Generating Multi-Objective Bilevel Optimization Problems with Multiple Non-Cooperative Followers

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ABSTRACT

This paper presents a test problem generator for multi-objective bilevel optimization problems with multiple non-cooperative followers. In this type of search space the leader and its followers can have multiple conflicting objectives and interactions between the leader and each one of the followers. The test problem generator can be used to instantiate test problems with user-controlled features such as the number of followers, convergence and interaction complexity.

CCS CONCEPTS

• **Computing methodologies** → **Heuristic function construction**; **Continuous space search**;

KEYWORDS

bilevel optimization, multi-objective, multiple followers

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1 INTRODUCTION

Multi-objective Bilevel Optimization (MBO) is an emergent and important area. A variety of engineering and scientific applications can be found from process optimization, game-playing, optimal control, strategy development, transportation problems, among others [6]. This kind of problems have been solved using different approaches which include classical and heuristic methodologies [2]. Particularly, the Evolutionary Computation community has proposed different approaches to tackle MBO problems by using both, evolutionary and swarm intelligence algorithms with successful results in artificial and real-world problems. Regarding single-objective multi-follower bilevel optimization problems, they have been studied, from classical approaches to metaheuristic-based proposals [6]. It is worth mentioning that the work on MBO problems with multiple followers (MBOMFPs) is scarce. A theoretical study and an interactive algorithm to solve a MBOMFP is reported in [5]. The usage of an evolutionary algorithm to deal with a MBOMFP is detailed in [3].

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2 MULTI-OBJECTIVE BILEVEL OPTIMIZATION WITH MULTIPLE FOLLOWERS

A multi-objective bilevel optimization problem with multiple interconnected followers can be defined as in [5]. Four types of interactions between followers are considered [7]:

- Non-cooperative: The followers are not sharing their decision variables among them, then, they do not share their objectives and constraints.
- Cooperative: The decision variables, objective functions and constraints are completely shared among followers.
- Semi-cooperative: Each follower has different objective functions and constraints but can share the decision variables.
- Reference-uncooperative: Each follower has individual decision variables but can use some variables as references from other follower.

In this work, we are focused on such MBOMFPs where followers are non-cooperative (MBONMFP), because it is the most simple (but not less complex) multi-follower bilevel optimization problem [8]. The formal definition of MBONMFP is to:
Minimize

$$F(\vec{x}, \vec{y}^{(1)}, \vec{y}^{(2)}, \dots, \vec{y}^{(n)}) = \begin{pmatrix} F_1(\vec{x}, \vec{y}^{(1)}, \vec{y}^{(2)}, \dots, \vec{y}^{(n)}) \\ F_2(\vec{x}, \vec{y}^{(1)}, \vec{y}^{(2)}, \dots, \vec{y}^{(n)}) \\ \vdots \\ F_{m_u}(\vec{x}, \vec{y}^{(1)}, \vec{y}^{(2)}, \dots, \vec{y}^{(n)}) \end{pmatrix} \tag{1}$$

subject to $(\vec{x}, \vec{y}^{(1)}, \vec{y}^{(2)}, \dots, \vec{y}^{(n)}) \in \Omega_0$ where $\vec{y}^{(i)}$, $(i = 1, 2, \dots, n)$ solves

$$\min_{\vec{y}^{(i)}} f^{(i)}(\vec{x}, \vec{y}^{(i)}) = \left(f_1^{(i)}(\vec{x}, \vec{y}^{(i)}), \dots, f_{m_i}^{(i)}(\vec{x}, \vec{y}^{(i)})^T\right) \tag{2}$$

Note that the i-th follower (LL) is only interacting with the leader (UL). Moreover, this allows us to extend previous procedures for proposing test problems [1], but considering multiple non-cooperative followers.

3 TEST PROBLEM CONSTRUCTION

We suggest the following test problem construction procedure for a MBONMFP having n non-cooperative followers. The proposed procedure contains the following mappings to meet the desired properties. Firstly, let us split both, upper and lower level decision vectors as follows: $\vec{x} = (\vec{x}_1, \vec{x}_2, \vec{x}_3) \in X_1 \times X_2 \times X_3 = X$ whilst the i-th follower decision vector is $\vec{y}^{(i)} = (\vec{y}_1^{(i)}, \vec{y}_2^{(i)}, \vec{y}_3^{(i)}) \in Y_1^{(i)} \times$

$Y_2^{(i)} \times Y_3^{(i)} = Y^{(i)}$. Now, for the i-th follower: $p^{(i)} : X_1 \times Y_1^{(i)} \to \mathbb{R}$ controls the interaction complexity between the leader and the i-th follower. $q^{(i)} : Y_2^{(i)} \to \mathbb{R}$ is defined to add convergence complexity to the i-th follower problem. $r_1^{(i)}, r_2^{(i)} : X_3 \times Y_3^{(i)} \to \mathbb{R}$ are used to control the Pareto-optimal front.

For the upper level optimization problem: $P : X_1 \times Y_1^{(1)} \times Y_1^{(2)} \times \cdots \times Y_1^{(n)} \to \mathbb{R}$ is defined to control the interaction between the followers and the leader. Here, $P(\vec{x}_1, \vec{y}_1^{(1)}, \ldots, \vec{y}_1^{(n)}) = 0$ if and only if $(\vec{x}_1, \vec{y}_1^{(1)}, \ldots, \vec{y}_1^{(n)})$ is part of a feasible solution and is related to an optimal solution at the upper level. $Q : X_2 \to \mathbb{R}$ is used to control the convergence complexity within the leader problem. $R_1,\ R_2 : X_3 \times Y_3^{(1)} \times Y_3^{(2)} \times \cdots \times Y_3^{(n)} \to \mathbb{R}$ denote conflicting objectives affected by both, leader and followers. Thus, the following optimization problem is defined to meet the properties presented in [1]. Minimize:

$$F(\vec{x}, \vec{y}^{(1)}, \vec{y}^{(2)}, \ldots, \vec{y}^{(n)}) = \tag{3}$$

$$\begin{pmatrix} P(\vec{x}_1, \vec{y}_1^{(1)}, \ldots, \vec{y}_1^{(n)}) + Q(\vec{x}_2) + R_1(\vec{x}_3, \vec{y}_3^{(1)}, \ldots, \vec{y}_3^{(n)}) \\ P(\vec{x}_1, \vec{y}_1^{(1)}, \vec{y}_1^{(2)}, \ldots, \vec{y}_1^{(n)}) + Q(\vec{x}_2) + R_2(\vec{x}_3, \vec{y}_3^{(1)}, \ldots, \vec{y}_3^{(n)}) \end{pmatrix} \tag{4}$$

subject to: $\vec{y}^{(i)} \in \arg\min_{\vec{y}^{(i)} \in Y^{(i)}} \{f^{(i)}(\vec{x}, \vec{y}^{(i)})\}$, where

$$f^{(i)}(\vec{x}, \vec{y}^{(i)}) = \begin{pmatrix} p^{(i)}(\vec{x}_1, \vec{y}_1^{(i)}) + q^{(i)}(\vec{y}_2^{(i)}) + r_1^{(i)}(\vec{x}_3, \vec{y}_3^{(i)}) \\ p^{(i)}(\vec{x}_1, \vec{y}_1^{(i)}) + q^{(i)}(\vec{y}_2^{(i)}) + r_2^{(i)}(\vec{x}_3, \vec{y}_3^{(i)}) \end{pmatrix} \tag{5}$$

Here, $p^{(i)}$ must satisfy the following: $\min_{\vec{y}^{(i)}} p^{(i)}(\vec{x}, \vec{y}^{(i)}) = 0$, for all $i = 1, \ldots, n$. Moreover, here

$$P\left(\vec{x}_1, \vec{y}_1^{(1)}, \vec{y}_1^{(2)}, \ldots, \vec{y}_1^{(n)}\right) = P'(\vec{x}_1) + \sum_{i=1}^{n} p^{(i)}\left(\vec{x}_1, \vec{y}_1^{(i)}\right) \tag{6}$$

which implies that $\min_{\vec{y}_1^{(i)}} P(\vec{x}_1, \vec{y}_1^{(1)}, \vec{y}_1^{(2)}, \ldots, \vec{y}_1^{(n)}) = P'(\vec{x}_1)$ if the solution $(\vec{x}_1, \vec{y}_1^{(1)}, \vec{y}_1^{(2)}, \ldots, \vec{y}_1^{(n)})$ corresponds to a feasible solution.

3.1 Example

Assume that $X_1 \subset \mathbb{R}^k$, $X_2 \subset \mathbb{R}^u$ and $X_3 \subset \mathbb{R}^2$ at the UL; and for the LL, $Y_1^{(i)} \subset \mathbb{R}^k$, $Y_2^{(i)} \subset \mathbb{R}^l$ and $Y_3^{(i)} \subset \mathbb{R}^2$. Here, k controls the number of variables interacting between follower i and the leader; u and l control the number of variables that add complexity in convergence at UL and LL, respectively. If D_{ul} and D_{ll} denotes the UL and LL number of decision variables, respectively, we suggest $k = \min\{D_{ul}, D_{ll}\}/2$, $u = D_{ul} - k - 2$, $l = D_{ll} - k - 2$.

In this example, let us consider an UL uni-frontal problem, each follower is also an uni-frontal problem. Note that, $X_1 = Y_1 = [-10, 10]^k$, $X_2 = [-10, 10]^u$, $Y_2 = [-10, 10]^l$ and $X_3 = Y_3 = [0, 1]^2$.

$$p^{(i)}(\vec{x}_1, \vec{y}_1^{(i)}) = \sum_{j=1}^{k}\left(\frac{x_{1,j}}{i} - y_{1,j}^{(i)}\right)^2, \quad q^{(i)}(\vec{y}_2^{(i)}) = \sum_{j=1}^{l}\left(y_{2,j}^{(i)}\right)^2$$

$$r_1^{(i)}(\vec{x}_3, \vec{y}_3^{(i)})) = \begin{cases} x_{3,1} + \lfloor 3y_{3,1}^{(i)} \rfloor + y_{3,2}^{(i)}, & \text{if } i \bmod 2 = 0 \\ x_{3,1} + \lfloor 3y_{3,1}^{(i)} \rfloor + \left(y_{3,2}^{(i)}\right)^2, & \text{otherwise} \end{cases}$$

$$r_2^{(i)}(\vec{x}_3, \vec{y}_3^{(i)})) = \begin{cases} x_{3,2} - \lfloor 3y_{3,1}^{(i)} \rfloor + 1 - \sqrt{|y_{3,2}^{(i)}|} & \text{if } i \bmod 2 = 0 \\ x_{3,2} - \lfloor 3y_{3,1}^{(i)} \rfloor + (y_{3,2}^{(i)} - 1)^2 & \text{otherwise} \end{cases}$$

$$P\left(\vec{x}_1, \vec{y}_1^{(1)}, \vec{y}_1^{(2)}, \ldots, \vec{y}_1^{(n)}\right) = \sum_{j=1}^{k}(x_{1,j} - 1)^2 + \sum_{i=1}^{n} p^{(i)}\left(\vec{x}_1, \vec{y}_1^{(i)}\right)$$

$$R_1^{(i)}(\vec{x}_3, \vec{y}_3^{(1)}, \vec{y}_3^{(2)}, \ldots, \vec{y}_3^{(n)}) = \lfloor (n+1)x_{3,1} \rfloor + x_{3,2} \tag{7}$$

$$R_2^{(i)}(\vec{x}_3, \vec{y}_3^{(1)}, \vec{y}_3^{(2)}, \ldots, \vec{y}_3^{(n)}) = -\lfloor (n+1)x_{3,1} \rfloor + 1 - \sqrt{\frac{x_{3,2}}{\theta}} \tag{8}$$

$$Q(\vec{x}_2) = \sum_{j=1}^{u}(x_{2,j} - 1)^2, \qquad \theta = 1 + \frac{9}{n} \sum_{\substack{i=1 \\ 1/3 \le y_{3,1}^{(i)} < 2/3}}^{n} y_{3,1}^{(i)}, \tag{9}$$

where θ is used to determine the LL variable values that let the leader find well distributed solutions at the UL Pareto-optimal front. Here, the i-th follower has non-dominated solutions for a given $\vec{x} \in X$ when $y_{1,j}^{(i)} = x_1/j$, $\vec{y}_2^{(i)} = \vec{0}$, and $\vec{y}_3^{(i)} \in Y_3^{(i)}$. Besides, the non-dominated solutions at the upper level are obtained when $\vec{x}_1 = \vec{1}$, $\vec{x}_2 = \vec{1}$, $\vec{x}_3 \in X_3$, $\vec{y}^{(i)}$ generate a non-dominated solution for the i-th follower and $1/3 \le y_{3,1}^{(i)} < 2/3$, $y_{3,2}^{(i)} \in [0, 1]$.

This problems was solved by a nested MOEA/D-DE [4] (with parameters $F = 0.5$, $CR = 1$, $\theta = 20$, $p_m = 1/10$, $H = 299$, $T = 20$) and obtained the following Inverted Generation Distance (IGD) values: minimum (0.2437), median (0.3108) and maximum (0.4414) from 11 independent runs (source code available at bi-level.org).

4 CONCLUSIONS

In this work, we have proposed a test problem generator for multi-objective bilevel optimization problems with multiple non-cooperative followers. From it, we exemplify the usage by proposing a scalable test problem in which the user can control the number of followers, the number of decision variables, the convergence complexity and the distribution of the Pareto-optimal front. After that, a nested MOEA/D-DE algorithm was used to solve this particular test instance, being unable to get a good approximation to the optimum Pareto front.

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