Two Comprehensive Performance Metrics for Overcoming the Deficiencies of IGD and HV

Liping Wang
Zhejiang University of Technology
Zhejiang, China
690589058@qq.com

Lin Zhang
Zhejiang University of Technology
Zhejiang, China
916289813@qq.com

Yu Ren
Zhejiang University of Technology
Zhejiang, China
774305752@qq.com

Qicang Qiu
Zhejiang Lab
Zhejiang, China
qiuqc@zhejianglab.com

Feiyue Qiu
Zhejiang University of Technology
Zhejiang, China
qfy@zjut.edu.cn

ABSTRACT

To overcome some deficiencies of inverted generational distance (IGD) and hypervolume (HV), two comprehensive metrics are proposed in this paper, the hypercube distance (HCD), a metric based on hypercubes, and the angle-based distance (AD) for calculating the cosine values of the angles between solutions, both proposed metrics don't need Pareto Front information and have low computational complexity.

CCS CONCEPTS

• **Theory of computation → Theory of randomized search heuristics**.

KEYWORDS

Multi-objective optimization, Performance measures

ACM Reference format:

Liping Wang, Lin Zhang, Yu Ren, Qicang Qiu and Feiyue Qiu. 2021. Two comprehensive performance metrics for overcoming the deficiencies of IGD and HV. In *Proceedings of the Genetic and Evolutionary Computation Conference 2021 (GECCO '21). ACM, New York, NY, USA, 2 pages*. https://doi.org/10.1145/3449726.3459451

1 INTRODUCTION

The two most popular comprehensive metrics for multi-objective evolutionary algorithms (MOEAs) are IGD [1] and HV [2], however, IGD requires Pareto Front information and may provide misleading results when the reference points are not reasonably set [3]; The computational complexity of HV is rather high and the reference point or shape of the PF may lead to some errors when calculating the HV [2]. In addition, when the convergence of solution set S is poor, it becomes difficult for both metrics to measure the diversity of S [4].

2 PROPOSED METRICS

2.1 Hypercube Distance

For each individual, the hypercube distance is made up of two parts: one is the hypercube occupied by each solution, which mainly reflects the diversity of the solution set, and the distance to the origin, which mainly reflects the convergence of the solution set. The HCD is defined as follows:

$$\mathrm{HCD}(S) = \frac{1}{|S|}\sum_{x\in S}\frac{hypercube(x)}{d(x)} \quad (1)$$

$$hypercube(x) = 2^m \times \min_{x,y\in S\wedge x\neq y}\left(\max_{x,y\in S\wedge x\neq y}(x_i - y_i)^m\right) \quad (2)$$

where $d(x) = \|x - (0,\cdots,0)\|$. In addition, $x_i - y_i$ is the difference between x and y on the m^{th} objective. A larger HCD is preferred, and its computational complexity is $O(m|S|^2)$.

Fig. 1(a) provides an illustrative example of how HCD is calculated. For solution x^1, we can construct a maximum cube where there is only a solution x^3 on the surface of the cube, then calculate the volume of the cube by calculating l_1, which represents half of the side length of the cube, where d_1 represents the distance from x^1 to the origin.

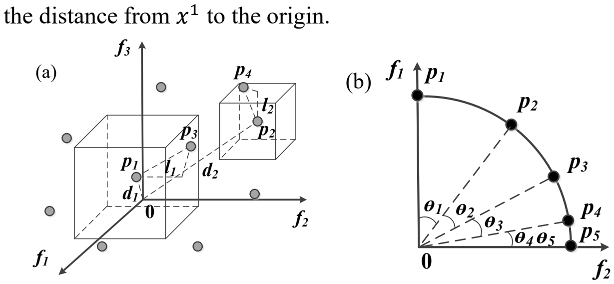

Figure 1: Illustration of how HCD is calculated. For instance, $\mathrm{HCD}(x^1) = 2^3 \times l_1/d_1$, $\mathrm{HCD}(x^2) = 2^3 \times l_2/d_2$. If $d(x) = 1$, then $AD(S) = \left(\sum_{i=1}^{4}\cos(\theta_i) + \cos(\theta_4)\right)/5$.

ACM ISBN 978-1-4503-8351-6/21/07.
https://doi.org/10.1145/3449726.3459451

2.2 Angular Distance

For each individual, the minimum angle between itself and the other individuals can reflect the diversity of the solution set, and the greater the larger degree of the angle, which can be compared by calculating the cosine value. Meanwhile, the maximum angle between the whole population and each objective can be used to evaluate the spread of the solution set, and thus the AD should be divided by the maximum cosine of the largest angles. The AD also needs to multiply the distance to the origin to measure the convergence of the solution set. The AD is defined as follows:

$$\mathrm{AD}(S) = \frac{\sum_{x \in S} \max_{y \in S \wedge x \neq y} \cos(x, y) \times d(x)}{|S| \times \prod_{i \in m} \max_{x \in S} \cos s(x, e^i)} \tag{3}$$

where $d(x) = \|x - (0, \cdots, 0)\|$, e^i is the unit vector on the i^{th} objective. The larger value of AD is preferred, and its computational complexity is $O(m|S|^2)$. Fig. 1(b) provides an illustrative example of how the AD is calculated.

3 RESULTS AND DISCUSSION

3.1 Theoretical study

To better understand the capacity of measuring the diversity of the metrics proposed, we calculated the values of six different solution sets with different diversities on the same Pareto Front. The solutions on $f_1 + f_2 + f_3 = 1$ and $f_1^2 + f_2^2 + f_3^2 = 1$ are shown in Fig. 2 and Fig. 3 respectively, all solutions have good convergence. The results in Table 1 show proposed metrics have strong consistency with IGD and HV when measuring diversity.

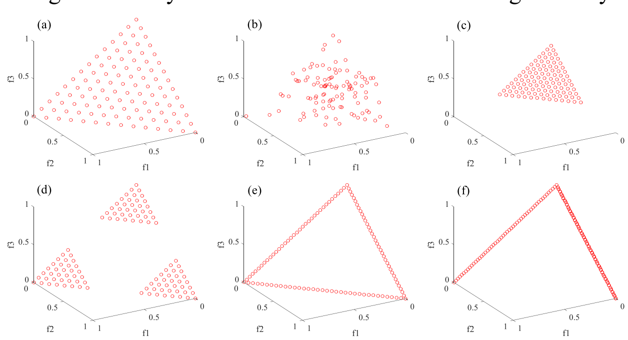

Figure 2: Six different solution set with PF $f_1 + f_2 + f_3 = 1$.

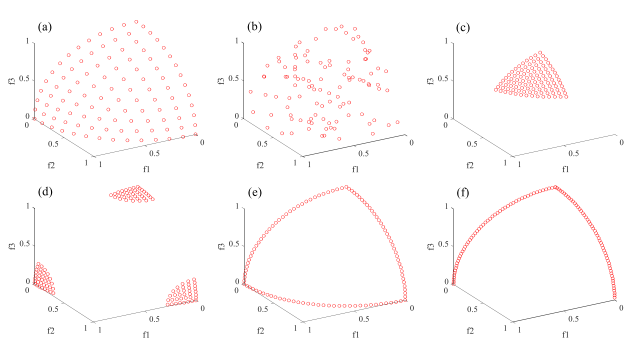

Figure 3: Six different solution set with PF $f_1^2 + f_2^2 + f_3^2 = 1$.

Table 1: Values and Ranks of different metrics in Fig. 2&3

Fig	HCD	AD	IGD	HV
2(a)	5.1047e-3 (1)	7.2096e-1 (2)	3.7950e-2 (1)	7.9290e-1 (1)
2(b)	1.4613e-3 (2)	6.6321e-1 (1)	5.8846e-2 (2)	7.4595e-1 (2)
2(c)	7.2829e-4 (4)	7.7088e-1 (3)	1.1774e-1 (3)	5.4833e-1 (6)
2(d)	1.0802e-3 (3)	8.0476e-1 (4)	1.2167e-1 (4)	6.7649e-1 (4)
2(e)	2.3255e-4 (5)	8.1096e-1 (5)	1.3458e-1 (5)	7.3908e-1 (3)
2(f)	7.0781e-5 (6)	8.1311e-1 (6)	2.0248e-1 (6)	6.5699e-1 (5)
3(a)	9.9580e-3 (1)	9.9208e-1 (1)	5.1625e-2 (1)	4.2090e-1 (1)
3(b)	2.5758e-3 (2)	1.0535e-0 (5)	7.3318e-2 (2)	3.8683e-1 (3)
3(c)	3.4501e-4 (5)	1.8457e-0 (6)	2.8739e-1 (6)	1.8647e-1 (5)
3(d)	4.7477e-4 (4)	9.9920e-1 (3)	2.4843e-1 (4)	9.5327e-2 (6)
3(e)	5.2894e-4 (3)	9.9899e-1 (2)	1.7139e-1 (3)	3.9843e-1 (2)
3(f)	1.5912e-4 (6)	1.0000e-0 (4)	2.5652e-1 (5)	3.2337e-1 (4)

Table 2: Values of different metrics on DTLZ1-4

$m = 3$		NSGA-II	MOEA/D	RVEA	IBEA
DTLZ1	HCD	2.5283e-3	**1.2927e-2**	1.2894e-2	7.9978e-3
	AD	3.7628e-1	3.6116e-1	**3.6106e-1**	5.0350e-1
	IGD	2.7259e-2	2.0570e-2	**2.0560e-2**	1.6778e-1
	HV	8.2454e-1	8.4156e-1	**8.4168e-1**	4.8063e-1
DTLZ2	HCD	2.9685e-3	**1.4245e-2**	1.4244e-2	4.7359e-3
	AD	1.0044e+0	**9.9043e-1**	9.9044e-1	9.9672e-1
	IGD	6.8681e-2	**5.4464e-2**	5.4464e-2	8.0562e-2
	HV	5.3277e-1	**5.5962e-1**	5.5961e-1	5.5764e-1
DTLZ3	HCD	3.0196e-3	1.4175e-2	**1.4203e-2**	4.8153e-4
	AD	1.0041e+0	9.9460e-1	**9.9327e-1**	1.0070e+0
	IGD	6.9971e-2	5.4898e-2	**5.4718e-2**	4.7630e-1
	HV	5.3152e-1	5.5407e-1	**5.5582e-1**	2.4839e-1
DTLZ4	HCD	2.8363e-3	1.0180e-2	**1.4772e-2**	4.8827e-3
	AD	4.3043e+0	6.3932e+0	**9.9057e-1**	9.9655e-1
	IGD	9.7642e-2	1.9798e-1	**5.9319e-2**	7.9437e-2
	HV	5.2031e-1	4.9254e-1	5.5590e-1	**5.5747e-1**

3.2 Empirical results

We also studied the performance of the MOEAs on DTLZ for three objectives. The simulation includes NSGA-II [5], MOEA/D [6], RVEA [7] and IBEA [8]. To make a fair comparison, 30 independent runs were performed for each MOEA with a maximum of 100, 000 function evaluations. In Table 2, Four metrics all have the same results on DTLZ2 and DTLZ3, which is proof of the practicability of the proposed metrics. For DTLZ4, our metrics show consistency with at least one traditional metric.

REFERENCES

[1] Carlos A. Coello Coello and Nareli Cruz Cortés. 2005. Solving multiobjective optimization problems using an artificial immune system. *Genetic programming and evolvable machines* 6, 2 (2005), 163-190.
[2] Eckart Zitzler, Lothar Thiele. 1998. Multiobjective optimization using evolutionary algorithms—a comparative case study. In *International Conference on Parallel Problem Solving from Nature*., Berlin, Germany, 292-301.
[3] Hisao Ishibuchi, Hiroyuki Masuda, Yuki Tanigaki and Yusuke Nojima. 2015. Modified distance calculation in generational distance and inverted generational distance. In *International conference on evolutionary multi-criterion optimization*. Guimarães, Portugal, 110-125.
[4] Zhenan He and Gary G. Yen. 2015. Visualization and performance metric in many-objective optimization. *IEEE Transactions on Evolutionary Computation* 20, 3 (2015), 386-402.
[5] Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal and T. Meyarivan. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE transactions on evolutionary computation* 6, 2 (2002), 182-197.
[6] Qingfu Zhang, & Hui Li. 2007. MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation* 11, 6 (2007), 712-731.
[7] Ran Cheng, Yaochu Jin, Markus Olhofer, Bernhard Sendhoff. (2016). A reference vector guided evolutionary algorithm for many-objective optimization. *IEEE Transactions on Evolutionary Computation* 20, 5 (2016), 773-791.
[8] Eckart Zitzler and Simon Künzli. 2004. Indicator-based selection in multiobjective search. In *International conference on parallel problem solving from nature*. Berlin, Germany, 832-842.