Optimization of Multi-Objective Mixed-Integer Problems with a Model-Based Evolutionary Algorithm in a Black-Box Setting

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CCS CONCEPTS

• **Computing methodologies** → *Search methodologies*; **Genetic algorithms**.

KEYWORDS

Evolutionary Algorithms, Mixed-Integer, Multi-Objective Optimization

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1 INTRODUCTION

Mixed-integer optimization, which focuses on problems where discrete and continuous variables exist simultaneously, is a well-known and challenging area for search algorithms. Mixed-integer optimization problems are especially difficult in a black-box setting where no structural problem information is available a-prior. In this paper we bring the strengths of the recently-proposed Genetic Algorithm for Model-Based mixed-Integer opTimization (GAMBIT) to the multi-objective (MO) domain, and determine whether the promising performance of GAMBIT is maintained. We introduce various mechanisms designed specifically for MO optimization resulting in MO-GAMBIT. We compare performance - in terms of the number of evaluations used - and runtime with alternative techniques, particularly linear scalarization and a selection of alternative MO algorithms. To this end, we introduce a set of objective functions which vary in composition in terms of discrete and continuous variables, as well as in the strength of dependencies between variables. Our results show that MO-GAMBIT can substantially outperform the alternative MO algorithms, thereby providing a promising new approach for multi-objective mixed-integer optimization in a black-box setting.

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2 GAMBIT AND MO-GAMBIT

MO-GAMBIT introduces new mechanisms to GAMBIT in order to handle multiple objectives. Some fundamental functionality of MO-GAMBIT however, is based on the existing, single-objective version of GAMBIT [5].

GAMBIT is a model-based EA designed for optimization of mixed-integer problems in a black-box setting. To overcome this initial lack of problem information, GAMBIT attempts to learn and exploit the underlying problem structure during execution, by means of estimating variable dependencies. To this end, GAMBIT utilizes a combination of a clustering mechanism with an integrated dependency models mechanism that constructs variable subsets which represent likely important building blocks. Performing variation by respecting the structural integrity indicated by these blocks, i.e, considering and treating variables in blocks in a joint fashion, can lead to efficient exploitation of problem structure, resulting in generating better new solutions faster. GAMBIT utilizes already existing approaches for learning and processing discrete and continuous variables as part of the integrated-models mechanism: The Linkage Tree Genetic Algorithm (LTGA) [6] and the Incremental Adapted Maximum-Likelihood Gaussian Model Iterated Density Estimation Evolutionary Algorithm (iAMaLGaM) [2] respectively for the discrete and continuous components and introduces new mechanisms to handle intra-variable domain depenencies.

In vast majority of cases a single solution cannot be considered a comprehensive solution to a MO problem, however. Instead, a typical solution to a MO problem is represented as a set of so-called Pareto-optimal solutions, which form a front of optimal solutions. A theoretically optimal solution to a MO problem is a set of such non-dominated solutions which form a Pareto-front. In order to estimate the Pareto front, MO-GAMBIT uses a variety of mechanisms. Specifically, **Elitist Archive** which uses a technique that adaptively changes the grid that governs the elitist archive so as to harbor a predefined maximum number of solutions, preventing occurrences of very similar solutions in the archive while promoting diversity [1] [4]. **Selection and Variation** mechanisms that allow MO-GAMBIT to specialize model-based optimization in different areas of the Pareto front by performing clustering in the objective space. **Multistart scheme** which dynamically introduces larger population sizes and more clusters, removing the need to explicitly set parameters such as cluster or population size.

3 BENCHMARKS AND ALGORITHMS

Each of the MO problems in Table 1 represents different features of a problem landscape, based on the objective components they are made up of, with varying degrees of inter- and intra-variable dependencies.

Table 1: Multi-Objective Problems

Name	Objectives
MO1	Obj1: $F_1(\mathbf{x_d}, \mathbf{x_c}) = F_{Onemax}(\mathbf{x_d}) + F_{Sphere}(\mathbf{x_c})$ Obj2: $F_2(\mathbf{x_d}, \mathbf{x_c}) = F_{Zeromax}(\mathbf{x_d}) + F_{Sphere-1}(\mathbf{x_c})$
MO2	Obj1:$F_1(\mathbf{x_d}, \mathbf{x_c}) = F_{Onemax}(\mathbf{x_d}) + F_{Sphere}(\mathbf{x_c})$ Obj2: $F_4(\mathbf{x_d}, \mathbf{x_c}) = F_{DT5}(\mathbf{x_d}) + F_{R.Ellip.}(\mathbf{x_c})$
MO3	Obj1: $F_3(\mathbf{x_d}, \mathbf{x_c}) = F_{DT5_{inv}}(\mathbf{x_d}) + F_{R.Ellip.-1}(\mathbf{x_c})$ Obj2: $F_4(\mathbf{x_d}, \mathbf{x_c}) = F_{DT5}(\mathbf{x_d}) + F_{R.Ellip.}(\mathbf{x_c})$

The following algorithms and variants were used for comparison with MO-GAMBIT.

Repeated GAMBIT with Objective Scalarization utilizes weighted scalarization of objectives with the single-objective GAMBIT: SO-GAMBIT-One-Norm and SO-GAMBIT-Infinity-Norm variants.

MO-iAMaLGaM models a Gaussian distribution using Pareto-dominance based solution raking, clustering the objective space and generating new solutions via sampling the clustered distribution.

NOMAD: The Nonlinear Optimization by Mesh Adaptive Direct Search algorithm implements the Mesh Adaptive Direct Search (MADS) algorithm, and is designed for black-box optimization under general nonlinear constraints in the MO setting [3].

4 RESULTS

MO-GAMBIT results show advantages of a dedicated multi-objective approach. On all three problems the overhead of MO mechanisms is outweighed by the resulting performance, as MO-GAMBIT outperforms both SO-GAMBIT variants.

From the results in it is clear that the MO-iAMaLGaM approach becomes very inefficient when facing problems with strong variable dependencies in the domain that they were not designed for. In terms of number of evaluations needed, NOMAD performs best on the MO1 problem, very quickly reaching high hyper-volume values. MO-GAMBIT requires more evaluations to reach the same values. However, in terms of runtime NOMAD is much slower than the remaining algorithms. With strong variable dependencies present in MO_2 andMO_3, MO-GAMBIT outperforms the remaining algorithms in terms of both number of evaluations and runtime needed to reach high values of the hyper-volume.

5 CONCLUSIONS

Our results show that single-objective scalarization-based approaches are less efficient and require additional parametrization. Direct extensions of multi-objective algorithms targeted at one type of variable (i.e. continuous or discrete) will likely also always fall short when faced with problem landscapes with strong variable dependencies in the variable domain they were not designed for, as illustrated with the MO-iAMaLGaM.

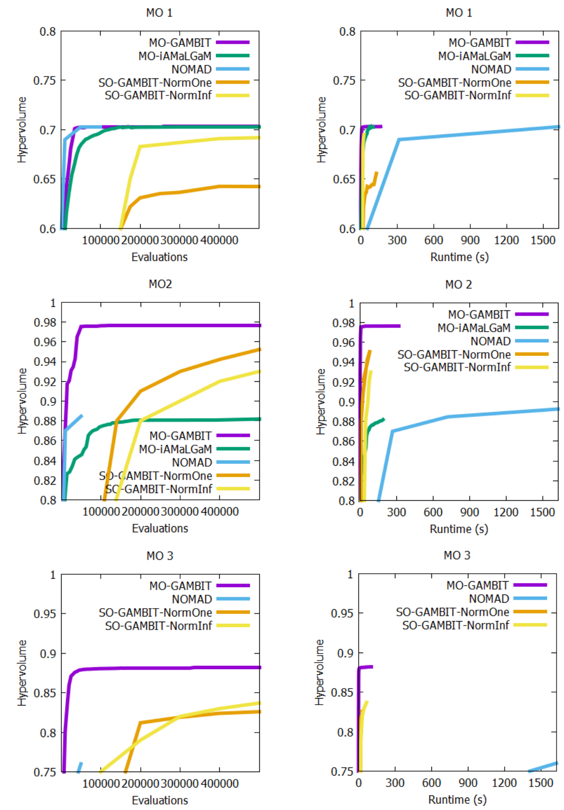

Figure 1: Number of black-box evaluations and run-times vs. hyper-volume of the selected MO algorithms on MO_1-MO_3

Overall, MO-GAMBIT achieved favorable results compared to alternatives, including a well-known mixed-integer optimization algorithm for MO problems, NOMAD. Good results were achieved on problems with and without variable dependencies, regardless of whether the Pareto front is convex or concave. We therefore believe that our results further motivate the use of MO-GAMBIT over all the alternatives considered in this paper for mixed-integer MO optimization.

REFERENCES

[1] P.A.N. Bosman and D. Thierens. 2003. The Balance Between Proximity and Diversity in Multiobjective Evolutionary Algorithms. *IEEE Transactions on Evolutionary Computation* 7, 2 (2003), 174–188.

[2] Peter A. N. Bosman, Jörn Grahl, and Dirk Thierens. 2008. Enhancing the Performance of Maximum-Likelihood Gaussian EDAs Using Anticipated Mean Shift. In *PPSN*. 133–143.

[3] Sébastien Le Digabel. 2011. Algorithm 909: NOMAD: Nonlinear Optimization with the MADS Algorithm. *ACM Trans. Math. Softw.* 37, 4, Article 44 (Feb. 2011), 15 pages.

[4] Ngoc Hoang Luong, Han La Poutré, and Peter A.N. Bosman. 2014. Multi-objective Gene-pool Optimal Mixing Evolutionary Algorithms. In *Proceedings of the 2014 Annual Conference on Genetic and Evolutionary Computation* (Vancouver, BC, Canada) *(GECCO '14)*. ACM, New York, NY, USA, 357–364.

[5] Krzysztof L. Sadowski, Dirk Thierens, and Peter A. N. Bosman. 2017. GAMBIT: A Parameterless Model-Based Evolutionary Algorithm for Mixed-Integer Problems. *Evolutionary computation, MIT Press* (2017), 1–27.

[6] D. Thierens and P.A.N. Bosman. 2011. Optimal Mixing Evolutionary Algorithms. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO '11)*. ACM, New York, NY, USA, 617–624.