

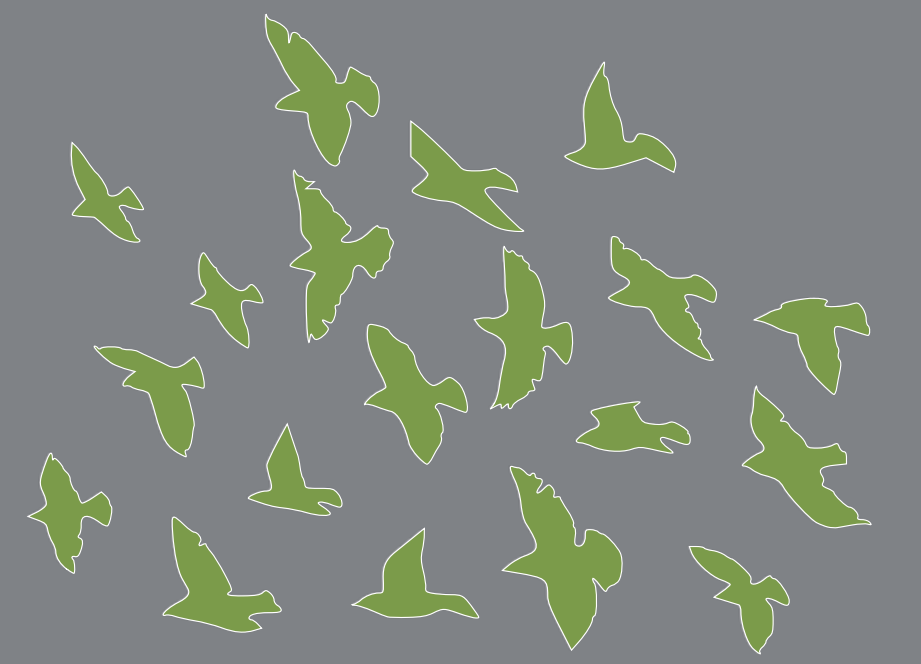
## Motivation

The design of Adaptive Fuzzy PSO (AFPSO) requires the implementation to provide rules linking the current state of the optimizer to tuned parameters. We aim to help the user in **designing the rules** given a **target benchmark** we call the **training set**.

A systematic process is proposed for designing and integrating fuzzy rulesets in **any population heuristic**.

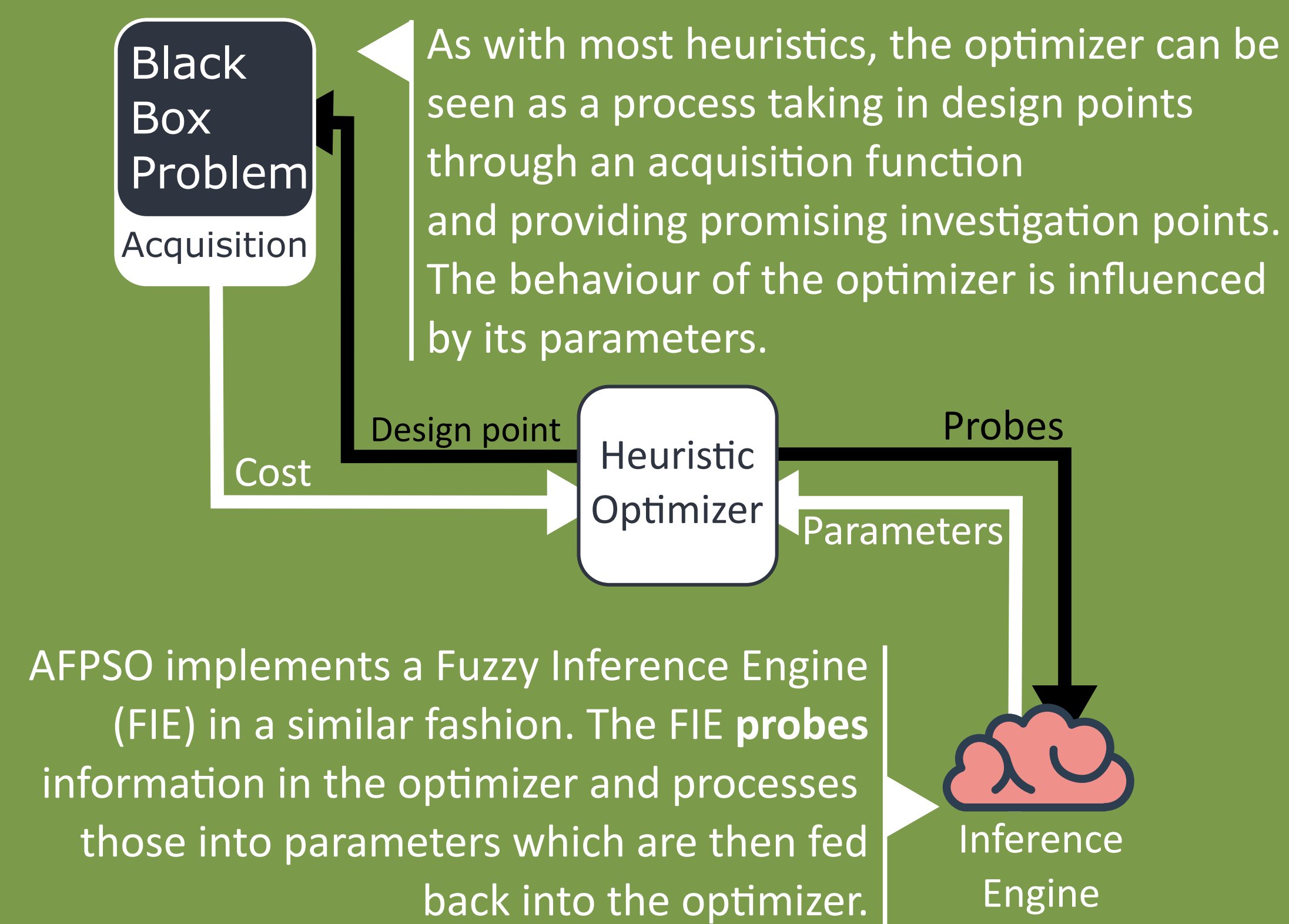
The framework is inspired by the **methodology** surrounding **neural networks**, using both training and validation benchmarks.

The system aims to automatically tailor an optimizer to a specific class of problems. Even if we seek specialization we also investigate generalization of produced optimizers to quantify overfitting using the validation benchmark.



## The principle

Although the framework is heuristic-agnostic, we will discuss results and implementation on a modern modification of **Particle Swarms** by J. Kennedy & R. Eberhart. The modifications include among others **inertia weight** and **random adaptive topology**. In PSO, efficient heuristic emerges from simple agent's behaviours. Tuning parameters of PSO during the optimization process using fuzzy logics was explored in Yuhui Shi et al.



## The implementation

### Agent's probes

An agent  $i$  knows the **proximity to its best friend**,

$$\delta_i = \frac{\|\vec{x}_i - \vec{b}_i\|}{\|\vec{u} - \vec{l}\|}$$

Labels:  $\vec{x}_i$  - Self and best friend's position,  $\vec{u}$  - Space diagonal

its rank modification or **improvement**

$$\phi_i = 4 \frac{\min(\max(r_i^t - r_i^{t-1}, -S/4), S/4)}{S}$$

Labels:  $r_i^t$  - Rank at iteration  $t$  in the swarm,  $S$  - Number of agents

and the **objective evaluation counter**.

### Search method

The search procedure is mostly unaltered except for parameters exposed to the FIE. In our case, PSO has a **weakly emergent** search procedure from the agents' simple behaviour. Each agent follows two attractors: its best solution and its neighbourhood's best solution. The neighbourhood is defined in the swarm's social network: the topology.

Two exposed parameters are, among **five others**:

- $\omega$  ▶ The tendency to stay on its track: **inertia**
- $h$  ▶ Probability to use Quantum PSO update: **hybridization**

### Rule Processing

Each controlled **parameter** has an associated fuzzy rules triplet which could read

**C** is LOW if **A** is LOW and **B** is HIGH  
**C** is MED if **A** is MED and **B** is MED  
**C** is HIGH if **A** is HIGH and **B** is LOW

In this example, a **higher probe** A reinforces **parameter** C while **high** B implies **low** C.

The ruleset gets compiled and is processed by the agent at each iteration to produce new parameters from the probes.



All agents in the swarm share the same FIE but they process their own probes.

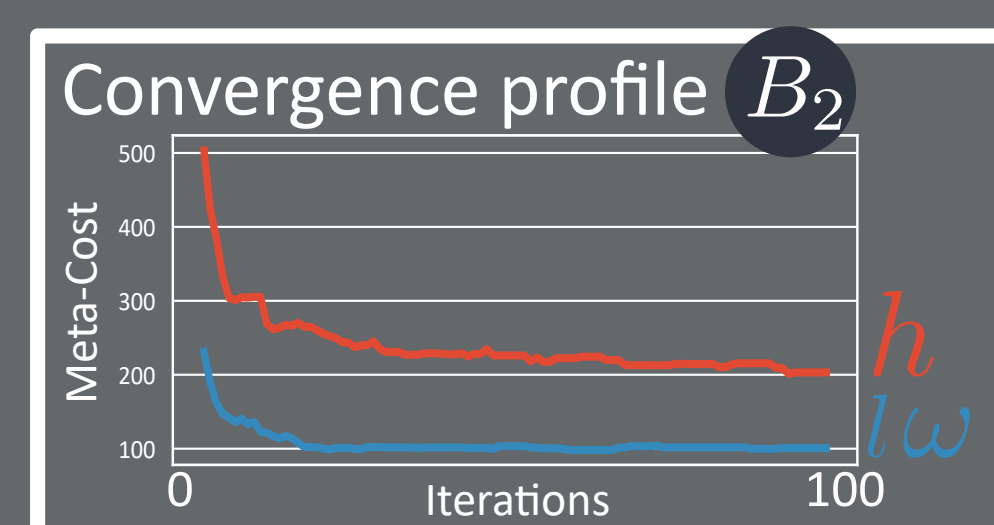
The concrete details of rules application are not explained here. We will however indicate critical choices: we used the **Zadeh operators** and **Sugeno inference** to implement our engine.

## Training & Validation

**Benchmark  $B_1$**   
 $f_1, f_2, \dots, f_n$   
 50 dimensions  
 40k evaluations

**Template**  
 Parameters to control (pairs)  
 $\omega, h$

The meta optimization process takes in pairs of parameters for which we want to design control. The optimization loop minimizes a meta-objective based on ranks regarding a database that collects all evaluations.



The same meta objective can be computed on another benchmark for the same design points that form the training profile. This leads to a validation profile which shows that both profiles converge!

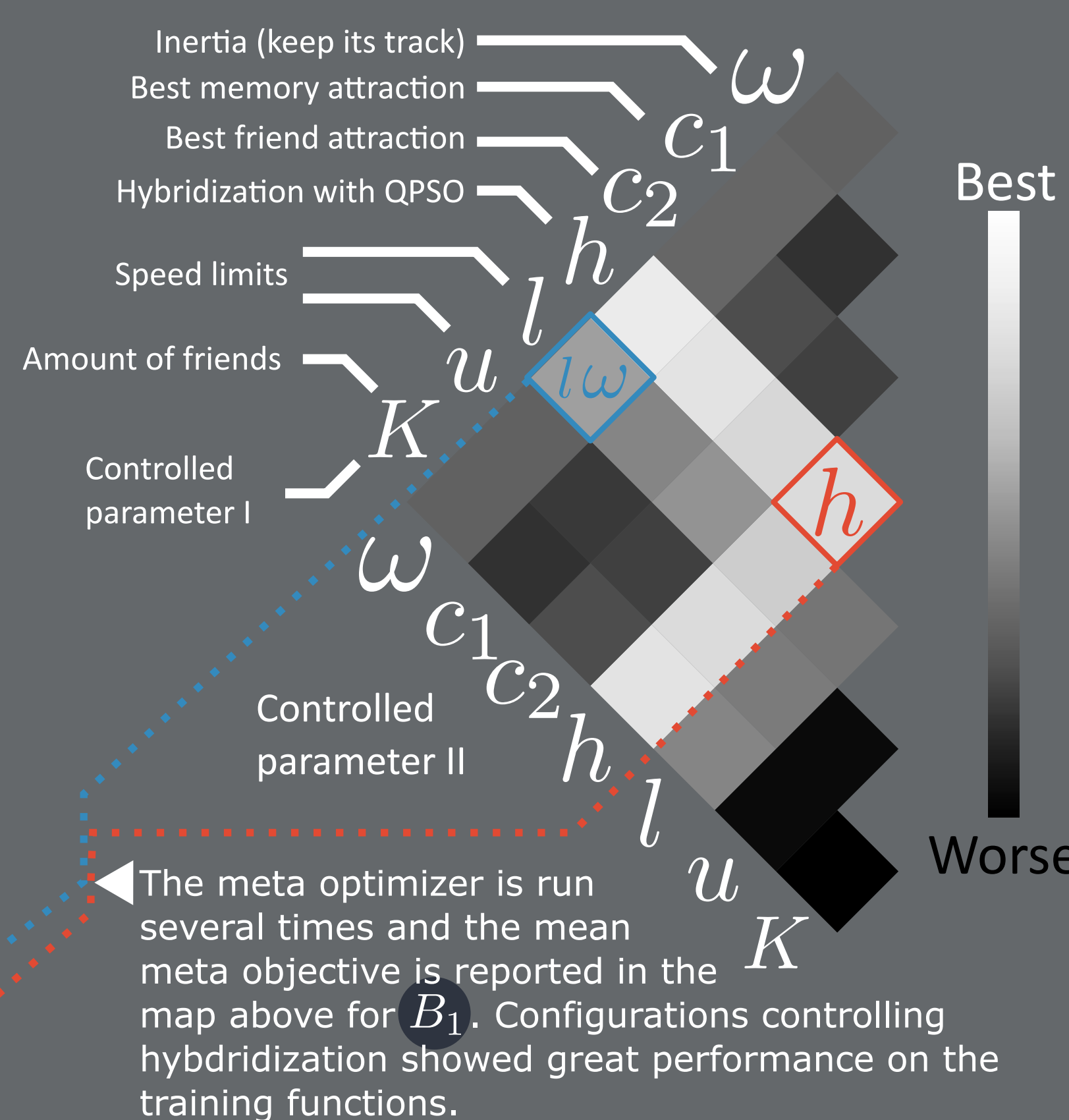
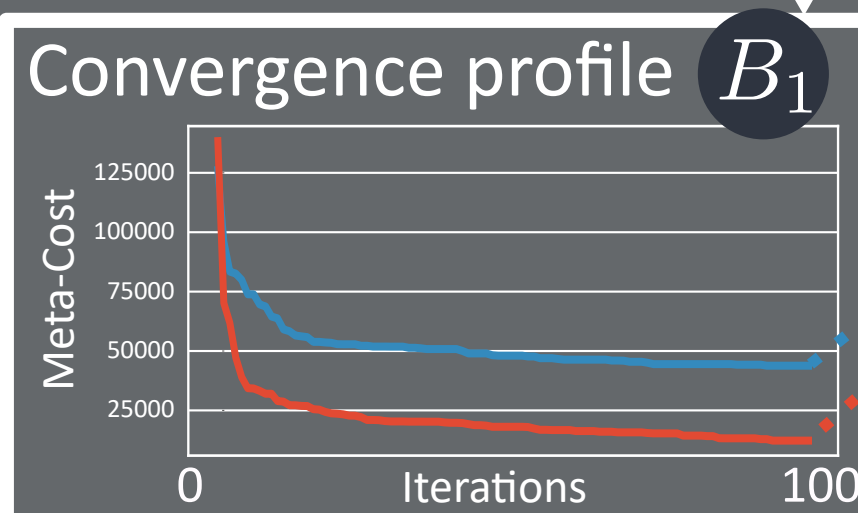
**Evaluation of optimizer**  
**Conditions chosen by user:**  
 Population: 40  
 Iterations: 1000  
 Ran on N different seeds

**Ranking**  
 Performance compared to known results for each function of the benchmark. The **mean of ranks** is used as a **meta objective**.

**New ruleset**  
 A rule is defined by  $r$ ,  
 $r \in D \subset \mathbb{N}^{6m}$   
 $m$  being the number of controlled parameters.

**GP+GBRT steps**  
 The meta cost is minimized using a **Gaussian Process with Gradient Boosted Regression Trees** surrogate.

The convergence profiles shows the evolution of the best meta objective on the training benchmark during optimization. Here, the profile shows 100 iterations of a meta optimization for two different prototypes.



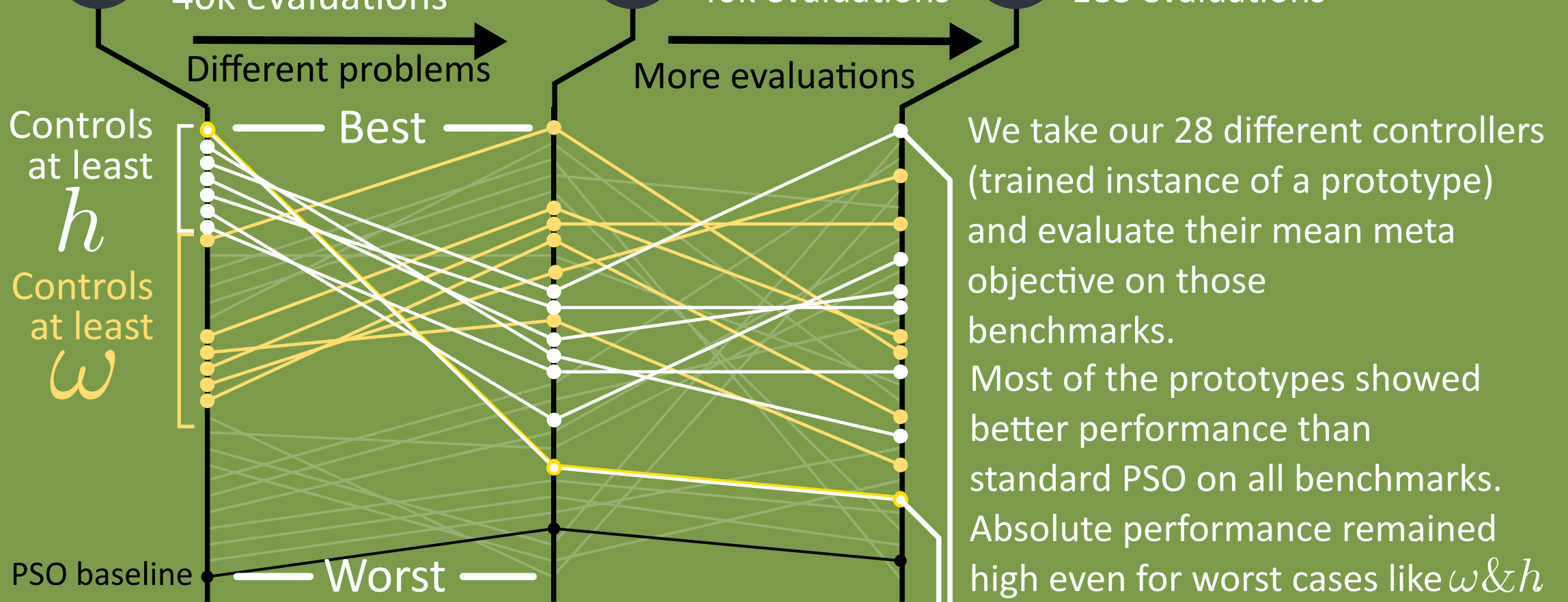
The meta optimizer is run several times and the mean meta objective is reported in the map above for  $B_1$ . Configurations controlling hybridization showed great performance on the training functions.

## Benchmark

**Training function set  $B_1$**   
 50 dimensions  
 40k evaluations

**GECCO'20 bound-constrained competition  $B_2$**   
 20 dimensions  
 40k evaluations

**$B_3$**   
 20 dimensions  
 1e8 evaluations



Any prototype with hybridization or inertia controls are good fitters for  $B_1$ .

Switching to  $B_2$ , inertia controls take the lead but ranks remain stable.

We take our 28 different controllers (trained instance of a prototype) and evaluate their mean meta objective on those benchmarks. Most of the prototypes showed better performance than standard PSO on all benchmarks. Absolute performance remained high even for worst cases like  $\omega \& h$ .

	$\omega \& h$	$l \& h$	PSO	GA	DE
cec_0	1469.0(1719.3)	0.0(0.0)	2.0e+08(2.0e+08)	5857.8(3251.9)	100.9(0.2)
cec_1	729.8(292.3)	267.5(260.8)	534.2(534.2)	1133.0(48.3)	2442.2(538.3)
cec_2	56.5(20.0)	40.1(7.6)	91.5(91.5)	728.2(3.1)	764.3(8.9)
cec_3	3.1(1.3)	1.4(0.4)	20.3(20.3)	1901.7(0.4)	1915.4(0.8)
cec_4	1161.7(1318.3)	148.4(117.2)	3950.3(3950.3)	2.0e+05(1.3e+05)	1967.8(173.6)
cec_5	12.4(12.2)	7.3(0.3)	45.1(45.1)	1601.9(0.7)	1613.0(2.4)
cec_6	244.1(99.8)	9.9(11.7)	3257.2(3257.2)	5.8e+04(3.2e+04)	2120.5(10.8)
cec_7	111.7(12.0)	103.8(2.0)	230.5(230.5)	2967.3(630.0)	2339.5(1.9)
cec_8	433.7(11.4)	417.8(7.8)	419.0(419.0)	2865.8(14.7)	2948.6(10.5)
cec_9	461.9(27.8)	424.8(21.1)	497.2(497.2)	2978.8(19.9)	2936.3(5.4)

**$B_1$**

	$\omega \& h$	$l \& h$	PSO	GA	DE
Sphere	0.0(0.0)	0.0(0.0)	0.1(0.1)	0.5(0.1)	0.4(0.1)
Ackley	0.0(0.0)	0.0(0.0)	9.8(9.8)	12.9(0.6)	20.4(0.2)
Rastrigin	0.0(0.0)	0.0(0.0)	209.0(209.0)	191.0(21.2)	483.3(28.8)
Rosenbrock	47.3(0.9)	47.4(0.4)	165.2(165.2)	664.6(126.1)	923.2(220.6)
Styblinski-Tank	181.9(51.3)	269.1(134.5)	392.5(392.5)	239.0(28.6)	699.7(31.4)
Schweifel	8124.9(1035.1)	8245.2(1603.1)	1.2e+04(1.2e+04)	5037.0(333.0)	1.4e+04(2.4e+02)
Griewank	0.0(0.0)	0.0(0.0)	1.2(1.2)	1.7(0.1)	1.8(0.4)
Chung-Reynolds	0.0(0.0)	0.0(0.0)	5.6e+09(5.6e+09)	6.3e+10(9.3e+09)	7.2e+10(2.8e+10)
Qing	9232.0(2198.6)	9017.3(2060.7)	4.0e+08(4.0e+08)	5.7e+09(8.3e+08)	6.4e+09(4.1e+09)
HappyCat	0.4(0.0)	0.5(0.1)	0.7(0.7)	0.7(0.0)	0.8(0.1)
Salomon	0.1(0.0)	0.2(0.0)	2.0(2.0)	4.4(0.4)	4.5(0.3)
XinSheYang	0.0(0.0)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0.0(0.0)
Bent Cigar	0.0(0.0)	0.0(0.0)	2.1e+10(2.1e+10)	8.3e+10(7.6e+09)	1.0e+11(4.9e+10)

▲ Absolute performance remains higher even on foreign benchmark  $B_3$ .

◀ Outstanding performance is achieved on the training benchmark  $B_1$ . With the exception of Schwefel function.

## References & Thanks

Yuhui Shi et al., *Fuzzy adaptive particle swarm optimization* doi: 10.1109/CEC.2001.934377.  
 Kennedy, J., & Eberhart, R. *Particle swarm optimization* doi: 10.1109/ICNN.1995.488968  
 Sun, J. et al. *PSO with particles having quantum behavior* doi: 10.1109/CEC.2004.1330875.  
 C. T. Yue et al., *Problem Definitions and Evaluation Criteria for the CEC 2020 Special Session and Competition on Single Objective Bound Constrained Numerical Optimization* available: <https://github.com/P-N-Suganthan/2020-Bound-Constrained-Opt-Benchmark>.



## Conclusions

We successfully designed and implemented a **framework** for **systematically designing FIEs** for heuristics. Such designed controllers showed **great specialization** of the training function but also **good generalization**, performing well of a past **GECCO competition**, far from training conditions. The work provides the methodology, from there, any implementation should design its own training and validation benchmark, knowing the problem at hand.