = ['B', 'U', 'U', 'U', 'U'] and N_f = 2, solutions 12345, 13245, 21345, 23145, 31245 and 32145 are all equivalent, because node 1 is burning and therefore in the first time step nodes 2 and 3 will become defended, regardless of the permutation of the numbers 1, 2 and 3. In the representation used in this paper the number of nodes defended in each time step is implicitly encoded in the permutation π in the following manner. Nodes to be defended in time step 1 are those with the numbers placed at the beginning of the permutation that are in the state 'U'. The list of nodes to be defended in time step 1 ends when nodes burning in the state S_0 are found in π. After that, nodes to be defended in time step 2 are found, after which nodes burning in time step 1 follow (see Figure 2). The P representation proposed in this paper is less redundant than the one using permutations in conjunction with the N_f parameter for the classical FFP. For example, the six solutions mentioned above are all equivalent in the classical FFP (if S_0 = ['B', 'U', 'U', 'U', 'U'] and N_f = 2), but when decoded using the P representation proposed here, they result in different nodes protected in time step 1. For the solutions 12345 and 13245 no nodes are defended in the first time step, because no non-burning nodes are placed before the element equal 1, which is burning in the initial state S_0. For the solution 21345 node 2 is defended in time step 1 and for the solution 31245 node 3 is defended in time step 1. Both solutions 23145 and 32145 result in nodes 2 and 3 being defended in time step 1. It is worth noticing that the P representation allows the number of defended nodes to be different in each time step. For example in Figure 2 two nodes are defended in time step 1 and three in time step 2.

Number of firefighters + Permutation (N+P) in which a permutation is used for determining the order in which nodes are defended in the graph and one integer is used for determining the number of firefighters (the number of nodes that become defended

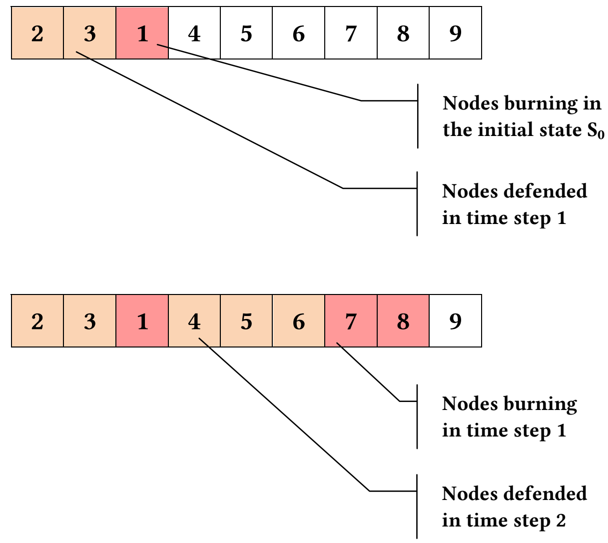

Figure 2: Permutation-based (P) representation.

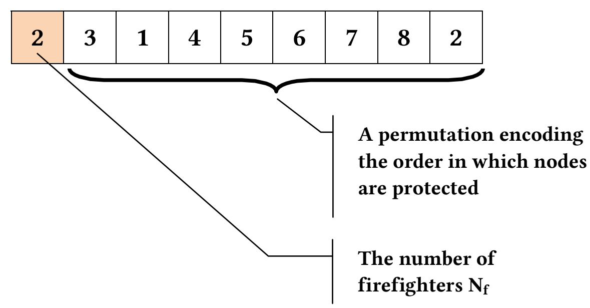

Figure 3: Number of firefighters + Permutation (N+P) representation.

per a time step) N_f. These two elements are combined in the genotype (Figure 3). In this representation the number of nodes defended in each time step is the same, equal N_f. The permutation part of the genotype in the N+P representation is interpreted in the same way as in the classical Firefighter Problem. At the beginning of each time step the first N_f elements which correspond to nodes in the 'U' state are selected from the permutation. The nodes of the graph G corresponding to the selected elements of the permutation are defended by changing their state to 'D'.

Time step (T) in which the genotype is an array of positive integers. Each entry in the genotype represents the number of the time step in which a given node is to be defended (Figure 4). Similarly as the N+P representation the T representation allows the number of defended nodes to be different in each time step. For example in Figure 4 three nodes are defended in time step 1, two nodes are defended in time step 2, and four nodes are defended in time step 3.

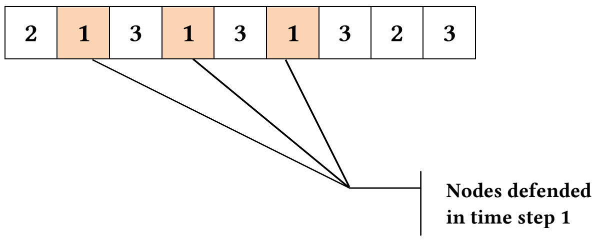

Figure 4: Time step (T) representation.

2.3 Genetic operators

In this paper three solution representations are proposed for the Parameterless Firefighter Problem. In order for evolutionary algorithms to work with these representations, crossover and mutation operators are necessary. In this paper the following operators were used.

For the **Permutation (P)** representation ten crossover operators were used: Cycle Crossover (CX) [31], Linear Order Crossover (LOX) [10], Merging Crossover (MOX) [30], Non-Wrapping Order Crossover (NWOX) [6], Order Based Crossover (OBX) [32], Order Crossover (OX) [13], Position Based Crossover (PBX) [32], Partially Mapped Crossover (PMX) [14], Precedence Preservative Crossover (PPX) [2, 3] and Uniform Partially Mapped Crossover (UPMX) [7]. Also, five mutation operators were used: Displacement, Insertion, Inversion, Scramble and Transpose.

For the permutation part of the **Number of firefighters + Permutation (N+P)** representation the same ten crossover and five mutation operators were used as for the Permutation (P) representation. The N_f parameter is represented as one integer in the genotype. This integer was selected randomly from one of the parents in the crossover operator and was mutated using uniform mutation.

For the **Time step (T)** representation single point, two point and uniform crossover operators were used. Also, two operators dedicated to this representation were tested, named MinCrossover and MaxCrossover which produced one offspring from the two parents by taking, respectively, a minimum or maximum at each position from each of the two parent genotypes. For mutation the Displacement, Insertion, Inversion, Scramble, Transpose and uniform mutation operators were used.

In the parameter tuning phase of the experiments these operators were tested individually and with an autoadaptation mechanism selecting operators with probabilities based on success rates of these operators [24].

3 EXPERIMENTS

The experiments presented in this paper were aimed at three goals:

- determining the best operators and algorithm parameterization for each solution representation,
- finding out which solution representation is the best, overall,
- comparing different multiobjective evolutionary algorithms.

In the experiments two algorithms were used: the MOEA/D [21, 34] and the NSGA-II [9], both with each of the three solution representations described in Section 2. These two algorithms were selected because they use two different approaches to multiobjective optimization. The NSGA-II is a Pareto-based algorithm in which solutions are selected according to Pareto dominance, and the MOEA/D is a decomposition-based algorithm in which solutions are selected using scalarized values of the objectives. Therefore, it is interesting to check if the choice of the optimization algorithm has any influence on which representation and genetic operators are the best. Also, these two algorithms are very often used in various applications, with the number of citations of the original papers exceeding 26 000 (the NSGA-II, [9]) and 4 000 (the MOEA/D, [21] and [34] combined) according to Google Scholar. The following sections describe the test instances, the parameter tuning method and the comparison of solution representations and algorithms.

3.1 Test instances

Test instances used in the experiments were based on REDS graphs [1]. This graph model was proposed in order to obtain spatial edge distribution resembling the distribution in a real-life social network. In REDS graphs denser cliques are separated by relatively sparse areas. Nodes of an REDS graph are placed randomly, with a uniform probability on the unit square $[0, 1] \times [0, 1]$. Three parameters are used to control the generation of edges: R - the maximum distance between connected vertices, E - the social energy, and S - the synergy parameter.

Each node in the graph is initially given an energy budget equal to E. Pairs of nodes separated by a distance not larger than R are randomly selected and for each selected pair $\langle v_i, v_j \rangle$ an attempt is made to generate an edge connecting these nodes. The base cost of an edge is equal to its length $D_{ij} = d(v_i, v_j)$, but if nodes v_i and v_j have k_{ij} neighbours in common, the cost of the edge is discounted by the factor of $\frac{1}{1+Sk_{ij}}$, where S is the synergy parameter. Also, the addition of the new edge between v_i and v_j may discount costs of other edges, because v_i can become a new common neighbour for v_j and one of its adjacent nodes or v_j can become a new common neighbour for v_i and one of its adjacent nodes. Thus, edge costs have to be recalculated for all edges connected to v_i and v_j. If these recalculated costs (including the cost of the newly created edge) do not exceed E for neither v_i nor v_j, the new edge is created.

In the experiments graphs with $N_v = 1000, \ldots, 10000$ were used. Table 1 lists the parameters used for generating the graphs: R, E and S. Apart from the graph parameters the time limit t_{max} for each instance is given, starting from 30 seconds for graphs with $N_v = 1000$ nodes and scaling linearly with the graph size.

Figure 5 shows an example of an REDS graph with $N_v = 5000$ nodes generated using the parameters shown in Table 1.

3.2 Operator selection and parameter tuning

The parameters of the algorithms were tuned by running both algorithms using each of the solution representations and with various settings of the parameters. Parameter tuning runs were performed on 30 Parameterless FFP instances with $N_v = 1000$ nodes with a running time limit of $t_{max} = 30$ seconds. Note, that in order to avoid overfitting, the parameter tuning was performed on different 30 instances than the ones used in Section 3.3 for comparing solution representations and algorithms. The performance of the algorithms was measured using the hypervolume (HV) indicator

Table 1: Parameters of graphs on which test instances were based and time limits used in the experiments.

N_v	R	E	S	t_{max}
1000	0.1000	0.15	0.5000	30.0
1250	0.0890	0.15	0.4470	37.5
1500	0.0820	0.15	0.4080	45.0
1750	0.0760	0.15	0.3780	52.5
2000	0.0700	0.15	0.3500	60.0
2250	0.0670	0.15	0.3330	67.5
2500	0.0630	0.15	0.3160	75.0
5000	0.0447	0.15	0.2236	150.0
10000	0.0316	0.15	0.1581	300.0

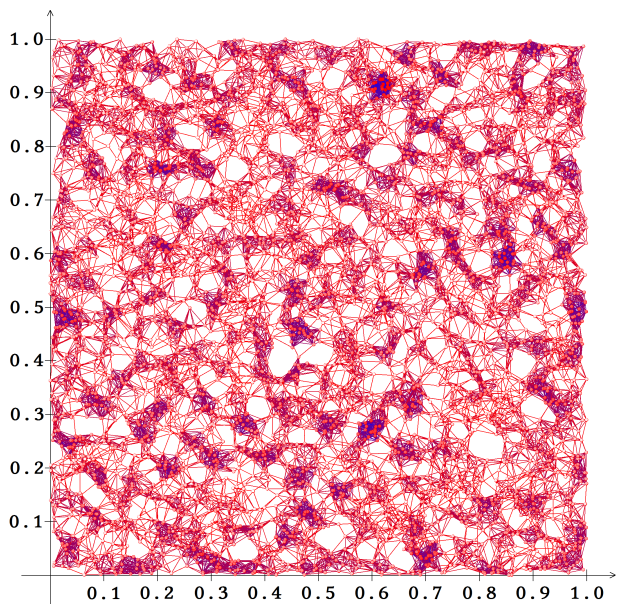

Figure 5: An example of an REDS graph with N_v = 5000 nodes generated using the parameters shown in Table 1.

[35] calculated for the set of nondominated solutions produced in each of the runs. From these 30 runs a median value was calculated.

Because of the large number of parameters the parameter tuning was split into three rounds in which, respectively, parameters of the evolutionary algorithms, crossover operators and mutation were adjusted. For both the MOEA/D and the NSGA-II the population size N_{pop} can be adjusted. The MOEA/D algorithm is also parameterized by T - the neighbourhood size, δ - the probability of selecting parents from the neighbourhood, n_r - the maximum number of neighbours replaced by a newly generated solution. In the first tuning round the parameters of the algorithms (N_{pop} for NSGA-II and N_{pop}, T, δ and n_r for the MOEA/D) were tuned using the grid search approach. The sets of tested values were $N_{pop} \in \{50, 100, 200, 500\}$, $\delta \in \{0.6, 0.7, 0.8, 0.9, 1.0\}$ and $n_r \in \{2, 4, 6, 8, 10\}$. Following the findings of the paper [23] the neighbourhood size was set to odd values $T \in \{13, 17, 21, 25, 29\}$ in order to avoid an asymmetric selective pressure.

After finding the best setting for population size and (in the case of the MOEA/D) other parameters, the selection of crossover operator was performed. The algorithms were run with each of the crossover operators available for a given solution representation and with a setting (denoted 'All') for which an autoadaptation mechanism [24] was used. The probability of crossover was changed in the range $P_{cross} \in \{0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. Similarly, mutation operators were selected and mutation probability was selected from the range of values $P_{mut} \in \{0.02, 0.04, 0.06, 0.08, 0.10\}$.

The results of parameter tuning for the bi-objective problem are summarized in Table 2 which presents, for each solution representation and each algorithm, the best population size N_{pop} and (in the case of the MOEA/D) other parameters, the best crossover and mutation operators and their probabilities.

A similar summary for the three-objective problem is presented in Table 3. Note, that the MOEA/D requires the population size for a three-objective problem to be a triangular number, because of the way in which weight vectors used for problem decomposition are arranged. For this reason in this round of tests slightly larger populations were used with $N_{pop} \in \{55, 105, 210, 528\}$.

From the operator selection and parameter tuning phase of the experiments some conclusions can be drawn regarding the best crossover operators for each of the representations. For the N+P (Number of firefighters + Permutation) representation the best crossover is the Position Based Crossover (PBX) operator [32]. For the T (Time step) representation the best crossover is the MinCross operator proposed in this paper. For the P (Permutation) representation in three out of four cases the LOX operator performed best, however, when the NSGA-II algorithm was used for the bi-objective problem, the autoadaptation mechanism ('All') produced the best results. There is no mutation operator that performed best in all cases, however the two best choices were to use the autoadaptation mechanism with all the mutation operators ('All') or the Scramble mutation alone.

3.3 Comparison of solution representations and evolutionary algorithms

This phase of experiments was aimed at finding the best solution representation and evolutionary algorithm for solving the Parameterless Firefighter Problem. In the experiments each of the two algorithms (the MOEA/D and the NSGA-II) was used with each of the three solution representations (Permutation (P), Number of firefighters + Permutation (N+P) and Time step (T)).

The parameters for each of these six cases were set to the best parameters obtained in Section 3.2 and the best set of genetic operators was used. Each solution representation and algorithm pair was tested on 30 instances of the Parameterless Firefighter Problem for the number of vertices $N_v = 1000, \ldots, 10000$. The maximum running time given in Table 1 was used as the stopping condition. For each of the 30 sets of nondominated solutions produced by a given algorithm with one of solution representations the value of the hypervolume indicator was calculated and the median from 30 runs was calculated.

Table 2: The best algorithm parameters, crossover and mutation operators and crossover and mutation probabilities for the bi-objective Parameterless FFP.

Representation	Algorithm	Parameters				Crossover	P_{cross}	Mutation	P_{mut}
Permutation (P)	MOEA/D	N_{pop} = 50	T = 29	δ = 0.7	n_r = 6	LOX	1.0	Scramble	0.02
	NSGA-II	N_{pop} = 200				All	1.0	All	0.08
Number of firefighters + Permutation (N+P)	MOEA/D	N_{pop} = 100	T = 29	δ = 1.0	n_r = 2	PBX	1.0	All	0.10
	NSGA-II	N_{pop} = 200				PBX	1.0	All	0.08
Time step (T)	MOEA/D	N_{pop} = 500	T = 29	δ = 0.8	n_r = 2	MinCross	0.5	Scramble	0.10
	NSGA-II	N_{pop} = 500				MinCross	1.0	Scramble	0.06

Table 3: The best algorithm parameters, crossover and mutation operators and crossover and mutation probabilities for the three-objective Parameterless FFP.

Representation	Algorithm	Parameters				Crossover	P_{cross}	Mutation	P_{mut}
Permutation (P)	MOEA/D	N_{pop} = 55	T = 29	δ = 1.0	n_r = 8	LOX	1.0	All	0.06
	NSGA-II	N_{pop} = 55				LOX	0.8	Scramble	0.10
Number of firefighters + Permutation (N+P)	MOEA/D	N_{pop} = 105	T = 29	δ = 0.6	n_r = 2	PBX	1.0	Displacement	0.08
	NSGA-II	N_{pop} = 55				PBX	1.0	Inversion	0.10
Time step (T)	MOEA/D	N_{pop} = 210	T = 13	δ = 0.9	n_r = 2	MinCross	0.5	All	0.08
	NSGA-II	N_{pop} = 55				MinCross	1.0	Inversion	0.10

Results for the bi-objective problem

Median results for the bi-objective problem are presented in Table 4 with the best (largest) value marked in bold.

From the values presented in Table 4 it is clear that the best hypervolume values were obtained by the MOEA/D algorithm using the N+P (Number of firefighters + Permutation) representation. To confirm that this result is statistically significant the following statistical testing procedure was performed for the values obtained for each graph size N_v separately. For 30 hypervolume values obtained using the best-performing algorithm and each of the other algorithms a paired Wilcoxon test [33] was performed yielding a p-value p_i for $i = 1, 2, 3, 4, 5$ (one p-value for each algorithm other than the best performing one). From these p-values the Family-Wise Error Rate (FWER) was calculated as $1 - \prod_1^5 (1 - p_i)$. This FWER value is the upper bound of the probability that for at least one of the compared algorithms the null hypothesis was wrongfully rejected (that is, that this algorithm actually performs equally well or better in spite of the observed evidence to the contrary). Low values of FWER confirm the high statistical significance of the observed superiority of the MOEA/D algorithm using the N+P (Number of firefighters + Permutation) representation over the other algorithms. Indeed, the highest FWER was $9.031 \cdot 10^{-4}$ obtained for $N_v = 1500$ and this value is lower than 0.001. Thus, it can be concluded that the MOEA/D algorithm using the N+P (Number of firefighters + Permutation) representation really produces the best results for the bi-objective Parameterless Firefighter Problem. On the other hand, it is worth noticing, that for each number of graph nodes N_v the second-best result was obtained by the NSGA-II algorithm using the same representation. Therefore, it is reasonable to assume that the N+P (Number of firefighters + Permutation) representation is indeed the best solution representation for the bi-objective Parameterless Firefighter Problem.

Results for the three-objective problem

The results of a similar analysis performed for the three-objective problem are presented in Table 5.

Clearly, it is not possible to select the best performing *algorithm* for the three-objective problem, because both the MOEA/D and the NSGA-II managed to produce the best results for certain graph sizes N_v. However, the results allow to choose the best *solution representation* which in the case of the three-objective problem is the N+P (Number of firefighters + Permutation) representation - the same as for the bi-objective problem. For the three-objective problem the FWER values are much higher than in the case of the bi-objective problem, and for many graph sizes N_v the statistical significance cannot be confirmed. Combined with the results for the bi-objective problem, however, the results presented in Table 5 strengthen the conclusion that N+P is the best representation.

4 CONCLUSIONS

In this paper a Parameterless Firefighter Problem was proposed, motivated by the fact that the number of graph nodes protected per a time step N_f has a strong impact on the difficulty of the Firefighter Problem instances. In order to allow optimization algorithms find good trade-offs between the N_f parameter and the number of graph nodes lost to fire the classical FFP was reformulated so that the number of graph nodes protected per a time step N_f is encoded in problem solutions. To achieve this, three different solution representations were proposed, one in which the N_f value is represented as a decision variable and two in which it is determined when the solution is decoded. The proposed solution representations were used with two multiobjective evolutionary algorithms: the MOEA/D and the NSGA-II and tested on Parameterless Firefighter Problem instances with the number of nodes between 1000 and 10000.

Table 4: Median hypervolume values from 30 runs attained by each solution representation and evolutionary algorithm for each graph size N_v for the bi-objective Parameterless FFP. The best (largest) value in each row is marked in bold.

N_v	Permutation (P)		N_f + Perm. (N+P)		Time step (T)		Family-Wise
	MOEA/D	NSGA-II	MOEA/D	NSGA-II	MOEA/D	NSGA-II	Error Rate
1000	982056.5	970671.5	**985410.5**	983029.5	967695.5	964810.0	$3.620 \cdot 10^{-5}$
1250	1533545.0	1492356.5	**1541137.5**	1537235.5	1508588.5	1506578.5	$1.272 \cdot 10^{-4}$
1500	2176013.5	2084052.0	**2198477.5**	2190384.0	2144016.5	2141100.0	$9.031 \cdot 10^{-4}$
1750	2945761.0	2794630.5	**3000800.0**	2972935.0	2933085.0	2932196.0	$7.014 \cdot 10^{-5}$
2000	3803169.5	3640489.5	**3919922.0**	3880674.0	3834869.0	3834609.0	$1.269 \cdot 10^{-5}$
2250	4785438.5	4579605.0	**4959531.5**	4903556.0	4857046.0	4854262.0	$1.269 \cdot 10^{-5}$
2500	5827676.5	5553248.5	**6113465.5**	6035517.5	5968693.0	5970898.0	$9.541 \cdot 10^{-6}$
5000	22169505.5	20620213.0	**24414291.5**	24235753.0	23959927.5	23847147.0	$8.672 \cdot 10^{-6}$
10000	81740030.5	75604814.0	**97776179.0**	97347454.5	95761864.0	89395384.0	$8.672 \cdot 10^{-6}$

Table 5: Median hypervolume values from 30 runs attained by each solution representation and evolutionary algorithm for each graph size N_v for the three-objective Parameterless FFP. The best (largest) value in each row is marked in bold.

N_v	Permutation (P)		N_f + Perm. (N+P)		Time step (T)		Family-Wise
	MOEA/D	NSGA-II	MOEA/D	NSGA-II	MOEA/D	NSGA-II	Error Rate
1000	953610516.0	935232586.5	**968258603.5**	889652125.5	893483867.5	730113850.0	0.0520
1250	1814368668.5	1756130281.0	**1870850430.0**	1671479941.5	1525211913.0	1388050477.5	0.0176
1500	3095128423.5	2889177852.5	**3211499922.0**	2796835187.0	2434920616.5	2255950641.5	0.0979
1750	4594296873.0	4446420416.5	**4879166742.0**	4326787175.0	3622807875.0	3545163978.5	0.0040
2000	6297965177.0	6035160940.5	**7102332970.0**	5868730658.5	5048301232.5	5124557726.0	0.1360
2250	8813502087.5	8191912579.5	**9531708081.0**	8289174146.5	7072694162.5	7238768842.0	0.0463
2500	11752739503.0	10608788569.0	**12327819503.5**	11145561694.5	9312510155.5	9482696949.0	0.0635
5000	63707019954.5	65787632113.0	71052731810.5	**78642727320.0**	68584248762.5	69371815103.5	0.1959
10000	448701138974.0	446261858669.0	512978666127.5	**560541442074.5**	519234500104.5	529539412451.0	0.0157

For the bi-objective Parameterless FFP the best results were obtained by the MOEA/D algorithm using the N+P (Number of firefighters + Permutation) representation. Statistical testing confirmed high statistical significance of this result. Also, the second-best results were obtained by the NSGA-II algorithm using the same representation. Therefore, it can be concluded that the N+P (Number of firefighters + Permutation) representation is indeed the best solution representation for the bi-objective Parameterless Firefighter Problem.

Also, the three-objective Parameterless FFP was studied, in which the total number of defended nodes has to be minimized in addition to the minimization of the number of the burnt nodes and the minimization of the number of nodes defended per a time step N_f. For this problem the best results were also obtained using the N+P representation, some using the MOEA/D and some using the NSGA-II.

Further work may include testing other metaheuristics with the proposed solution representations for the Parameterless FFP. Especially, Estimation of Distribution Algorithms (EDAs) may be of interest, because different probabilistic models may be applied to different solution representations. The proposed solution representation may also be useful in studies comparing the Firefighter Problem with other optimization problems involving threats spreading in graphs, such as epidemics prevention [27] or prevention of bankruptcies, for example using the threshold propagation model by Burkholz et al. [5]. In both these problems the amount of resources used to stop the threat (vaccine doses or financial reserves, respectively) is often represented as one of the objectives and not as a constraint imposed per a time step. Therefore, the Parameterless FFP may be better suited for tests of optimization methods involving the FFP and these two problems than the original formulation for which the value of the N_f parameter has to be predetermined for each problem instance.

ACKNOWLEDGMENT

Calculations have been carried out using resources provided by Wroclaw Centre for Networking and Supercomputing, grant No. 407 (http://wcss.pl).

REFERENCES

[1] Alberto Antonioni, Seth Bullock, and Marco Tomassini. 2014. REDS: an energy-constrained spatial social network model. In *ALIFE 14: The Fourteenth International Conference on the Synthesis and Simulation of Living Systems*, Hod Lipson, Hiroki Sayama, John Rieffel, Sebastian Risi, and Rene Doursat (Eds.). MIT Press.

[2] Christian Bierwirth, Dirk C. Mattfeld, and Herbert Kopfer. 1996. On Permutation Representations for Scheduling Problems. In *Proceedings of the 4th Internat. Conf. on Parallel Problem Solving from Nature*. Springer-Verlag, 310–318.

[3] Joe L. Blanton, Jr. and Roger L. Wainwright. 1993. Multiple Vehicle Routing with Time and Capacity Constraints Using Genetic Algorithms. In *Proceedings of the 5th Internat. Conf. on Genetic Algorithms*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 452–459.

[4] Christian Blum, M. J. Blesa, C. García-Martínez, F. J. Rodríguez, and M. Lozano. 2014. The Firefighter Problem: Application of Hybrid Ant Colony Optimization

Algorithms. In *Evolutionary Computation in Combinatorial Optimisation*, Christian Blum and Gabriela Ochoa (Eds.). LNCS, Vol. 8600. Springer Berlin Heidelberg, 218–229.

[5] Rebekka Burkholz, Matt Leduc, Antonios Garas, and Frank Schweitzer. 2016. Systemic risk in multiplex networks with asymmetric coupling and threshold feedback. *Physica D* 323-324 (2016), 64–72.

[6] Vincent A. Cicirello. 2006. Non-wrapping Order Crossover: An Order Preserving Crossover Operator That Respects Absolute Position. In *Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, USA, 1125–1132.

[7] V. A. Cicirello and S. F. Smith. 2000. Modeling GA performance for control parameter optimization. In *GECCO-2000: Proceedings of the Genetic and Evolutionary Computation Conference: A Joint Meeting of the Ninth Internat. Conf. on Genetic Algorithms (ICGA-2000) and the Fifth Annual Genetic Programming Conference (GP-2000), July 10-12, 2000, Las Vegas, Nevada*, L.D. Whitley (Ed.). Morgan Kaufmann Publishers, 235–242.

[8] Vitor Costa, Simone Dantas, Mitre C. Dourado, Lucia Penso, and Dieter Rautenbach. 2013. More fires and more firefighters. *Discrete Appl. Math.* 161, 16-17 (2013), 2410–2419.

[9] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation* 6 (2002), 182–197.

[10] E. Falkenauer and S. Bouffouix. 1991. A Genetic Algorithm for Job Shop. In *Proceedings of the 1991 IEEE Internat. Conf. on Robotics and Automation*. 824–829.

[11] Ohad N. Feldheim and Rani Hod. 2013. 3/2 firefighters are not enough. *Discrete Applied Mathematics* 161, 1-2 (2013), 301–306.

[12] Stephen Finbow, Andrew King, Gary MacGillivray, and Romeo Rizzi. 2007. The firefighter problem for graphs of maximum degree three. *Discrete Mathematics* 307, 16 (2007), 2094–2105.

[13] D. Goldberg. 1989. *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison Wesley.

[14] David E. Goldberg and Robert Lingle Jr. 1985. Alleles, loci, and the traveling salesman problem. In *Proceedings of the First Internat. Conf. on Genetic Algorithms and Their Applications*, John J. Grefenstette (Ed.). Lawrence Erlbaum Associates, Publishers, 154–159.

[15] B. Hartnell. 1995. Firefighter! An application of domination. In *20th Conference on Numerical Mathematics and Computing*.

[16] Bin Hu, Andreas Windbichler, and Günther R. Raidl. 2015. A New Solution Representation for the Firefighter Problem. In *Evolutionary Computation in Combinatorial Optimization*, Gabriela Ochoa and Francisco Chicano (Eds.). LNCS, Vol. 9026. Springer, 25–35.

[17] Angel A. Juan, Javier Faulin, Scott E. Grasman, Markus Rabe, and Gonçalo Figueira. 2015. A review of simheuristics: Extending metaheuristics to deal with stochastic combinatorial optimization problems. *Operations Research Perspectives* 2 (2015), 62–72.

[18] Andrew King and Gary MacGillivray. 2010. The firefighter problem for cubic graphs. *Discrete Mathematics* 310, 3 (2010), 614–621.

[19] Jiangxu Kong, Weifan Wang, and Xuding Zhu. 2012. The surviving rate of planar graphs. *Theoretical Computer Science* 416 (2012), 65–70.

[20] Jiangxu Kong, Lianzhu Zhang, and Weifan Wang. 2014. The Surviving Rate of Digraphs. *Discrete Math.* 334 (2014), 13–19.

[21] Hui Li and Qingfu Zhang. 2009. Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II. *IEEE Trans. Evol. Comput.* 13, 2 (2009), 284–302.

[22] P. Lipinski. 2017. Solving the Firefighter Problem with two elements using a multi-modal Estimation of Distribution Algorithm. In *2017 IEEE Congress on Evolutionary Computation (CEC)*. 2161–2168.

[23] Krzysztof Michalak. 2014. The effects of asymmetric neighborhood assignment in the MOEA/D algorithm. *Applied Soft Computing* 25 (2014), 97–106. https://doi.org/10.1016/j.asoc.2014.07.029

[24] Krzysztof Michalak. 2015. The Sim-EA Algorithm with Operator Autoadaptation for the Multiobjective Firefighter Problem. In *Evolutionary Computation in Combinatorial Optimization*, Gabriela Ochoa and Francisco Chicano (Eds.). LNCS, Vol. 9026. Springer, 184–196.

[25] Krzysztof Michalak. 2017. ED-LS - A heuristic local search for the multiobjective Firefighter Problem. *Applied Soft Computing* 59 (2017), 389 – 404. https://doi.org/10.1016/j.asoc.2017.05.049

[26] Krzysztof Michalak. 2017. Estimation of Distribution Algorithms for the Firefighter Problem. In *Evolutionary Computation in Combinatorial Optimization: 17th European Conference, EvoCOP 2017, Amsterdam, The Netherlands, April 19-21, 2017, Proceedings*, Bin Hu and Manuel López-Ibáñez (Eds.). Springer International Publishing, Cham, 108–123.

[27] Krzysztof Michalak. 2018. Informed Mutation Operator Using Machine Learning for Optimization in Epidemics Prevention. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO '18)*. ACM, New York, NY, USA, 1294–1301. https://doi.org/10.1145/3205455.3205647

[28] Krzysztof Michalak and Joshua D. Knowles. 2016. Simheuristics for the Multiobjective Nondeterministic Firefighter Problem in a Time-Constrained Setting. In *Applications of Evolutionary Computation: 19th European Conference, EvoApplications 2016, Porto, Portugal, March 30 – April 1, 2016, Proceedings, Part II*, Giovanni Squillero and Paolo Burelli (Eds.). Springer International Publishing, 248–265.

[29] S. A. Moeller and P. Wang. 2002. Fire control on graphs. *Journal of Combinatorial Mathematics and Combinatorial Computing* 41 (2002), 19–34.

[30] Christine L. Mumford. 2006. New Order-based Crossovers for the Graph Coloring Problem. In *Proceedings of the 9th Internat. Conf. on Parallel Problem Solving from Nature*. Springer-Verlag, Berlin, Heidelberg, 880–889.

[31] I. M. Oliver, D. J. Smith, and J. R. C. Holland. 1987. A Study of Permutation Crossover Operators on the Traveling Salesman Problem. In *Proceedings of the Second Internat. Conf. on Genetic Algorithms on Genetic Algorithms and Their Applications*. Lawrence Erlbaum Associates Inc., Hillsdale, NJ, USA, 224–230.

[32] Gilbert Syswerda. 1991. Schedule Optimization Using Genetic Algorithms. In *Handbook of Genetic Algorithms*, Lawrence Davis (Ed.). Van Nostrand Reinhold, New York, NY.

[33] Frank Wilcoxon. 1945. Individual Comparisons by Ranking Methods. *Biometrics Bulletin* 1, 6 (1945), 80–83.

[34] Qingfu Zhang and Hui Li. 2007. MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. *IEEE Transactions on Evolutionary Computation* 11, 6 (2007), 712–731.

[35] Eckart Zitzler, Lothar Thiele, Marco Laumanns, Carlos M. Fonseca, and Viviane Grunert da Fonseca. 2002. Performance Assessment of Multiobjective Optimizers: An Analysis and Review. *IEEE Transactions on Evolutionary Computation* 7 (2002), 117–132.