Adversarial Bandit Gene Expression Programming for Symbolic Regression

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ABSTRACT

Gene expression programming (GEP) is a commonly used approach in symbolic regression (SR). However, GEP often falls into a premature convergence and may only reach a local optimum. To solve the premature convergence problem, we propose a novel algorithm based on an adversarial bandit technique, named AB-GEP. AB-GEP segments the mathematical space into many subspaces. It leverages a new selection method, AvgExp3, to enhance the population jump between segmented subspaces while maintaining the population diversity. AvgExp3 dynamically estimates a subspace by rewards generated from AB-GEP without any assumption about the distribution of subspace rewards, making AB-GEP choose the appropriate subspace that contains the correct results. The experimental evaluation shows that the proposed AB-GEP method can maintain the population diversity and obtain better results than canonical GEPs.

CCS CONCEPTS

• **Computing methodologies** → **Genetic programming**; *Machine learning approaches*;

KEYWORDS

adversarial bandit, symbolic regression, gene expression programming

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1 INTRODUCTION

For a given dataset $\{X, Y\}$, the goal of symbolic regression (SR) is to find a symbolic function $f(X) = Y'$ that can minimize the distance between Y' and Y in the mathematical expression space. Although many genetic programming (GP) algorithms have shown to be efficient in SR, they tend to converge prematurely and fall into a local optimum owing to the decline in population diversity. For maintaining a healthy population diversity, [3] adopts a static space partition method (**SPJ-GEP**) to let population jump between segmented subspaces. However, SPJ-GEP implementing the UCB and ϵ-greedy strategy to make the population jump between subspaces will become invalid over time. To avoid the performance degradation, we propose a new adversarial bandit based gene expression programming (**AB-GEP**) algorithm. As shown in Figure 1, AB-GEP first partitions the mathematical space into k subspaces. Then, it leverages a new subspace selection method (called **AvgExp3**) which modifies the adversarial bandit Exp3 [2] to choose one subspace. AvgExp3 evaluates the subspace according to the rewards dynamically generated by exploiting subspaces, so it can get the evaluation result more accurately. Next, AB-GEP uses the crossover operator to make the population jump between subspaces. After the population jumps into a subspace, the population recombines with the historically best individual in the subspace to exploit this subspace quickly.

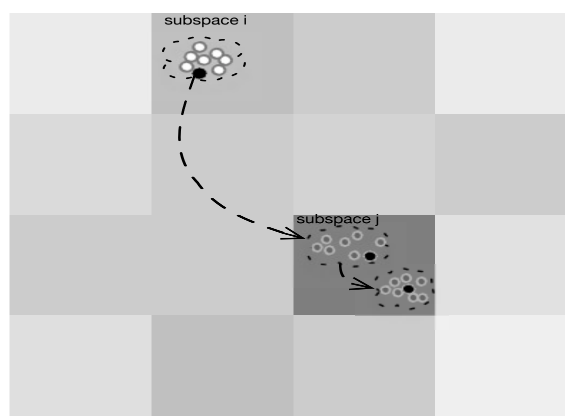

Figure 1: AB-GEP framework

2 ADVERSARIAL BANDIT GEP

The space-partition tree [3] divides the mathematical expression space Ω into k subspaces $\{\omega_1, \omega_2, \ldots \omega_k\}$. If each subspace ω_i is regarded as an arm in a k-armed bandit, the partitioned subspaces

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$\{\omega_1, \omega_2, \dots \omega_k\}$ are the k-armed bandit environment (adversary). Then SR is similar to the k-armed adversarial bandit problem, i.e., balancing between subspace exploration and subspace exploitation to discover the best mathematical expression that is fitted to the dataset.

2.1 Choosing Subspace

AB-GEP obtains the probability (P_{t,ω_i}) of choosing each subspace ω_i at round (generation) t according to the probability distribution $\mathcal{P}_t = \{P_{t,\omega_1}, ..., P_{t,\omega_k}\}$. P_{t,ω_i} is calculated by Equation 1.

$$P_{t,\omega_i} = \frac{e^{\alpha \hat{S}'_{t-1,\omega_i}}}{\sum_{j=1}^{k} e^{\alpha \hat{S}'_{t-1,\omega_i}}}. \tag{1}$$

where α is the learning rate and $\hat{S}'_{t-1,\omega_i}$ is the average estimated reward of subspace ω_i by the end of round $t-1$. $\hat{S}'_{t-1,\omega_i}$ is calculated by Equation 2.

$$\hat{S}'_{t,\omega_i} = \frac{(t-1)\hat{S}'_{t-1,\omega_i} + 1 - \frac{\mathbb{I}\{\omega_x=\omega_i\}\left(1-\frac{1}{1+f^*_{\omega_x}}\right)}{P_{t,\omega_i}}}{t} \tag{2}$$

where $f^*_{\omega_x}$ is the fitness of the best individual in subspace ω_x.

2.2 Crossover

The crossover goal is to allow the population jump from the original subspace ω_i to a selected subspace ω_j. Meanwhile, the crossover exploits ω_j. The following two steps, jump and exploitation, are the same as the steps in [3]. For example, given an individual '$\{+-\} - \times xxxxxx$' in ω_{+-}, it jumps to ω_{++} after the head of its code is replaced with '$\{++\}$' by crossover. Then, it recombines with the best individual in ω_{++} to exploit ω_{++}. The jump of the population between subspaces avoids falling into a local optimum and maintains the population diversity.

3 EXPERIMENTS

For evaluating the proposed algorithm **AB-GEP**, we import the other three baseline GEP methods: **GEP**, **SL-GEP**, and **SPJ-GEP**. To obtain the performance metrics of the four algorithms, each of the algorithms runs ten times on twelve SR benchmark problems [1]. Moreover, the results are shown in Table 1. Observing the comparative results, we conclude that AB-GEP outperforms GEP, SPJ-GEP, and SL-GEP owing to the following two reasons. One reason is that, AB-GEP can find more correct results on Nguyen benchmark. The other reason is that AB-GEP can obtain smaller RMSEs than the other three algorithms on most benchmarks. In addition, Figure 2 shows the subspace selection method AvgExp3 in AB-GEP is more reasonable than that in SPJ-GEP, which lets AB-GEP obtain more accurate results than SPJ-GEP.

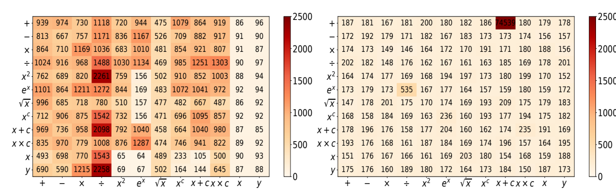
4 CONCLUSION

In this paper, we propose a novel algorithm AB-GEP to solve SR. AB-GEP has a novel subspace selection method AvgExp3 based on the adversarial bandit method. The subspace selection method can find and choose a high-value subspace that contains better results with high probability. Using Avg-Exp3 as the subspace selection method, AB-GEP can make the population jump between subspaces while maintaining the population diversity to avoid falling into a local optimum. The experimental results show that the performance of SPJ-GEP surpasses that of the other three GEPs.

Table 1: performance metrics

Function	GEP		SPJ-GEP		SL-GEP		AB-GEP	
	CR	RMSE	CR	RMSE	CR	RMSE	CR	RMSE
Koza-2	80%	.0059	**100%**	.0079	**100%**	**.0043**	**100%**	.0084
Nguyen-2	50%	.0142	70%	.0059	80%	.0071	**100%**	**.0024**
Nguyen-4	10%	.0256	20%	.0236	20%	.0238	**40%**	**.0206**
Nguyen-5	70%	.0098	**100%**	**.0065**	**100%**	.0073	**100%**	.0072
Nguyen-6	40%	.0123	**100%**	.0024	**100%**	.0025	**100%**	**.0014**
Nguyen-7	70%	.0091	60%	.0097	40%	.0111	**100%**	**.0082**
Nguyen-10	50%	.0137	80%	.0057	50%	.0102	**100%**	**.0018**
Vladis-1	0%	.0526	0%	.0531	0%	.0606	0%	**.0427**
Vladis-2	0%	.0913	0%	**.0637**	0%	.1132	0%	.0759
Vladis-3	0%	.4801	0%	**.4369**	0%	.6739	0%	.5753
Vladis-7	0%	1.3756	0%	1.3540	0%	1.4081	0%	**1.2765**
Vladis-8	0%	.3369	0%	**.2676**	0%	.6237	0%	.3487
best	0	0	3	4	3	1	**7**	**7**

CR represents the proportion of finding the correct result whose fitness computed by RMSE is less than 0.01.



(a) AB-GEP on Vladis-1 (b) SPJ-GEP on Vladis-1

Figure 2: comparison of AB-GEP and SPJ-GEP on selecting subspaces. Each small square represents a subspace. The darker the color, the more times the subspace is selected

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REFERENCES

[1] James McDermott, David R. White, Sean Luke andLuca Manzoni, et al. 2012. Genetic programming needs better benchmarks. ACM Press, 791. https://doi.org/10.1145/2330163.2330273

[2] Tor Lattimore and Csaba Szepesvári. 2020. *Bandit algorithms*. Cambridge University Press.

[3] Qiang Lu, Shuo Zhou, Fan Tao, Jake Luo, and Zhiguang Wang. 2021. Enhancing gene expression programming based on space partition and jump for symbolic regression. *Information Sciences* 547 (2021), 553–567.