

# Optimisation Algorithms for Parallel Machine Scheduling Problems with Setup Times

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## ABSTRACT

Parallel machine scheduling is a problem of high practical relevance for the manufacturing industry. In this paper, we address a variant in which an unweighted combination of earliness, tardiness and setup times aggregated in a single objective function is minimised. We compare an Evolutionary Algorithm (EA) approach with a variant of local search implementing a probabilistic Best Response Dynamic algorithm (p-BRD) inspired by game theoretic considerations. Our p-BRD algorithm achieved promising results outperforming the EA on a series of test sets.

## CCS CONCEPTS

• **Theory of computation** → **Scheduling algorithms**; • **Applied computing** → *Industry and manufacturing*; • **Computing methodologies** → *Planning and scheduling*;

## KEYWORDS

Best Response Dynamics, Machine Scheduling with Setup Times

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## 1 INTRODUCTION

Optimising parallel machine scheduling (PMS) has a long history of research and is of high practical relevance for the manufacturing industry. Minimising the time it takes a product to pass through manufacturing affects the revenue of the company and improves customer satisfaction. The objective is to assign jobs to machines so that the resulting schedule completes all jobs in full with minimal deviation from due dates.

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In this work, we address a variant of PMS with an unweighted combination of earliness, tardiness and setup times aggregated into a single objective function (AOF). Criteria are of equal importance which can be changed by adding weighting factors.

The order of jobs determines setup times and has significant impact on the completion time of each job. Therefore, we address a parallel machine environment incorporating sequence dependant setup times, earliness and tardiness and not only focus on minimising the makespan.

PMS is well known to be NP-hard and is a typical case for applying meta-heuristics such as Evolutionary Algorithms (EAs) [6]. Local-search algorithms have also been applied to scheduling problems providing good results (e.g. [5]). We extend this strand with a new variant inspired by game theory including a non deterministic stochastic element. We call this approach *p-Best Response Dynamic* (p-BRD). For comparison of solution quality and runtime we also implemented a simple EA applying both to a series of test sets.

## 2 ALGORITHMS

Best Response Dynamics is an iterative algorithm for searching Nash equilibria in  $n$  player non-cooperative games. We view PMS as a game in which agents representing jobs seek an optimal position on a machine while minimising payoff determined by AOF (see Algorithm 1). While searching for an optimal solution agents mutually exchange their positions. Input to p-BRD is a set of jobs to be scheduled on available machines and a setup matrix. In an initialisation phase, doubly linked lists of job agents are constructed for each machine. Job agents are first sorted in ascending order by due date and maximum production times. During the game, each job agent  $a$  has a set of options to change its position in the schedule. It can either swap position with another agent  $a'$  if machines match and production times allow the change without interfering with start and end dates of other agents or - in case that  $a'$  cannot run on the machine  $a$  is assigned to - change its position by moving behind  $a'$  if  $a'$  has no successor. A swap is performed if it improves AOF. To avoid being stuck in a local minimum too early, non improving swaps increasing AOF are accepted with probability  $p$ . p-BRD stops as soon as no more swaps occur meaning the game has reached a Nash equilibrium.

A straightforward EA (tournament selection/replacement, single point crossover and swap mutation) in which fitness of each

**Algorithm 1** p-BRD

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1: repeat
2:   for  $J_j$  in  $J$  do
3:     for  $J_k$  in  $J \setminus \{J_j\}$  do
4:       if machines of  $J_j$  and  $J_k$  are compatible then
5:         if (swap( $J_j, J_k$ ) improves AOF) or (rand<p) then
6:           swap( $J_j, J_k$ )
7:           break
8:       else if if  $J_j$  fits on machine of  $J_k$  and  $J_k$  last then
9:         if moving  $J_j$  after  $J_k$  improves AOF then
10:          move  $J_j$  after  $J_k$ 
11:    $p = p \cdot d$ 
12: until no more swaps

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**Table 1: Statistical measures for the experiments**

Data	GA			p-BRD		
	Mean	STD	RT	Mean	STD	RT
URG	7587.8	224.6	9776ms	7467.01	156.8	4397ms
SPRD	3803.2	178.2	7075ms	3868.6	135.4	100ms
LRG	26999.2	820.1	35519ms	22700.2	500.1	5955ms
OL30	464.91	26.09	6217ms	443.23	27.72	3984ms

individual is determined by evaluating AOF for the corresponding production schedule is used for assessing the results.

### 3 DATA SETS

Three exemplary test sets representing realistic production setups are generated with a method controlled by a set of parameters (see [2]). Data set URGENCY contains a considerable amount of jobs, relatively few machines and jobs with early due dates provoking high delay and an increased importance of setup times. In contrast the jobs in data set SPREAD vary in their due dates requiring a sequence focusing on the deviation of the jobs from their due dates. Data set LARGE measures scalability with a large number of jobs. Additionally, the OLIVER 30 benchmark for the travelling salesmen problem [4] is reformulated as a scheduling problem by interpreting distances as set up times.

### 4 RESULTS

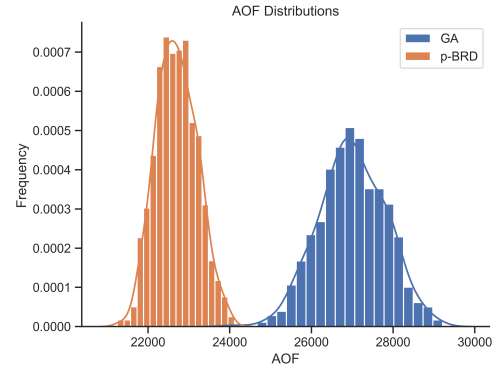
Tree-structured Parzen Estimators [1] were applied to fine tune parameter settings for p-BRD and EA<sup>1</sup>.

For data set LARGE (see Figure 1), p-BRD performs better than EA with a smaller standard deviation (see Table 1).

For data set URGENCY the differences are less pronounced in terms of mean but with lower standard deviation for p-BRD.

The result distribution for data set SPREAD has a low standard deviation for p-BRD, due to the small probability for accepting temporarily worse solutions ( $p = 0.0012$ ). Standard deviation of EA is larger with a better mean than p-BRD.

<sup>1</sup>Final parameters and data sets are available here <https://git.thm.de/fktl79/probabilistic-brd>.

**Figure 1: Comparison of AOF on data set LARGE.**

For the tests on OLIVER30 AOF was modified to include the setup time between the last and the first job. p-BRD reached a mean tour length of 424.39 with a standard deviation of 0.39 (Optimal length is 419 [3]). The EA scatters more widely between 423 and 565, with a mean significantly larger than that of p-BRD (see Table 1)

In conclusion p-BRD produces better results for two of the machine scheduling problems and comparable ones on the third, while being significantly faster in each case.

### 5 CONCLUSION AND FUTURE WORK

For lack of publicly available benchmark tests for our PMS problem newly generated test sets are created. The experiments showed that p-BRD produced good results with better runtime behaviour than a standard EA. Therefore, p-BRD can be considered feasible for real world application. Additional research on p-BRD may investigate more elaborate swapping strategies. Applicability on further problems will be in focus as well as an extension of the heuristics that closes gaps in the timeline of the production schedule.

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