Multi-Objective Parameter-less Population Pyramid in Solving the Real-World and Theoretical Problems

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ABSTRACT

Many real-world problems are notoriously multi-objective and NP-hard. Hence, there is a constant striving for optimizers capable of solving such problems effectively. In this paper, we examine the Multi-Objective Parameter-less Population Pyramid (MO-P3). MO-P3 is based on the Parameter-less Population Pyramid (P3) that was dedicated to solving single-objective problems. P3 employs linkage learning to decompose the problem and uses this information during its run. P3 maintains many different linkage information sets, which is the key to effectively solve the problems of the overlapping nature, i.e., the problems whose variables form a large and complicated network of dependencies rather than additively separable blocks. MO-P3 inherits the features of its predecessor and employs both linkage learning and linkage diversity maintenance to effectively solve hard multi-objective problems, which includes both: well-known test problems and NP-hard real-world problems.

CCS CONCEPTS

• **Computing methodologies** → **Artificial intelligence**;

KEYWORDS

Multi-objective genetic algorithms, Linkage learning, Parameter-less population pyramid, Process manufacturing optimisation

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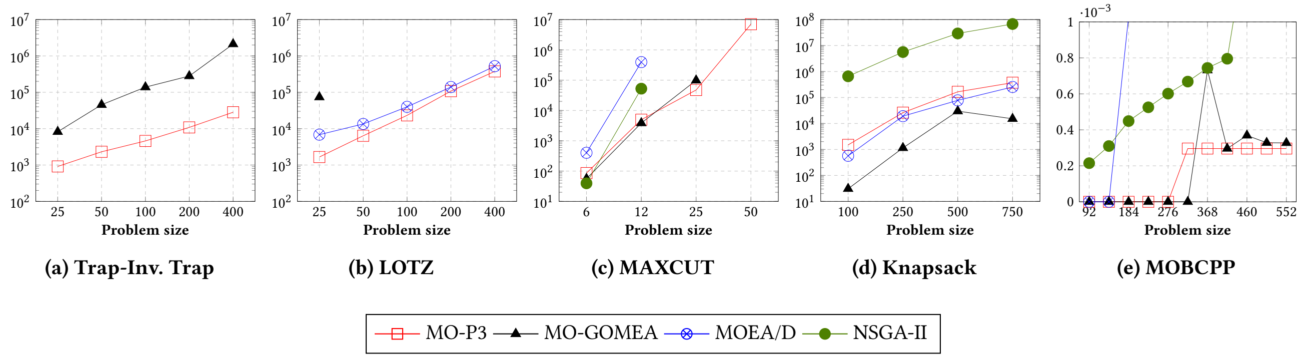
1 INTRODUCTION

In this paper, we consider binary-encoded multi-objective optimisation (MO) problems. We are to minimize m objective functions $f_i(x), i \in \{0, 1, \ldots, m-1\}$. For each binary solution vector x, we obtain the objective value vector $f(x) = (f_0(x), f_1(x), \ldots, f_{m-1}(x))$. We say that x^1 *dominates* x^2 if and only if $f_i(x^1) \leq f_i(x^2)\ \forall i \in \{0, 1, \ldots, m-1\}$ and $f(x^1) \neq f(x^2)$. If a solution is not dominated by another solution, it is called a Pareto-optimal solution. A set of all Pareto-optimal solutions $\mathcal{P}_S$ may be large or even infinite (the latter in case of real-coded search spaces). Therefore, usually in MO, the objective is to find a satisfiable approximation of the Pareto-optimal front $\mathcal{P}_F$ that is a set of objective value vectors of all Pareto-optimal solutions.

In single-objective optimization of discrete problems, many state-of-the-art Genetic Algorithms (GAs) employ linkage learning (LL) [1, 3, 7]. LL is supposed to discover an underlying problem structure. This knowledge is then used to improve the evolutionary search. As shown in [6], the quality of the problem decomposition may be crucial to solving GA-hard problems effectively. However, in MO, the use of LL is a more challenging task. For instance, in bi-objective optimization, we may consider only the first or only the second objective, or we can use the weight vector to combine two objectives and produce a single-objective problem. Each of these problems may have a different underlying structure. Thus, each of these problems may require different problem decomposition information (linkage).

The above difficulties are one of the reasons why a problem decomposition is not a frequent choice for improving the effectiveness of MO-dedicated methods. In [4], the Multi-objective Gene-pool Optimal Mixing Evolutionary Algorithm (MO-GOMEA) is proposed. In each iteration, MO-GOMEA clusters its population based on the individuals' distances in the objective space. For each of such subpopulations, linkage is discovered separately. As shown in [4], an LL-enhanced GA may obtain excellent results while applied to solving MO problems. Therefore, in this paper, we analyze MO-P3 that was originally proposed in [5].



(a) Trap-Inv. Trap (b) LOTZ (c) MAXCUT (d) Knapsack (e) MOBCPP

Figure 1: Scalability of MO-P3 and competing methods (Fig.1a-1c - median FFE until optimal PF; Fig.1d and 1e - median IGD)

2 MO-P3

MO-P3 is based on P3 [3] that is its single-objective predecessor. In P3, the population structure resembles a pyramid. At each iteration, a new individual is added to the first level of a population pyramid. Then, the new individual is mixed with the individuals on the subsequent pyramid levels using Optimal Mixing (OM) [4, 5]. Whenever OM improves the individual, its improved copy is added to the higher pyramid level. Thus, we may say that the new individual climbs up the pyramid. Each pyramid level has its separate linkage. During OM, P3 uses the linkage that refers to the level of the individual the new individual is mixed with.

The general procedure of MO-P3 is similar to P3. At the beginning of each iteration, MO-P3 randomly chooses the weight vector and transforms the MO problem into a single-objective one. Then, an individual climbs up the pyramid in the same way as in P3. Thanks to the maintenance of many linkages (inherited from P3), MO-P3 is capable of supporting high *linkage diversity* [7]. This issue has been recently identified as crucial for the effective optimization of so-called overlapping problems [5, 7]. MO-P3 inherits this feature from P3, but it also uses it differently. Instead of dividing the population into different clusters (e.g., MO-GOMEA [4]), in MO-P3, we assume that at least some part of the various linkages maintained by MO-P3 is good enough to successfully mix the individual that is heading towards the part of Pareto front (PF) defined by the weight vector chosen at the beginning of the iteration. The results presented in the next section show that MO-P3 is competitive to the state-of-the-art methods.

3 RESULTS

To check the performance of MO-P3, we compare it with MO-GOMEA, NSGA-II [2], and MOEA/D [8]. We employ the same theoretical problems as in [4] and the Multi-Objective Bulk Commodity Production Problem (MOBCPP) from [5]. As quality measures, we use the number of Fitness Function Evaluations (FFE) necessary to find an optimal PF (Fig.1a-1c) and the Inverted Generational Distance (IGD, Fig.1d and 1e) [4, 5]. The lower values of these measures refer to the results of a higher quality.

As presented in Fig. 1, MO-P3 outperforms the competing methods for trap-inverted trap problem, Leading Ones Trailing Zeroes (LOTZ) and MAXCUT. For all these problems, MO-P3 requires the lowest FFE to find an optimal PF. Note that some of the competing methods cannot find the optimal PF even for small instances of the considered benchmarks. MO-P3 is outperformed only for the MO-knapsack problem and only by MO-GOMEA (the second considered MO-dedicated GA that employs LL). Finally, for the MOBCPP, both LL-using methods significantly outperform both the remaining competing GAs. Note that MO-P3 performs better than MO-GOMEA, especially for the larger problem sizes.

4 CONCLUSION

MO-P3 is a novel MO-dedicated GA that employs LL and is competitive to other state-of-the-art GAs. The experiments show that the use of problem-decomposition techniques in MO is a promising direction for obtaining the high-quality results.

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