Advanced Mine Optimisation under Uncertainty Using Evolution

William Reid
Maptek Pty. Ltd
Adelaide, Australia

Aneta Neumann
Optimisation and Logistics
The University of Adelaide
Adelaide, Australia

Simon Ratcliffe
Maptek Pty. Ltd
Adelaide, Australia

Frank Neumann
Optimisation and Logistics
The University of Adelaide
Adelaide, Australia

ABSTRACT

In this paper, we investigate the impact of uncertainty in advanced mine optimisation. We consider Maptek's software system Evolution which optimizes extraction sequences based on evolutionary computation techniques and quantify the uncertainty of the obtained solutions with respect to the ore deposit based on predictions obtained by ensembles of neural networks. Furthermore, we investigate the impact of staging on the obtained optimized solutions and discuss a wide range of components for this large scale stochastic optimisation problem which allow us to mitigate the uncertainty in the deposit while maintaining high profitability.

KEYWORDS

Evolutionary algorithms, open pit mine optimization, open pit mine production scheduling, uncertainty, mine planning, staging

ACM Reference Format:
William Reid, Aneta Neumann, Simon Ratcliffe, and Frank Neumann. 2021. Advanced Mine Optimisation under Uncertainty Using Evolution. In *2021 Genetic and Evolutionary Computation ConferenceCompanion (GECCO '21 Companion), July 10–14, 2021, Lille, France.* ACM, New York, NY, USA, 9 pages. https://doi.org/10.1145/3449726.3463135

1 INTRODUCTION

Evolutionary algorithms provide great flexibility in dealing with a wide range of optimisation problems. This includes highly constrained problems as well as problems involving dynamic and/or stochastic components. Their wide applicability has made evolutionary computing techniques popular optimisation techniques in areas such as engineering, finance, and supply chain management [4].

The area of mining where the goal is to extract ore in a cost efficient way poses large scale optimisation problems, and evolutionary computation techniques have successfully been applied in this area [1, 8, 21, 23]. We consider the problem of mine planning and focus on uncertainties which highly impact the mine planning process. Mine planning is one of the key optimisation problems in mining and a wide range of approaches have been developed over the years. The classical article of Lerchs and Grossmann [13] introduced the basic problem formulation and provided a dynamic programming approach. Over the years, a wide range of mine planning approaches taking different characteristics of this important real-world optimisation problem into account have been studied in the literature. This includes integer programming approaches based on block scheduling [14, 18] and heuristic techniques that are able to deal with various characteristics such as uncertainties of the problem [7, 12, 17]. Different software products for carrying out mine planning and extraction sequences are available [15, 16].

We discuss the mine planning problem in the light of Maptek's mine planning optimisation software Evolution [15]. Evolution is representative of a new breed of commercially available approaches to mine planning, using evolution algorithms to find near-optimal solutions in the face of non-linearity and arbitrarily complex constraints. There are challenging linear programming approaches that can exactly optimise approximate linear models of complex systems. As a result, Evolution is increasingly being used on large data sets and complex problems by large mining corporations globally to plan their life-of-mine extraction sequences.

We introduce and evaluate a new approach which allows the effect of uncertainties in the source geological data on which extraction sequences are based to be economically quantified. In mine planning the classical goal is to maximize net present value (NPV) over the life of the mine. This optimisation task involves deciding on whether to process given blocks of minerals based on the estimated amount of ore it contains. This estimate of ore is highly uncertain for most of the material that needs to be considered due to the expense of measurement prior to mining. Point samples from drilling must be interpolated for most of the material. In our approach, uncertainty is quantified by an ensemble of neural network interpolations that predict the grade of ore in the different blocks for process and recovery considerations.

A crucial part when running the Evolution software is the staging of blocks. This process divides the mine planning task into different stages that are processed sequentially. Given that the number of blocks is very large, usually in the hundreds of thousands, the staging process enables an efficient optimisation process by constraining the time at which a block can be processed. On the other hand, the staging heavily influences the quality of the solution to be obtained overall and at different periods in the extraction sequence. We exemplify this by showing different staging setups and the resulting profits that are obtained in the periods of a mine plan. Furthermore, staging has the potential to deal with uncertainties as it is possible to combine certain and uncertain areas of the mine into a stage which can lead to a reduction in the overall uncertainty in critical time periods of the life of a mine. Our experimental investigations explore the uncertainty of typical mining schedules

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than the author(s) must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.
GECCO '21 Companion, July 10–14,2021, Lille, France
© 2021 Copyright held by the owner/author(s). Publication rights licensed to ACM.
ACM ISBN 978-1-4503-8351-6/21/07.
https://doi.org/10.1145/3449726.3463135

and compare the impact of different staging approaches on NPV and uncertainty in the different time periods of the life of a mine.

In Section 2 we define the problem and describe the mineral resource block model. We present Maptek's Evolution software in Section 3 and introduce uncertainty qualification based on ensembles of neural networks in Section 4. In Section 5, we discuss the implications of staging on optimised solutions. We present our experimental results on uncertainty quantification and staging in Section 6. Finally, we discuss important open problems and research gaps.

2 PROBLEM FORMULATION

The open-pit mine production scheduling problem, known as the open-pit mine block sequencing problem, is an important problem in open-pit mine planning as it determines the material that contributes to the sustainable utilization of mineral resources over the life of the mine [10]. The overall goal is to find an optimal extraction sequence maximizing net present value (NPV). Decisions on block scheduling in the open-pit mine production scheduling problem have to take into account several constraints [6, 9, 10]. The set of constraints includes amongst others blending constraints, stockpile related constraints, logistic constraints, mill feed and mill capacity, reserve constraints and slope constraints.

Stockpiling is the process of storing mined material that will eventually go through the processing plant, but is deferred for mill capacity or economic reasons. Product blending can be used to ensure processed material meets a minimum grade, and can result in the schedule reflecting simultaneous mining from different areas of the pit, or using lower grade material from stockpiles to achieve a target grade. These have not been reflected below as they are not always required.

In the following, we describe a basic model of the open pit scheduling problem similar to the one given in [22] and it should be noted that the actual implementation might require the consideration of additional constraints and/or variations of the objective function. A block model is given by a set of blocks B, and a set of destinations D are then assigned to each block during the optimisation process. Each block $i \in B$ has a certain number of parameters such as density, tonnage, ore grade, etc. These parameters permit to determine the economic value of every block $i \in B$ at a given time. We denote $T = \{1, \ldots, t_{\max}\}$ the set of time periods where $t_{\max} = |T|$ is the number of time periods. For each block $i \in B$ we assume a revenue of r_i^t and a cost c_i^t if block i is sent at time t for processing. On the other hand, it encounters a mining cost m_i^t if it is send to waste. In addition, the slope requirements for the set of blocks and other mining constraints are described by a set of precedence constraints P as follows. The pair $(i, j) \in P$ describes a scenario where a block i must be extracted by time t if block j needs to be extracted at time t. More precisely, P is the set of precedence constraints, and we have $(i, j) \in P$ if block i a has to be mined before block j.

The value $(r_i^t$ - $c_i^t)$ is given for each block $i \in B$ at time $t \in T$ that the block i is sent to processing plant, and produces a cost m_i^t if the block is sent to waste dump. We denote by τ_i the tonnage of block i. For each period t maximum limits M^t on the amount of material that can be mined are imposed. Similar, for each period t the maximum processing capacity P^t i.e., the amount of ore that is milled have to be met.

In our model we use two types of decision variables for each block $i \in B$ and time $t \in T$. The first type are the variables associated to the extraction for processing purposes for each block. A binary variable x_i^t is one if block i is extracted and processed in the period t and zero otherwise. The second variable type describes the decision relating to the disposal of a block. The binary variable w_i^t is one if block i is extracted and sent to waste in the period t and zero otherwise.

The simplified mathematical model can be summarized as follows:

$$\textbf{max} \quad NPV(x, w) = \sum_{t \in T} \left(\frac{1}{(1+d)^{t-1}} \sum_{i \in B} \left((r_i^t - c_i^t) x_i^t - m_i^t w_i^t \right) \right) \tag{1}$$

$$\textbf{s.t.} \quad \sum_{t=1}^{T} x_i^t + w_i^t \leq 1 \qquad \forall\, i \in B \tag{2}$$

$$(x_j^t + w_j^t) \leq \sum_{r=1}^{t} (x_i^r + w_i^r) \qquad \forall (i, j) \in P, t \in T \tag{3}$$

$$\sum_{i \in B} \tau_i (x_i^t + w_i^t) \leq M^t \qquad \forall\, t \in T \tag{4}$$

$$\sum_{i \in B} \tau_i x_i^t \leq P^t \qquad \forall\, t \in T \tag{5}$$

$$x, w \in \{0, 1\}^{|B| \cdot |T|} \tag{6}$$

In this formulation, the objective function (1) seeks to maximise net present value of the solution (NPV) based on the a given discount rate d. Constraints (2) ensures that is not possible to choose two different destinations for a block i and that a block is only chosen at one point in time. Crucial constraints are the *precedence constraints* (3) which determine the extraction process of each block i from surface down to the bottom of the ore deposit. In order to provide for access and the stability of the pit walls it is not possible to mine a given block in a given time in the case that blocks in a defined pattern above have not already been extracted. The *mining constraints* (4) ensure that the total weight of blocks mined during each period do not exceed the available extraction equipment capacity i.e., the mining capacity. Similarly, the amount of material that can be processed in each period is restricted by the *processing constraints* (5).

2.1 The resource block model

The mineral resource block model is a simplified representation of an ore resource. The volume is represented by a rectangular, three-dimensional array of blocks that contain estimates of data such as dimensions, volume, spatial reference points, density of the material, grades of each block and the type of material. The reserve model additionally considers technical and economical properties in compliance with, for example, the JORC Code [3] (i.e., system for the classification of minerals, exploration results, mineral resources and ore reserves [2]). Each in-pit block contains financial variables such as recovery cost, mining cost, processing cost, and rehabilitation costs. The information can often be reduced to the ore grade and the ore tonnage available in each block. Ultimately, this set of blocks are divided into two distinct subsets: ore blocks that are sent to a mill (potentially via a stockpile) and the remaining waste blocks. Waste blocks are not processed, but still need to be mined due the precedence constraints on ore blocks. Each block has an economic value at a given time period t which represents the net present value that is associated with this particular block at time t given by its revenues and associated processing cost. A waste block has negative financial value that is incurred by the cost of mining the block.

Blocks are grouped into a two layer hierarchy of stages and benches. Stages are assigned in the resource block model and methods for doing this are discussed. The benches within each stage are the sets of blocks within each horizontal layer of the block model.

Shells often form the basis for stages, and are calculated as subsets of blocks that can be mined at economic break-even over a range of likely ore prices. Without consideration for the time value of money, shells are economically optimal subsets of the block model that honour precedence constraints. The shells can be generated using the well-known Lerchs-Grossmann algorithm [13]. The ultimate pit limit is the outer-most shell chosen based on the highest likely ore price and defines the total subset of blocks to be considered by a scheduling algorithm.

In the real world, the extraction of the ore based on sequentially mining Lerchs-Grossman shells is often impracticable. The mining shells are often discontinuous and consists of small number of blocks in between shells. Due to the placement of machines and controlled blasting of rock during mining, it is not profitable to mine small numbers of blocks in disconnected places. In order to tackle mining problems practically, we apply a stage design taking into account the representation of the valuable blocks.

3 MAPTEK'S EVOLUTION SOFTWARE

Evolution is a suite of products offered by Maptek that provides scheduling engineers with tools to produce optimal extraction sequences constrained by many practical considerations. These products work together to collectively span multiple time horizons, ranging from:

- life-of-mine: yearly scheduling showing returns for an entire mine's life.
- medium term: monthly scheduling used to optimise extraction sequence and equipment for a portion of the mine's life.
- short term: operational scheduling used to inform daily operations on required material volumes and grades, as well as assign given equipment to specific tasks.

For the purpose of our investigation, Maptek Evolution Strategy was used. Strategy is used for life-of-mine economic optimisation that aims to maximise net present value of the resource (NPV) by optimally ordering the stage/bench extraction sequence. It employs a dynamic cut-off grade policy for both the stockpiling and wasting of material.

Strategy's only required input is a block model as described. Along with the block model, Strategy requires yearly mining capacities and yearly plant capacities, as well as costs for mining, processing and selling the target ore. The optimisation weighs the cost of mining against the value of the product being sold, dynamically choosing on a per stage, per bench basis of when and how to best handle materials of different composition to maximise economic returns.

Strategy employs evolutionary algorithms to perform this optimisation. A dynamic cut-off grade policy introduces non-linearity into the optimisation and evolutionary algorithms can handle this. The algorithm has been shown to perform robustly in acceptable time frames on very large whole-of-mine sized resource models. Another strength of an evolutionary algorithm approach is that the best solution is available at any given point in the run time. This means that a solution can be inspected for a given computation budget and released as 'good enough' without requiring guaranteed optimality.

Strategy is designed to always produce valid solutions. A *valid* solution adheres to precedence and capacity constraints. Mutation and crossover operators are designed to preserve these constraints so provided seed genomes produce sequences that exhibit them, then the evolutionary algorithm will preserve them during selection and generation. A *feasible* solution is not only valid, but also practical and economically viable to mine. This will typically require cash flows to remain positive after an initial start-up and with material movements aiming to consistently utilise mining equipment at or near full capacity.

A feasible solution isn't always the solution with the highest NPV for the resource model. This is because not all economic considerations are captured in this value. For example, solutions with periods of negative cash flow may make financing the operation infeasible even if this is for the benefit of larger future profits in net present terms. In this situation a solution that evens out revenues in each period will be considered feasible compared with another that may have a higher NPV.

Staging - the assignment of stages to divide the block model into sequentially mined subsets of blocks - is required by Evolution. It is used extensively in traditional manual scheduling but is also a useful tool to preserve feasibility whilst optimising for NPV. Staging is a discrete process and has a non-linear effect on NPV. Variability in the assigned domain of blocks and the ore grade within them also has a non-linear effect on the NPV of a given extraction sequence and this aspect is now able to be explored in the software from the work presented here.

4 UNCERTAINTY QUANTIFICATION USING NEURAL NETWORKS

A mineral resource block model is inherently uncertain because the estimations of ore grade in each block are derived from samples obtained by drilling into the rock and the spacing of drilling is typically much wider than the block resolution. Many interpolation methods exist for combining samples in the local neighbourhood of a block to estimate its grade value. Those with a basis in geological processes are favoured. These methods typically involve a categorisation of the rock mass into *lithological, structural or alteration domains.* Such domains typically delineate separate mineralisation processes that have occurred under different physical conditions, such that the concentration of minerals or metals in one domain is considered independent to that in another. The domain classification then guides the choice of samples and the weight to give to them in a geostatistical interpolation method such as Kriging [11] or inverse distance weighted averaging [24] to derive an estimate of the grade for each block in the block model.

Quantifying the uncertainty of any estimation process involving domains is complicated to do analytically because complex 3D geometries are often involved with the interfaces between domains [5, 19]. Conditional simulation is a commonly used technique in geostatistics to quantify the grade uncertainty in a multi-domain reserve block model [20]. This method can be thought of as a sensitivity analysis of grade value estimations and their distribution throughout the model based on varying one or several of the parameters involved in one or several geostatistical interpolation methods. The range through which to vary the parameters and the methods to include in the simulation are all subject to qualitative choices and are difficult to approach for a novice in the field.

Here we employ a simpler technique to quantify the locations and ranges of uncertainty which has been introduced in [25]. The method produces domain and grade predictions that are adequate to illustrate the economic sensitivity of different extraction sequences based on the spatially varying uncertainty that arises from a population of different interpolations of the input sample data. The method allows variation in domain boundaries and grade distributions within the domains to be explored. The approach in summary is to train a deep neural network to fit the input sample data in its three spatial dimensions, an arbitrary number of continuous dimensions corresponding to grades of elements of interest and a categorical dimension corresponding to the domain. The fit is performed from random starting weights and employs standard deep learning model fitting techniques based on gradient descent to minimise a differentiable error function until the goodness-of-fit is within a specified tolerance. By exploiting the property of such networks to converge on a set of weights from different random initialisations, we compute an ensemble of trained networks that statistically interpolate the input data equally well.

Maptek's DomainMCF machine learning geological modelling software was used to produce an ensemble of ten models based on this technique. These models are then evaluated into each block in a block model to produce a population of grade and domain predictions for each block. These ensembles are the basis for quantifying the economic uncertainty of optimised extraction sequences.

5 STAGING

Staging is an important component that serves as an input to Evolution. It divides the optimisation problem into stages that are processed sequentially. Staging is therefore a partitioning of the given set of blocks B into stages $S_1, \ldots, S_k$ where each block in stage S_i is processed before each block in stage P_j iff $i < j$. For the given partitioning $S_i \cap S_j = \emptyset$ iff $i \neq j$ and $\cup_{i=1}^{k} S_i = B$ is required. In practice, the requirement of processing a stage completely before starting the next one is not strict and there might be some time overlaps in processing different stages. Note that a trivial staging is obtained by using only one stage S_1 and setting $S_1 = B$ which implies that all blocks belong to the same stage. The drawback of this is that all blocks have to be considered at the same time which does not break down the optimisation problem. Furthermore, benches are scheduled completely in a single stage and breaking benches down by dividing them into different stages can lead to better overall solutions.

Several methods have been proposed that do not use staging in mine optimisation and schedule blocks directly. These are collectively known as direct block scheduling methods. They remain an active area of research because they still have not solved the benefits afforded by staging methods around some aspects of practicality even though they may yield an extraction sequence of higher NPV. For example, staging methods inherently create a controlled number of active mining areas and so afford a direct means of localising where equipment needs to be and where it needs to go next on relatively long time frames. This enables roads to be constructed in time and without interfering with other passages, distances over which equipment needs to be moved to be minimised and other similarly advantageous considerations for practical mining. Direct block scheduling methods require explicit constraints to gain these advantages and without them will often result in extraction sequences that involve mining small pockets of material from impractically distant locations.

Staging helps to take a problem from hundreds of millions of variables into a few thousand that can be optimised with complex non-linear relationships. It introduces practical constraints on the order of blocks in a schedule by constraining the stages and benches in each period. Strategy uses stages and considers combinations of stage and bench. A stage and bench combination is all the blocks that share the same stage and position on the Z axis (the bench). When considering what blocks to mine in each period, the optimisation process will sum the values of each block in a stage and bench and mine it in one large chunk. Without staging, Strategy would be forced to mine the entirety of each bench before moving on, without consideration of domain or grade. This will result in significant mining of waste before any ore could be reached. In effect, staging allows high value ore to be targeted early on in the process, leaving the movement of waste from other stages to later in the process.

The creation of these stages is used as a loose guide for a logical mining sequence. Typically, stage 1 will be mined before stage 2, but there will be some overlap. As mining hits ore in stage 1, the start of stage 2 - which will often be removal of waste blocks to get to future ore blocks - would begin. This simultaneous mining of ore and waste in stages ensures a consistent positive cash flow. Through the process of staging, a number of practical mining rules are adhered to. There is an assumption that a completed stage design will follow some basic mining practices. These are - but not limited to - the rules outlined in Section 3 which Strategy uses to produce valid mining solutions.

6 EXPERIMENTAL INVESTIGATIONS

Our approach on uncertainty quantification has been implemented into Evolution and is part of the latest software release. We now investigate the uncertainty of optimised solutions introduced by the neural network approach. We interpret the solutions obtained from a mine planning perspective in terms of economic risk. Furthermore,

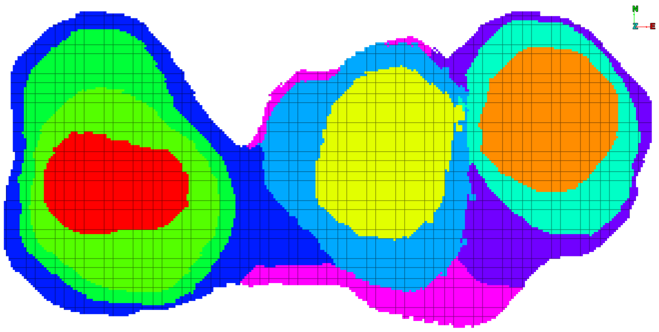


Figure 1: Example of model staging. This model has 9 total stages, created from 21 individual shells.

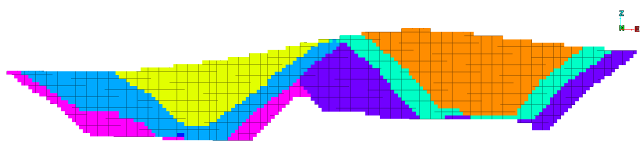


Figure 2: A cut along the Y axis of the model showing the eastern section of the staged pit.

we investigate optimised solutions for different staging approaches and show that this leads to significantly different results.

6.1 Problem setup

Using the same input drilling data, we generated 10 individual block models using the neural network technique. We will call this collection of models an ensemble, with each individual model being a member of the ensemble.

We then combined the ensemble into a single model, known as the aggregate model. This aggregate model took the median domain between the ensemble members, and the mean grade for the selected domain. We used this aggregate to create an ultimate pit using Lerchs-Grossmann [13] pit optimisation. From here, we could begin to stage the model.

During testing we recognised a number of issues when comparing results from different staging techniques. The largest being that in order to compare the staging between tests, constraints manually applied to positively drive feasibility in one should not then negatively drive feasibility in another. Typically, a mining engineer would manually apply constraints of this nature such as:

- Bench turnover: The number of bench's to be mined per period
- Stage availability: Holding a stage back from the optimiser in order to achieve some aspects of feasibility

and many more. The application of these constraints is very specific to each individual staging and so could not sensibly remain constant across all our tests. As such, no such additional constraints were placed on the optimisation. Instead, each test shared the same 'calendar'. A calendar sets maximum tonnages for mining capacity, plant processing and also economics such as ore price, processing, mining and selling costs, and other constants on a per-period basis.

For our investigations we used representative mining values for the copper ore under consideration; a processing cost of \$15 AUD per tonne, and a mining cost of \$4.20 AUD per tonne. Mining capacity was set to 25 million tonnes per period, with periods defined in yearly increments. The resulting block model had 190 000 blocks, and given the significant over-burden meant the processing plant was only required from the middle of period 3. Plant capacity began at 5 million tonnes per period, before being upgrade to 9 million tonnes in period 9. There were no constraints placed on stockpiling capacity. Included in the economics was a minimum cut-off grade of 0.25% copper, discount rate $d = 8\%$ and a price per tonne of \$7 673 AUD. Selling cost (the administrative cost involved in selling the ore) was set at \$1 AUD per tonne of produced metal, and rehabilitation cost (the cost to rehabilitate the site at the end of the mine life) was also set at \$1 AUD per tonne of mined material. The deposit included a number of different copper rich domains. These domains ranged in copper recovery - the amount of material that can realistically be recovered by milling - from 75% to 92%. All of these quantities are representative of sensible values for this deposit at the time of writing. Slight variations for each value do not change the conclusions that we draw. To begin our tests, we used the individual shells generated by the Lerchs-Grossmann pit optimisation. These shells guided subsequent staging approaches. Figure 1 and 2 show an example of a staged model.

As part of the optimisation process, the Strategy engine picks a per-period stage/bench extraction sequence for the aggregate model and its associated staging. Within the constraints of the staging, this sequence is near-optimal for maximising the per-period discounted profit of the in-ground reserve. Once optimisation is complete, the engine takes each individual model and replays the same stage/bench sequence, evaluating the per-period economics. This results in us getting back 11 different evaluations; 1 for the aggregate model, and 10 in total for the ensemble. This allows us to plot the economics for each member of the ensemble on a per-period basis for comparison.

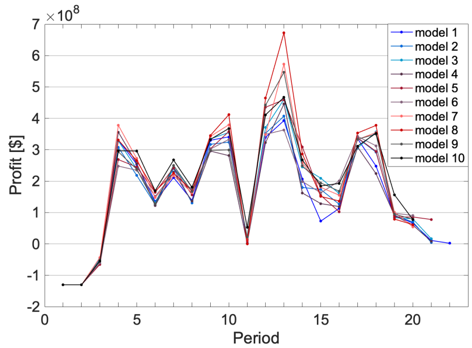
The block properties that vary due to geological uncertainty are domain and grade. The assigned domain influences the processing method and associated cost this may result in a decision to waste a block instead of process it. Changes in grade can force an ore block to now sit below cut-off, thus classifying it as waste. Reclassifications like this are thought to be where uncertainty will have the biggest economic impact.

6.2 Staging approaches

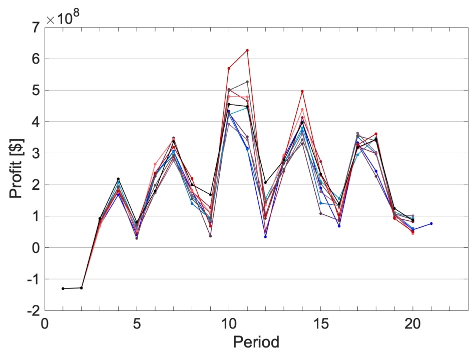
With the problem now consistently formulated independently of staging, we started to consider how stage design could affect the resulting charts. With this in mind we opted for 4 different stage designs, each with 6 individual stages. Note that each staging mechanism is practical and potentially feasible.

Lazy staging. This stage design involved aggregating sequential Lerchs-Grossmann shells into stages. This represents an NPV-maximising stage design without any consideration of geological uncertainty.

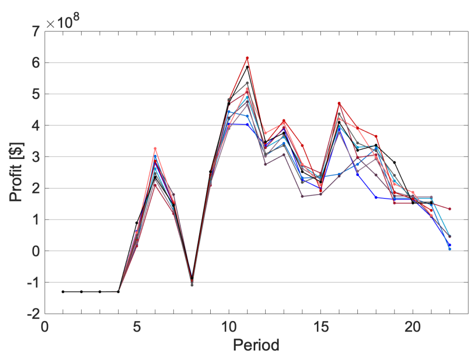
Expected staging. For this stage design, a professional mining engineer with knowledge of the deposit staged the model based on a number of factors: maximising NPV whilst keeping the plant



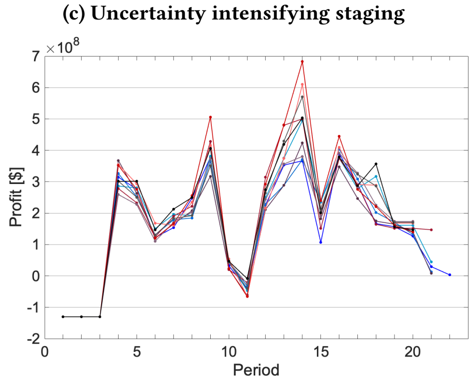
(a) Lazy staging



(b) Expected staging

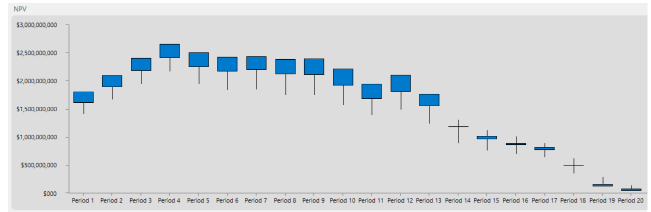


(c) Uncertainty intensifying staging

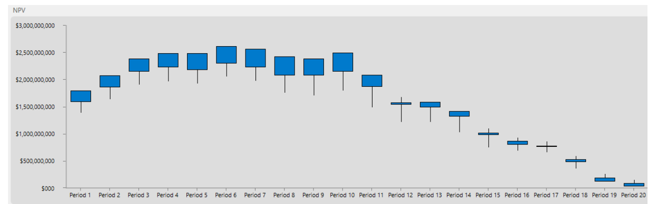


(d) Uncertainty mitigating staging

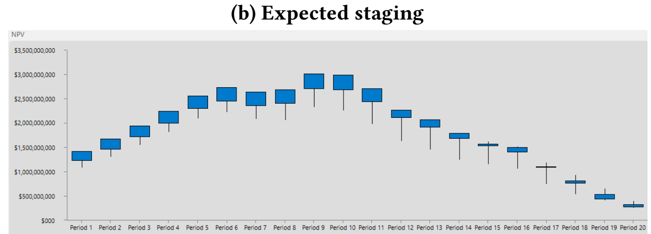
Figure 3: Profit per period (undiscounted) per ensemble member for four different staging approaches.



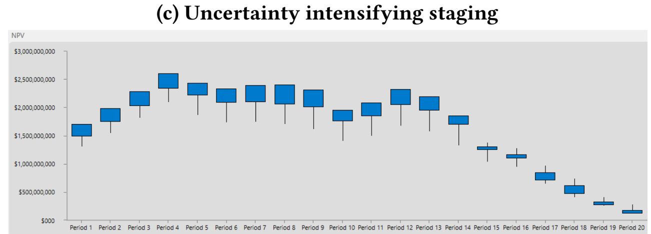
(a) Lazy staging



(b) Expected staging



(c) Uncertainty intensifying staging



(d) Uncertainty mitigating staging

Figure 4: Box and whisker plot showing the remaining resource NPV per period for four different staging approaches.

full and maintaining a positive cash flow across all periods, again without consideration of geological uncertainty.

Uncertainty intensifying staging. To best try and see an effect from uncertainty, this staging isolates all ore blocks with a grade standard deviation greater than 1% to only two stages. The remaining 4 stages consisted the ore blocks with a grade standard deviation of less than or equal to 1% and the waste blocks.

Uncertainty mitigating staging. With the results of the last three stage designs in mind, the last attempt was to try and level out uncertainty between the 6 stages. By grouping pockets of uncertainty together with certain ore, the hope was the combination would result in a reduction in overall per-period uncertainty. The mitigation was done manually based on visual inspection of grade variance.

6.3 Experimental results

For our investigations, we refer to average NPV to represent the models feasibility, and profit range to represent what effect uncertainty has on the overall schedule. The results of four different staging approaches with respect to per period profit are shown in Figure 3 and the NPV whisker plots are shown in Figure 4. The figures show screenshots of our approach implemented into Evolution. The overall summary of the results in terms of the average NPV and the total profit range (difference of highest total profit and lowest total profit predicted by the ensemble) is as follows.

- Lazy staging: average NPV of $1.585B, total profit range of $1.84B.
- Expected staging: average NPV of $1.569B, total profit range of $1.929B.
- Uncertainty intensifying staging: average NPV of $1.206B, total profit range of $1.98B.
- Uncertainty mitigating staging: average NPV of $1.485B, total profit range of $1.99B.

Figure 3 shows the profit results of 4 different stage designs for 20 periods. The results for each of the 10 ensemble members are shown as colored lines. The spread of the lines shows the range of economic uncertainty introduced by the effects of geological uncertainty in each period. Comparison between the charts highlight how staging can mitigate or intensify this. Additional details on the minimum, maximum, average and standard deviation of profit values per time period are given in Table 1.

In Figure 3 (a) we see the results in terms of the profit achieved at each period for lazy staging approach for 10 models. In this setting, the obtained function values for period $t = 4, 10, 13$ are the highest (\$378M, \$411M, \$673M), respectively, and the lowest profit value of $-\$0.5M$ is obtained in the period $t = 11$. The obtained variation in profit due to uncertainty is the highest for $t = 13$. Note that in practical mining scheduling often the profit at the beginning of mining is negative due to mining of covering waste blocks. Figure 3 (b) shows the results for the expected staging approach. The obtained profit is highest (\$570M, \$627M, \$497M) in the period $t = 10, 11, 14$, respectively. In contrast to the previous approach, we observe that the highest variation is obtained for period $t = 11$. Figure 3 (c) shows the uncertainty intensifying staging approach generates the worst profit values overall and obtains the highest profit values (\$482M, \$615M, \$471M) for period $t = 10, 11, 16$, respectively. More surprising, not only is the variation highest for $t = 16$ but we also observe high profit uncertainty at the end of the life of the mine, namely in the period $t = 17, 18, 19$. Finally, Figure 3 (d) shows the profit obtained using uncertainty mitigating staging approach. In this scenario, where geological uncertainty was considered in the stage design, the highest profit \$683M, occurred later than in previous approaches in the period $t = 14$ and the lowest negative profit $\$ - 66M$, in the period $t = 11$ except the first three periods. The obtained uncertainty range shows the similar course as in the Figure 3 (a) i.e., the greatest variability is observed in the period $t = 14$ where also the highest profit occurred.

We now consider the four different staging approaches with respect to NPV. In Figure 4 we show the NPV of the remaining reserve for each time period. When looking at an individual period, the NPV contribution to the overall objective function (1) represents a discount at time t. However when calculating the NPV of a mining schedule at time t as done in Figure 4, the value is calculated by summing the discounted per-period profit for all future periods starting at time t. In the box and whisker charts, we see period 1 with a high NPV, and it continues to grow as we remove waste material and uncover profit earning ore. As we begin to mine the ore and sell it in the later periods, the NPV decreases over time tending towards zero. Specific to the box and whisker plots, NPV is modelled giving us a confidence range. As the deposit is mined, the confidence remains the same until large areas of uncertainty are mined out.

Figure 4 (a) shows that the lazy staging approach obtained the highest NPV value of $2.70B in period $t = 4$ - very early in the life of the mine. Although highest NPV value $2.60B is achieved later in period $t = 6$ using the expected staging approach, we observe that from period $t = 12$ until the end of the mine the interquartile range of the remaining reserve is very small (see Figure 4 (b)). As expected in Figure 4 (c), the NPV value $3.05B using the uncertainty intensifying staging approach obtained the highest value at the latest period $t = 9$. Finally, Figure 4 (d) shows the highest NPV values $2.55B for period $t = 4$ under the mitigating staging approach. We observed higher range of different NPV values until period $t = 9$. Indeed, active attempts to spread uncertainty across stages and periods seems to have inflated overall economic uncertainty. This result is counter intuitive and requires further investigation and possibly a rethink on how best to deal with geological uncertainty via staging approaches.

Table 1 shows further details on the uncertainties. Although lazy staging approach achieves the highest mean profit \$479M in period $t = 13$, the standard deviation is \$94M which indicates the highest uncertainty among all periods. We see the same picture applying to the expected and uncertainty mitigating staging approach with the highest standard deviation (\$102M, \$108M) in the period $t = 11, 14$, respectively. In contrast, the uncertainty intensifying staging approach attains its highest standard deviation of \$82M in the period $t = 16$ which is only $1/5$ of maximal standard deviations of the other approaches.

7 DISCUSSION AND OPEN PROBLEMS

We have presented an approach for visualizing and quantifying economic uncertainty in mine planning based on geological uncertainty derived from a neural network approach to obtain multiple interpolations of the same input data. The approach allows mine planners to consider the effects of what they do not know during crucial planning periods and possibly take manual steps to mitigate downside risk. We pointed out the impact of staging on solutions mine plans produced by Evolution. Our results were successful in showing how different staging mechanisms can effect NPV, as well as per-period uncertainty.

However, our results do not yet inform how an algorithm could be designed to assist with this. A manual approach to staging with the intent of spreading economic uncertainty over several periods as a way of mitigating its effect on feasibility is not conclusively beneficial. We maintain that this is a worthy goal of further research. Even if a staging approach is shown to reduce NPV, if it can conclusively and consistently be shown to mitigate financial uncertainty arising from geological uncertainty using this problem model it will present an attractive alternative for mining operations and investors alike.

Table 1: Maximum (max), minimum (min), mean (mean), and standard deviation (std) in terms of profit ($) for four staging approaches.

	PROFIT															
	Lazy staging (1)				Expected staging (2)				Uncertainty intensifying staging (3)				Uncertainty mitigating staging (4)			
t	max	min	mean	std	max	min	mean	std	max	min	mean	std	max	min	mean	std
1	-130250000	-130250000	-130250000	0	-130250000	-130250000	-130250000	0	-130250000	-130250000	-130250000	0	-130250000	-130250000	-130250000	0
2	-130250000	-130250000	-130250000	0	-127686754	-128559124	-128076490	243683	-130250000	-130250000	-130250000	0	-130250000	-130250000	-130250000	0
3	-43003516	-65238038	-53494867	7500500	93001540	67567509	80687311	8226357	-130250000	-130250000	-130250000	0	-130250000	-130250000	-130250000	0
4	377666413	247890113	313721272	39142232	218149045	168226841	189805079	14313558	-129794911	-130240649	-130145265	144482	367832346	259189975	314227365	35783141
5	296127619	218012252	251672750	22165892	81011752	28819070	58600949	16406517	88693807	14915434	43601463	22286506	301647149	227163533	265172607	24613966
6	169539650	121938844	144194954	18338564	265053007	173179102	215034038	31266553	326229459	209235340	265896134	36358016	168098131	110723541	132181782	17534232
7	267020158	211454610	235159816	16766316	348489523	276429252	311037144	24907895	179146701	118519573	145160118	16673732	212831164	153862905	177876120	17947427
8	180310851	130687008	158126583	17171760	219158586	139847250	176464398	22352770	-84961879	-108577862	-94198476	6891741	255338344	184280502	218793008	26598564
9	345554357	295667982	320060358	19276574	168164965	36817077	97146590	35213116	252598223	208935015	230983938	13853225	506256332	317132886	392349639	51441530
10	411434723	281275037	344964781	38143934	569627084	392546645	460447937	52548953	481754826	389184095	437471124	35326818	53862053	20211592	37589529	11688541
11	52438199	-486424	15677302	14878463	626869693	311816368	431274119	101642852	614589821	402742016	501763759	65218777	-8531588	-65753828	-40946358	19497027
12	464568298	322983719	383684339	50020522	206731579	34644790	116739055	50845327	375660147	275456244	324942187	28357615	314692580	211746767	252115211	33940321
13	672523177	363047995	478726114	93598849	293783740	242667129	268556693	17400222	415836690	306678450	369831970	33855380	481632112	288395595	384683655	68455008
14	308380653	162121061	236215069	47591367	496790988	329700939	392946083	48819158	336055118	173434173	252025141	43405378	682758260	366744813	490368808	107628521
15	209445109	72629627	161148010	38802588	273540015	108530275	199842357	48473147	247701092	180485404	218347143	22579828	238932187	106568283	195479268	41676834
16	200415396	102244809	147342990	33477893	156055558	67903050	116403470	28051739	470789884	238359062	385135403	82014550	444685994	347422047	388895101	26157688
17	353293421	305559809	329706772	15778922	363243715	295126639	330782107	20721886	392264967	243191352	314440700	51228895	327148952	246849901	292070447	23650898
18	377405951	223747316	316250729	51127697	361215825	226612396	310344316	45730689	365001442	170015704	299139066	56015194	356645297	164372257	240253022	67935562
19	156685046	79124295	97223858	21918299	124470905	92660186	106904342	8835889	281719505	152039480	199074742	40268631	173314238	151690481	163403152	7332068
20	90566777	54415740	72449646	11856505	100881640	44903667	71143444	21069309	186475785	152039480	165831289	10749314	173314238	127021392	151076659	16539977

We now discuss important challenges when optimising extraction sequences in the face of geological uncertainty in the context of using the Evolution Strategy software. We concentrate on the problem of staging. Because the optimiser used in Evolution depends on the given stage design, an important problem is to formulate a stage design that leads to a high quality schedule: in terms of maximising NPV, in terms of feasibility and now also in terms of economic mitigation of geological uncertainty. The overall goal - a remaining open problem - is to produce stage designs in an automated way that lead to feasible extraction sequences exhibiting high NPV compared with others and low economic uncertainty in critical periods of the mine plan.

From our tests, the first observation is that uncertainty in our setting can be split into two categories: domain uncertainty and grade uncertainty. Domain uncertainty typically manifests by a block switching domain between different members of the ensemble. This effect would appear to have little influence on economic uncertainty. The impact of uncertainty observed in our experimental results is primarily related to grade uncertainty. Uncertainty in ore grade around the cut-off grade has a considerable effect on NPV. Most mines are scheduled based on their minimum cut-off grade, and there are typically large reserves of ore at or near this grade. If this ore was to fluctuate in grade by even a fraction of a percent, it could then be classified as waste. The effect of this re-classification is non-linear and can result in tens of millions of dollars being lost in unprofitable mining. This suggests that uncertainty in ore near the cut-off grade has a disproportionate effect.

Our attempts to mitigate uncertainty showed that in order to reduce the economic risk of mining highly uncertain high grade ore, it needs to be mined in the same period with highly certain, high grade ore. It could also be mined in the same period with a large deposit of highly certain low to medium grade ore. This suggests that it is not just grade, but also tonnage that needs to be considered when attempting to mitigate uncertainty across a schedule. Further work is required here.

It would be very valuable to design automated staging approaches that lead to a feasible extraction sequence with a high NPV while minimizing the risk associated with geological uncertainty. An avenue for future work is to take the quantified economic effect of uncertainty presented here into the fitness function used as part of the evolutionary algorithm optimiser in Strategy. It could do this by processing the full ensemble simultaneously. Additional constraints around the uncertainty for critical time periods of low cash flow could be introduced that prevent the mine plan from having a high uncertainty at such times.

ACKNOWLEDGEMENT

This work has been supported by the South Australian Government through the Research Consortium "Unlocking Complex Resources through Lean Processing".

REFERENCES

[1] Mohammad Reza Bonyadi and Zbigniew Michalewicz. 2016. Evolutionary Computation for Real-World Problems. In *Challenges in Computational Statistics and Data Mining*, Stan Matwin and Jan Mielniczuk (Eds.). Studies in Computational Intelligence, Vol. 605. Springer, 1–24. https://doi.org/10.1007/978-3-319-18781-5_1

[2] Michael J. Bourassa. 2021. *Practice Guides Mining*. Lexology Getting the Deal Through, London, UK. https://www.lexology.com/gtdt/guides/mining

[3] The Australasian Joint Ore Reserves Committee. 2012. The Australasian Code for Reporting of Exploration Results, Mineral Resources and Ore Reserves. (2012). http://www.jorc.org/docs/JORC_code_2012.pdf

[4] Shubhabrata Datta and J. Paulo Davim (Eds.). 2019. *Optimization in Industry, Present Practices and Future Scopes*. Springer. https://doi.org/10.1007/978-3-030-01641-8

[5] Sridhar Dutta, Sukumar Bandopadhyay, Rajive Ganguli, and Debasmita Misra. 2010. Machine Learning Algorithms and Their Application to Ore Reserve Estimation of Sparse and Imprecise Data. *J. Intell. Learn. Syst. Appl.* 2, 2 (2010), 86–96. https://doi.org/10.4236/jilsa.2010.22012

[6] Daniel G. Espinoza, Marcos Goycoolea, Eduardo Moreno, and Alexandra M. Newman. 2013. MineLib: a library of open pit mining problems. *Ann. Oper. Res.* 206, 1 (2013), 93–114. https://doi.org/10.1007/s10479-012-1258-3

[7] Ryan C. Goodfellow and Roussos G. Dimitrakopoulos. 2016. Global optimization of open pit mining complexes with uncertainty. *Appl. Soft Comput.* 40 (2016), 292–304.

[8] Maksud Ibrahimov, Arvind Mohais, Sven Schellenberg, and Zbigniew Michalewicz. 2014. Scheduling in iron ore open-pit mining. *The International Journal of Advanced Manufacturing Technology* 72, 5 (2014), 1021–1037. https://doi.org/10.1007/s00170-014-5619-8

[9] Thys B. Johnson. 1969. Optimum production scheduling. In *Proceedings of the 8th International Symposium on Computers and Operations Research, Salt Lake City*. 539–562.

[10] Bruce A. Kennedy. 1990. *Surface mining*. Society for Mining, Metallurgy & Exploration.

[11] Danie G. Krige. 1951. *A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand*. Chemical, Metallurgical and Mining Society of South Africa. http://www.saimm.co.za/Conferences/DanieKrige/DGK01b.pdf

[12] Amina Lamghari and Roussos G. Dimitrakopoulos. 2020. Hyper-heuristic approaches for strategic mine planning under uncertainty. *Comput. Oper. Res.* 115 (2020), 104590.

[13] Helmut Lerchs and Ingo F. Grossmann. 1965. Optimum design of open-pit mines. *Canadian Mining and Metallurgical Bulletin, CIM Bulletin, Montreal, Canada* (1965), 47–54. https://store.cim.org/en/optimum-design-of-open-pit-mines

[14] Orlando Rivera Letelier, Daniel G. Espinoza, Marcos Goycoolea, Eduardo Moreno, and Gonzalo Muñoz. 2020. Production Scheduling for Strategic Open Pit Mine Planning: A Mixed-Integer Programming Approach. *Oper. Res.* 68, 5 (2020), 1425–1444.

[15] Maptek Pty. Ltd. 2021. Maptek Evolution. https://www.maptek.com/products/evolution/index.html.

[16] MineMax Pty. Ltd. 2021. Minemax Scheduler. https://www.minemax.com/products/scheduler/.

[17] Luis Montiel and Roussos G. Dimitrakopoulos. 2017. A heuristic approach for the stochastic optimization of mine production schedules. *J. Heuristics* 23, 5 (2017), 397–415.

[18] Gonzalo Muñoz, Daniel G. Espinoza, Marcos Goycoolea, Eduardo Moreno, Maurice Queyranne, and Orlando Rivera Letelier. 2018. A study of the Bienstock-Zuckerberg algorithm: applications in mining and resource constrained project scheduling. *Comput. Optim. Appl.* 69, 2 (2018), 501–534.

[19] Seyyed Ali Nezamolhosseini, Seyed Hossein Mojtahedzadeh, and Javad Gholamnejad. 2017. The Application of Artificial Neural Networks to Ore Reserve Estimation at Choghart Iron Ore Deposit. *Analytical and Numerical Methods in Mining Engineering* 6 (2017), 73–83. http://anm.yazd.ac.ir/article_959_318b75919d244435649bd4933f6f74fb.pdf

[20] Julian M. Ortiz, Oy Leuangthong, and Clayton V. Deutsch. 2004. A MultiGaussian Approach to Assess Block Grade Uncertainty. In *CIM Conference and Exhibition, Edmonton 2004, May 9–12*. 1–10.

[21] Yuki Osada, R. Lyndon While, Luigi Barone, and Zbigniew Michalewicz. 2013. Multi-mine Planning using a Multi-objective Evolutionary Algorithm. In *IEEE Congress on Evolutionary Computation*. IEEE 2013, 2902–2909.

[22] Jose Saavedra-Rosas, Enrique Jeivez, J. Miguel Amaya, and Nelson Morales. 2016. Optimizing open-pit block scheduling with exposed ore reserve. *Journal of the Southern African Institute of Mining and Metallurgy* 116, 7 (2016), 655–662. https://doi.org/10.17159/2411-9717/2016/v116n7a7

[23] Sven Schellenberg, Xiaodong Li, and Zbigniew Michalewicz. 2016. Benchmarks for the Coal Processing and Blending Problem. In *Proceedings of the 2016 on Genetic and Evolutionary Computation Conference, Denver, CO, USA, July 20 - 24, 2016*, Tobias Friedrich, Frank Neumann, and Andrew M. Sutton (Eds.). ACM, 1005–1012. https://doi.org/10.1145/2908812.2908945

[24] Donald Shepard. 1968. A two-dimensional interpolation function for irregularly-spaced data. In *Proceedings of the 23rd ACM National Conference, 1968*. 517–524.

[25] Steve Sullivan, Chris Green, Derek Carter, Hugh Sanderson, and James Batchelor. 2019. Deep Learning - A New Paradigm for Orebody Modelling. *the AusIMM Mining Geology Conference, Perth, Western Australia 25-26 November, 2019* (2019). https://www.maptek.com/pdf/insight/Maptek_Deep_Learning_A_New_Paradigm_for_Orebody_Modelling.pdf