Towards Higher Order Fairness Functionals for Smooth Path Planning

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ABSTRACT

Smoothness of mobile and vehicle navigation has become relevant to ensure the safety and the comfortability of riding. The robotics community has been able to render smooth trajectories in mobile robots by using non-linear optimization approaches and well-known fairness metrics considering the curvature variations along the path. In this paper, we introduce the possibility of computing smooth paths from observed mobile robot trajectories from higher order non-linear fairness functionals. Our approach is potential to enable the generation of simple and computationally-efficient path planning smoothing for navigation in mobile robots.

CCS CONCEPTS

• **Computing methodologies Evolutionary robotics**;

KEYWORDS

Path Smoothing, Optimization, Curve Fitting, Fairing, Smoothness Functionals, Mobile Robots

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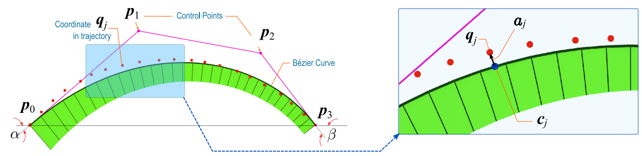
1 INTRODUCTION

With the advent of recent technologies, the applications of mobile robots have found capitalization opportunities in resource ad labour intensive fields, such as agriculture, logistics, forestry and manufacturing.

Path planning with smoothness considerations has attracted the attention of the community due to the straightforward implications for safety and comfortability [1, 2]. To generate smooth trajectories, the conventional approaches have used the idea of placing knots and fitting data points to curves encoded by polynomials and kernels, in which trajectory tracking ensures compliance with the generated smooth path. The above-mentioned problem is basically a non-linear optimization task, and its solution is amenable to gradient-free approaches, e.g. Particle Swarm Optimization [3], Differential Evolution [4, 5] and Estimation of Distribution Algorithms [6].



(a) Curvature Profile (b) Vector $\boldsymbol{a}_j$

Figure 1: Basic idea of the main elements in our approach.

The above-mentioned approaches rendered the attractive performance considering smoothness in a number of design and path planning scenarios. In this paper, we evaluate the feasibility to generate smooth paths considering the curve fitting and fairing to observed mobile robot trajectories with quasi-linear variation of curvature as a byproduct of higher order derivatives on the curvature. Thus, we propose the approach to generate trajectory profiles that pass through anchoring points, in which smooth trajectories are computed by considering the *curvature* and the *fitting* to fair curves given inputs of trajectories of real-world robotic hardware.

2 PROPOSED APPROACH

Smooth paths are computed by solving the following

$$\min_{p_i} \; F = E + \lambda H \tag{1}$$

, where p_i is the i-th control point of a Bézier curve $r(u)$ (Fig. 1), E is the fitting error of the curve $r(u)$ to given input trajectory coordinates q_j, $j = 1, 2, ..., m$, H is the smoothness (fairness metric) of the curve $r(u)$, and λ is the user-defined constant to balance fairing and fitting. Also, the fitting error is defined by the sum of distances of points in the trajectory to the given curve

$$E = \sum_{j=1}^{m} \|a_j\|^2, \tag{2}$$

$$a_j = q_j - c_j \tag{3}$$

, where

- m is the number of input trajectory points,
- a_j is the vector pointing from c_j to q_j; in which c_j is a point in the curve $r(u)$ (described below),
- $q_j \in \mathbb{R}^2$ is the j-th input trajectory coordinate obtained from the robot's trajectory measurements,
- $c_j \in \mathbb{R}^2$ is the nearest point in the curve $r(u)$ to the input coordinate q_j, which is obtained by the following:

$$\min_{u} \; \|q_j - r(u)\| \tag{4}$$

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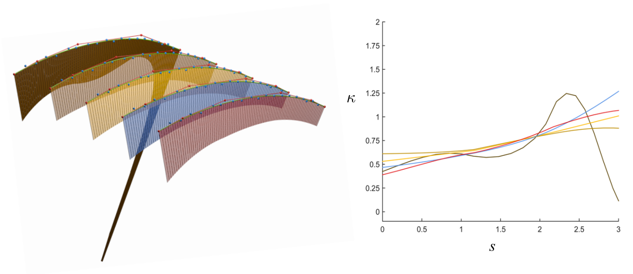

Figure 2: Curvature profile of a path. In the left, various curves for the same set of trajectory input points. In the right, the curvatureκ as a function of arc length for each of the curve in the left.

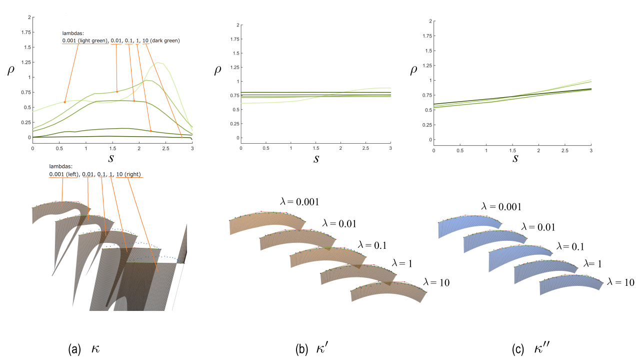

Figure 3: Curvature Profiles. In the top, the radius of curvature as function of curve length. In the bottom, the curvature profiles for each value of constant λ.

, where $u \in [0, 1]$, and $\| \cdot \|$ denotes the Euclidean norm.

Whereas the fitting quality is measured by the error metric E, the smoothness of the curve $r(u)$ is computed by the mapping $H : \kappa \longrightarrow \mathbf{R}$:

$$H(\kappa) = \int \Big(h(\kappa)\Big)^2 ds, \tag{5}$$

, where κ is the curvature of the curve $r(u)$, s is the arc length, and h is either curvature κ, $\kappa' = \frac{d\kappa}{ds}$ or $\kappa'' = \frac{d^2\kappa}{ds^2}$.

The basic idea of using the curvature profile as a smoothness metric in robot navigation is to aim at the monotonous transition between origin and destination. Fig. 2 shows an example of the curvature profile of a Bézier curve. Here, in the left, the points from a trajectory and the control points of the Bézier curve are presented. In the right, the metric κ as a function of arc length s. The quasilinear nature of the curvature plot implies the monotonous variation of the trajectory. Although the function $h_1 = \kappa$ is well-known in the literature, in this paper we evaluate the feasibility of using higher order derivatives of the above-mentioned function.

3 COMPUTATIONAL EXPERIMENTS

In order to show the performance of our proposed approach, we evaluated our algorithms in Matlab under a plural number of values of $\lambda = \{0.001,\ 0.01,\ 0.1,\ 1,\ 10\}$. The solution of Eq. 1 is done by Ratio-Based Differential Evolution[7] under convergence tolerance on the solution and function at 10^{-3}. Although using other gradient-free algorithms is potential, the throughout comparison is out of the scope of this paper. As for input trajectory, we considered an arbitrary trajectory over a quadratic domain. Solutions converged to reach the above-stated tolerance in the order of seconds.

To portray the kind of curve configurations, Fig. 3 shows the curvature profiles of the smoothed paths for arbitrary input points and for distinct fairness functionals. Here, Fig. 3-(a) shows the case when $h_1 = \kappa$, Fig. 3-(b) shows the case when $h_2 = \kappa'$ and Fig. 3-(c) shows the case when $h_3 = \kappa''$. Note that in Fig. 3, $\rho = \frac{1}{\kappa}$ denotes the radius of curvature as a function of curve length s. Also, Fig. 3 shows in the bottom the curvature profiles for each kind of fairness functional and constant λ. As shown by the results of Fig. 3, we observe the following facts:

- Our approach shows the potential to compute curves with quasi-linear variation of radius of curvature for distinct fairness functionals and values of λ. Achieving the quasi-linear variation of radius of curvature implies the smooth navigation and the comfortability during driving.
- The fairness functionals (b) $h_2 = \kappa'$ and (c) $h_3 = \kappa''$ shows the improved robustness to the choice of constant λ, implying that higher order fairness functionals balance the fairing and fitting reasonably well with no significant effect in undermining the quality of smoothness. In all such cases, the quasi-linear variation of the radius of curvatures are obtained.
- It is possible to obtain smooth curves with quasilinear variation of curvature for any user-defined λ when (b) $h_2 = \kappa'$ and (c) $h_3 = \kappa''$ are used.

Our above results offer the building blocks to further advance towards developing data-driven path planning, path smoothing algorithms and control algorithms with comfortability and safety considerations.

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