Theoretical Analyses of Multi-Objective Evolutionary Algorithms on Multi-Modal Objectives (Hot-off-the-Press Track at GECCO 2021)

Benjamin Doerr*
Laboratoire d'Informatique (LIX)
École Polytechnique, CNRS
Institut Polytechnique de Paris
Palaiseau, France

Weijie Zheng*
Guangdong Provincial Key Laboratory of Brain-inspired Intelligent Computation
Department of Computer Science and Engineering
Southern University of Science and Technology
Shenzhen, China

ABSTRACT

Previous theory work on multi-objective evolutionary algorithms considers mostly easy problems that are composed of unimodal objectives. This paper takes a first step towards a deeper understanding of how evolutionary algorithms solve multi-modal multi-objective problems. We propose the OneJumpZeroJump problem, a bi-objective problem with single objectives isomorphic to the classic jump function benchmark. We prove that the simple evolutionary multi-objective optimizer (SEMO) cannot compute the full Pareto front. In contrast, for all problem sizes n and all jump sizes $k \in [4..\frac{n}{2}-1]$, the global SEMO (GSEMO) covers the Pareto front in $\Theta((n-2k)n^k)$ iterations in expectation. To improve the performance, we combine the GSEMO with two approaches, a heavy-tailed mutation operator and a stagnation detection strategy, that showed advantages in single-objective multi-modal problems. Runtime improvements of asymptotic order at least $k^{\Omega(k)}$ are shown for both strategies. Our experiments verify the substantial runtime gains already for moderate problem sizes. Overall, these results show that the ideas recently developed for single-objective evolutionary algorithms can be effectively employed also in multi-objective optimization.

This Hot-off-the-Press paper summarizes "Theoretical Analyses of Multi-Objective Evolutionary Algorithms on Multi-Modal Objectives" by B. Doerr and W. Zheng, which has been accepted for publication in AAAI 2021 [9].

*Both authors contributed equally to this work and both act as corresponding authors.

This work was supported by a public grant as part of the Investissement d'avenir project, reference ANR-11-LABX-0056-LMH, LabEx LMH. This work was also supported by Guangdong Basic and Applied Basic Research Foundation (Grant No. 2019A1515110177); Guangdong Provincial Key Laboratory (Grant No. 2020B121201001), the Program for Guangdong Introducing Innovative and Enterpreneurial Teams (Grant No. 2017ZT07X386); Shenzhen Peacock Plan (Grant No. KQTD2016112514355531); and the Program for University Key Laboratory of Guangdong Province (Grant No. 2017KSYS008).

GECCO '21 Companion, July 10–14, 2021, Lille, France

ACM ISBN 978-1-4503-8351-6/21/07.
https://doi.org/10.1145/3449726.3462719

CCS CONCEPTS

• **Theory of computation → Theory and algorithms for application domains**; *Theory of randomized search heuristics.*

KEYWORDS

Multi-objective evolutionary algorithms, multi-modal objectives, running time analysis, theory.

ACM Reference Format:
Benjamin Doerr and Weijie Zheng. 2021. Theoretical Analyses of Multi-Objective Evolutionary Algorithms on Multi-Modal Objectives (Hot-off-the-Press Track at GECCO 2021). In *2021 Genetic and Evolutionary Computation Conference Companion (GECCO '21 Companion), July 10–14, 2021, Lille, France.* ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3449726.3462719

SUMMARY OF OUR RESULTS

Many real-world applications contain multiple conflicting objectives. For such problems, a single best solution cannot be determined. Therefore, the task is to compute a set of solutions each of which cannot be improved without worsening in at least one objective (Pareto optima). With their population-based nature, multi-objective evolutionary algorithms (MOEAs) have been successfully applied here [25]. Similar to the situation in the theory of single-objective evolutionary algorithms, rigorous theoretical analyses of MOEAs fall far behind their successful applications in practice.

In order to reveal the working principles of MOEAs, the research has resorted to multi-objective, especially bi-objective, counterparts of well-analyzed single-objective benchmark functions used in evolutionary computation theory. For example, in the problems COCZ [18] and OneMinMax [13], the two objectives are both (conflicting) variants of the classic OneMax benchmark. The classic benchmark LeadingOnes was used to construct the LOTZ [17] and WLPTNO [22] problems. These multi-objective benchmark problems are among the most intensively studied [2, 6, 7, 12, 15, 16, 20]. We note that these problems are *unimodal* in the sense that from each set of solutions P a set P' witnessing the Pareto front can be computed by repeatedly selecting a solution from P, flipping a single bit in it, adding it to P, and removing dominated solutions from P. They are thus relatively easy to solve.

As in the theory of single-objective evolutionary computation, multi-modal problems are much less understood also in the theory of evolutionary multi-objective optimization. To the best of our knowledge, there is not a single work discussing in detail how

MOEAs cope with multimodality. There are works that contain multimodal problems, but they are using these problems mainly to study other research questions or the multimodality is only minor [3, 11, 19, 21].

Our contributions. This work aims at a deeper understanding how MOEAs cope with multi-objective problems with natural, well-analyzed, multi-modal objectives. In the theory of single-objective evolutionary computation, the class of JUMP function is a natural and intensively used multi-modal function class [10] that has inspired many interesting results including that larger mutation rates, crossover, and estimation of distribution algorithms as well as ant-colony optimizers help in the optimization of multi-modal functions [see, e.g., 1, 4, 5, 8, 14]. Hence, in this paper, we design a bi-objective counterpart of the JUMP function with problem size n and jump size k, called $\text{ONEJUMPZEROJUMP}_{n,k}$. It consists of one JUMP function w.r.t. the number of ones and one JUMP function w.r.t. the number of zeros. We compute its disconnected Pareto front.

We prove for all n and $k \in [2..\frac{n}{2}]$ that the simple evolutionary multi-objective optimizer (SEMO) cannot find the Pareto front, but that the global SEMO (GSEMO) finds the Pareto front in $O((n-2k)n^k)$ iterations in expectation. We show a matching lower bound of $\Omega((n-2k)n^k)$ for $k \in [4..\frac{n}{2}-1]$. Here and in the remainder, the asymptotic notation only hides constants independent of n and k.

We also consider two approaches that showed advantages in single-objective multi-modal problems. Via the heavy-tailed mutation proposed in [8], we improve the expected runtime of the GSEMO by a factor of $k^{\Omega(k)}$ to $O((n-2k)(en)^k/k^{k+0.5-\beta})$, where $\beta > 1$ is the power-law distribution parameter. Via a suitable adaptation of the stagnation detection strategy from [23] to multi-objective optimization, we obtain an expected runtime of $O((n-2k)(en)^k/k^k)$, again a $k^{\Omega(k)}$ factor improvement over the classic GSEMO and reducing the runtime guarantee for the heavy-tailed GSEMO by a (small) factor of $\Omega(k^{\beta-0.5})$. Our experiments show that these are not only asymptotic differences, but that roughly a factor-5 speed-up with heavy-tailed mutation and a factor-10 speed-up with stagnation detection can be observed already for jump size $k = 4$ and problem sizes n between 10 and 50.

Impact and further discussion. Overall, this work suggests that the recently developed ideas to cope with multimodality in single-objective evolutionary optimization can be effective in multi-objective optimization as well. In this first work in this direction, we only concentrated on mutation-based algorithms. The theory of evolutionary computation has also observed that crossover and estimation-of-distribution algorithms can be helpful in multi-modal optimization. Investigating to what degree these results extend into multi-objective optimization is clearly an interesting direction for future research.

Also, we only covered very simple MOEAs in this work. Analyzing more complex MOEAs such as the successful decomposition-based MOEA/D [15, 16, 19, 24] would be highly interesting. This would most likely require an adaptation of our benchmark problem. Since the difficult-to-find extremal points of the front are just the solutions of the single-objective sub-problems, and thus the two problems that naturally are part of the set of subproblems regarded by the MOEA/D, this algorithm might have an unfair advantage on the ONEJUMPZEROJUMP problem.

REFERENCES

[1] Riade Benbaki, Ziyad Benomar, and Benjamin Doerr. 2021. A rigorous runtime analysis of the 2-MMAS_{ib} on jump functions: ant colony optimizers can cope well with local optima. In *GECCO 2021*. ACM.

[2] Chao Bian, Chao Qian, and Ke Tang. 2018. A general approach to running time analysis of multi-objective evolutionary algorithms. In *IJCAI 2018*. IJCAI, 1405–1411.

[3] Dimo Brockhoff, Tobias Friedrich, Nils Hebbinghaus, Christian Klein, Frank Neumann, and Eckart Zitzler. 2007. Do additional objectives make a problem harder?. In *GECCO 2007*. ACM, 765–772.

[4] Duc-Cuong Dang, Tobias Friedrich, Timo Kötzing, Martin S. Krejca, Per Kristian Lehre, Pietro S. Oliveto, Dirk Sudholt, and Andrew M. Sutton. 2018. Escaping local optima using crossover with emergent diversity. *IEEE Transactions on Evolutionary Computation* 22 (2018), 484–497.

[5] Benjamin Doerr. 2019. A tight runtime analysis for the cGA on jump functions: EDAs can cross fitness valleys at no extra cost. In *GECCO 2019*. ACM, 1488–1496.

[6] Benjamin Doerr, Wanru Gao, and Frank Neumann. 2016. Runtime analysis of evolutionary diversity maximization for ONEMINMAX. In *GECCO 2016*. ACM, 557–564.

[7] Benjamin Doerr, Bojana Kodric, and Marco Voigt. 2013. Lower bounds for the runtime of a global multi-objective evolutionary algorithm. In *CEC 2013*. IEEE, 432–439.

[8] Benjamin Doerr, Huu Phuoc Le, Régis Makhmara, and Ta Duy Nguyen. 2017. Fast genetic algorithms. In *GECCO 2017*. ACM, 777–784.

[9] Benjamin Doerr and Weijie Zheng. 2021. Theoretical analyses of multi-objective evolutionary algorithms on multi-modal objectives. In *AAAI 2021*. AAAI. To appear. Full version at https://arxiv.org/abs/2012.07231.

[10] Stefan Droste, Thomas Jansen, and Ingo Wegener. 2002. On the analysis of the (1+1) evolutionary algorithm. *Theoretical Computer Science* 276 (2002), 51–81.

[11] Tobias Friedrich, Nils Hebbinghaus, and Frank Neumann. 2010. Plateaus can be harder in multi-objective optimization. *Theoretical Computer Science* 411, 6 (2010), 854–864.

[12] Oliver Giel. 2003. Expected runtimes of a simple multi-objective evolutionary algorithm. In *CEC 2003*, Vol. 3. IEEE, 1918–1925.

[13] Oliver Giel and Per Kristian Lehre. 2010. On the effect of populations in evolutionary multi-objective optimisation. *Evolutionary Computation* 18, 3 (2010), 335–356.

[14] Václav Hasenöhrl and Andrew M. Sutton. 2018. On the runtime dynamics of the compact genetic algorithm on jump functions. In *GECCO 2018*. ACM, 967–974.

[15] Zhengxin Huang and Yuren Zhou. 2020. Runtime analysis of somatic contiguous hypermutation operators in MOEA/D framework. In *AAAI 2020*, Vol. 34. AAAI, 2359–2366.

[16] Zhengxin Huang, Yuren Zhou, Zefeng Chen, and Xiaoyu He. 2019. Running time analysis of MOEA/D with crossover on discrete optimization problem. In *AAAI 2019*, Vol. 33. AAAI, 2296–2303.

[17] Marco Laumanns, Lothar Thiele, and Eckart Zitzler. 2004. Running time analysis of multiobjective evolutionary algorithms on pseudo-boolean functions. *IEEE Transactions on Evolutionary Computation* 8, 2 (2004), 170–182.

[18] Marco Laumanns, Lothar Thiele, Eckart Zitzler, Emo Welzl, and Kalyanmoy Deb. 2002. Running time analysis of multi-objective evolutionary algorithms on a simple discrete optimization problem. In *PPSN 2002*. Springer, 44–53.

[19] Yuan-Long Li, Yu-Ren Zhou, Zhi-Hui Zhan, and Jun Zhang. 2016. A primary theoretical study on decomposition-based multiobjective evolutionary algorithms. *IEEE Transactions on Evolutionary Computation* 20, 4 (2016), 563–576.

[20] Edgar Covantes Osuna, Wanru Gao, Frank Neumann, and Dirk Sudholt. 2020. Design and analysis of diversity-based parent selection schemes for speeding up evolutionary multi-objective optimisation. *Theoretical Computer Science* 832 (2020), 123–142.

[21] Chao Qian, Ke Tang, and Zhi-Hua Zhou. 2016. Selection hyper-heuristics can provably be helpful in evolutionary multi-objective optimization. In *PPSN 2016*. Springer, 835–846.

[22] Chao Qian, Yang Yu, and Zhi-Hua Zhou. 2013. An analysis on recombination in multi-objective evolutionary optimization. *Artificial Intelligence* 204 (2013), 99–119.

[23] Amirhossein Rajabi and Carsten Witt. 2020. Self-adjusting evolutionary algorithms for multimodal optimization. In *GECCO 2020*. ACM, 1314–1322.

[24] Qingfu Zhang and Hui Li. 2007. MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation* 11, 6 (2007), 712–731.

[25] Aimin Zhou, Bo-Yang Qu, Hui Li, Shi-Zheng Zhao, Ponnuthurai Nagaratnam Suganthan, and Qingfu Zhang. 2011. Multiobjective evolutionary algorithms: A survey of the state of the art. *Swarm and Evolutionary Computation* 1, 1 (2011), 32–49.