# An operation to promote diversity in Evolutionary Algorithms in a Dynamic Hybrid Island Model


## ABSTRACT

Currently, there is a considerable variety of Evolutionary Algorithms (EAs) and due to their performances some of them become more popular. EAs can be implemented in different ways, such as the Island Model (IM). However, despite the good performance of some EAs and the possibilities of varying their implementations, they can converge to a local optimum mainly because of the loss of diversity in the population. This work proposes an operation for a dynamic hybrid IM (D-IM), aiming to promote diversity to the population if it is converging to a certain portion of the search space. Thus, the D-IM reacts to the possible local convergence of its population, in addition to adjust the topology according to the EAs in the islands. The results demonstrated that the proposed operation can improve the efficiency of the D-IM search process and be competitive for solving bounded constrained optimization problems.



## CCS CONCEPTS

• **Mathematics of computing** → **Evolutionary algorithms**; • **Computing methodologies** → *Self-organization*; **Parallel algorithms**.

## KEYWORDS

Evolutionary Algorithms, Island Model, Local convergence, Diversity of population.

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## 1 INTRODUCTION

Several EAs have been proposed in the last decades. They are very attractive alternatives to solve complex optimization problems. Some of them become more popular due to their reported performances. However, even these EAs can converge to local optima and do not find the global optimum of the problem.

One of the possible causes for local convergence of EAs is the reduction of exploration of new regions in the search space with the merging of a great number of the candidate solutions to few regions already found. These regions may be good, but may be distant from the global optimum. This type of evolutionary behavior can also cause premature convergence. The intensification starts early and the population is kept in the same region for a considerable part of the computational cost defined for the EA execution. In short, an EA tends to be more robust and efficient if it establishes a balance between the exploration of the search space and the exploitation of the best regions identified in the search process.

Different strategies have already been proposed to control the diversity of the EAs population along the search process. Most of them are focused on the selection of solutions to be involved in the operations and the adjustment of the parameters of those operations [23]. Another alternative adopted to promote diversity in the population of EAs is the implementation through the IM. In this case, the population is divided into sub-populations called islands, which evolve individually by their own algorithms and communicate periodically through the migration process. In this case, despite the possibility of local convergence in each island, it can occur in different regions of the search space [12, 15]. In addition, the immigrant solutions can indicate new regions to be explored.

In a hybrid IM, different EAs are applied in the islands. Given the considerable number of proposed EAs and the difficulty to choose just one, this alternative may be very important in solving the problem. In addition, the IM structure may be dynamic and adaptive, as is the case of the D-IM. In D-IM, the topology and distribution of solutions between islands is dynamically adjusted over migrations. Such adjustment is defined according to the attractiveness of each island, calculated from characteristics of its algorithm.

In general, the strategies proposed to control the diversity of the IM population are also focused on the selection operation. In this case, in the selection of migrant solutions and the respective islands of destination.

This work proposes the operation DIV-OP for the D-IM with the objective to promote diversity to its population if it is converging to a small region of the search space, even using different EAs. In addition to D-IM adjusting its topology and distributing solutions according to the EAs in its islands, the DIV-OP restarts part of its population if a possible local convergence is identified. In the experiments, DIV-OP was evaluated through an D-IM based on intensifying EAs. The obtained results demonstrated that the proposed intervention can improve the efficiency of the D-IM search process.

## 2 ADAPTIVE DYNAMIC ISLAND MODEL

The IM is an alternative to implement EAs to run in parallel computational environments. In IM, the population is divided in subpopulations called islands. Each island evolves in parallel with others by its own EA. They are connected by a topology and periodically they exchange solutions through the migration operation.



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In a hybrid IM, different EAs are applied in the islands. The implementation of IM also requires decisions on [1, 13, 16, 18]: *Number of islands* ($I$), *Migration topology*, *Migration rate* ($\xi$), *Migration frequency* ($F_{mig}$) and *Migration policy*.

In [7], it was proposed a dynamic hybrid IM identified in this work as D-IM. Initially, the islands in D-IM are fully connected by weighted uni-directional connections. At each migration, the weight of each connection is dynamically adjusted in $[0, 1]$ according to the attractiveness of the destination island to the source island, based on its EA features.

In D-IM proposed in [7], the islands attractiveness are defined according to the convergence rate of their EAs. In [8], it was proposed a strategy to evaluate the islands attractiveness according to the quality of solutions produced by their EAs.

In D-IM, the solutions are actually moved from an island to another in migration. In this case, the number of solutions directed to each island may be different. The idea is that the islands with more suitable EAs maintain larger numbers of candidate solutions.

## 3 OPERATION PROPOSED FOR DYNAMIC ISLAND MODEL

Generally, the IM produces better solutions than the individual EAs applied in its islands. One of the reasons is the diversity that it can promote by the distribution of the population, accompanied by the migration of the solutions . Even so, the IM may converge prematurely to a local optimum. Works such as [12], [2] and [11] proposed mechanisms to promote diversity to the IM population, in general, based on the selection of migrant solutions and their respective destination islands.

This section presents the DIV-OP, an additional operation for D-IM. The DIV-OP aims to promote diversity to the D-IM population through the inclusion of new regions of the search space in the evolutionary process by replacing/restarting some solutions. This intervention will be performed only if it is identified that the current population covers less than a certain portion of the total search space.

By the DIV-OP, after each D-IM migration, the worst solutions in each island are replaced by new random ones, limited to the portion $\chi \in (0, 1]$ of its population, where $\chi$ is defined by the user. The solutions restart is performed only if $div < spc$, where $spc \in (0, 1]$, where $spc$ is also defined by the user. The $spc$ is the minimum portion of the search space to be covered by the D-IM population, even divided in islands, to classify its diversity as satisfactory. The $div$ value is given by

$$div = DIV/SPC, \tag{1}$$

where $DIV$ is

$$DIV = \frac{\sum_{k=1}^{I} Mdist_k}{I}, \tag{2}$$

where $Mdist_k$ is the mean distance between solutions in the island $k$, given by

$$Mdist_k = \frac{\sum_{i=1}^{PS_k} \sum_{j=1}^{PS_k} MD_{i,j}}{PS_k \times PS_k}, \qquad k = 1, 2, ..., I, \tag{3}$$

where $PS_k$ is the population size of island $k$ in the current migration and $MD_{i,j}$ is the value in row $i$ and column $j$ of the symmetric matrix $MD$. Each value $MD_{i,j}$ is the Euclidean distance between solutions $s_i$ and $s_j$ in the population of island $k$. In this case, each value $MD_{i,j}$ is given by

$$MD_{i,j} = \|s_i - s_j\|. \tag{4}$$

The $SPC$ value in (1) is given by

$$SPC = \|b_u - b_l\|, \tag{5}$$

where $b_u \in \mathbb{R}^D$ and $b_l \in \mathbb{R}^D$ are respectively the vectors of upper and lower bounds of each variable $\in \mathbb{R}^D$ according to the problem.

In D-IM, an island is removed if its population size is reduced to 0. In such case, $I$ in (2) and (3) is the number of islands still present in the D-IM topology.

Besides the D-IM topology adjustment according to the EAs applied in its islands, DIV-OP analyzes the evolution of the population as a whole. Then, if is necessary to avoid a possible local convergence, identified by comparing $div$ with $spc$, an opportunity to search new solutions is created by replacing the portion $\chi$ of the population.

## 4 MATERIALS AND METHODS

### 4.1 Differential Evolution

Differential Evolution (DE) was proposed in [20] and became one of the most popular EAs. In DE, the population composed by $NP$ $D$-dimensional vectors, where $NP$ is defined by the user, is submitted to the operations named mutation, crossover and selection. In mutation, for each vector $x_i$ ($i = 1, 2, ..., NP$) in the population, a mutant vector $v_i$, is produced and given by

$$v_i = x_{r1} + F \times (x_{r2} - x_{r3}), i = 1, 2, ..., NP, \tag{6}$$

where $r1$, $r2$ and $r3 \in \{1, 2, ..., NP\}$ are random indexes of vectors (candidate solutions), mutually different and also different from $i$, $F \in (0, 2]$ is a DE parameter, whose value is defined by the user [20].

In crossover, the solutions $v_i$ and $x_i$ are combined to produce the solution $u_i$, given by

$$u_{i,j} = \begin{cases} v_{i,j}, & \text{if } \mathrm{rand}(j) \leq CR \text{ or } j = \mathrm{rand}(i) \\ x_{i,j}, & \text{if } \mathrm{rand}(j) > CR \text{ and } j \neq \mathrm{rand}(i) \end{cases}, \tag{7}$$

where $j = 1, 2, ..., D$, rand($j$) is the $j$-th random real value $\in [0, 1]$, rand($i$) is a random integer value $\in \{1, 2, ..., D\}$. In (7), $CR \in [0, 1]$ is another DE parameter to be defined by the user [20].

The selection operator defines which solution between $u_i$ and $x_i$ will compose the DE population. They are compared by their objective function values $f(\cdot)$. For a minimization problem, if $f(u_i) < f(x_i)$, $x_i$ will be replaced by $u_i$, otherwise $x_i$ will be kept in the population [20].

In [20], some variants of DE and a scheme to name them were also proposed. In this way, a DE instance is identified as DE/$x$/$y$/$z$, where $x$ is the strategy adopted to define the vectors involved in mutation, $y$ is the number of difference operations in mutation and $z$ is the crossover scheme.

The popularity of DE is due to its simplicity and reported efficiency. Another feature known about DE and its variants is their fast convergence, generally due to their intensifying trend.

DE was the basis of many works in the literature, some of them proposing variants for it. The DE-based algorithms generally take positions among the winners when participating of competitions as those for EAs to solve bounded constrained optimization problems, frequently promoted in the Congress on Evolutionary Computation (CEC). This is the case of the EAs called L-SHADE, SPS-L-SHADE-EIG, DEsPA, CCLSHADE, jSO, LSHADE-RSP, DE variants proposed in [3, 5, 10, 17, 19, 22] respectively, which won or were second place in competitions occurred in CEC editions from 2014 to 2018. These EAs were proposed based on JADE and SHADE, DE variants proposed in [24] and [21] respectively. Basically, these DE variants applied a scheme to adaptively adjust the parameters $F$ and $CR$ from JADE and an operation to adaptively change the population size from L-SHADE.

Due to the mentioned reasons, the DE variants cited in the previous paragraph were considered in this work for the evaluation of DIV-OP in D-IM. The values for their respective parameters were defined according to the recommendations of the authors, except the population size. Three different implementations of the basic DE, corresponding to the variants DE/$rand$/1/$bin$, DE/$best$/1/$bin$ and DE/$best$/2/$bin$ according to [20], identified in this work as DE-1, DE-2 and DE-3 respectively, were also used in the experiments, for which the valures $F = 0.5$ and $CR = 0.9$ were defined.

### 4.2 D-IM configuration

Besides the definition of EAs to be applied in the D-IM islands, other parameters values should be defined, such as: (i) $M = 1$; (ii) $\theta = 0.05$, (iii) Migration rate ($\xi$) = 10%. Migration frequency ($F_{mig}$) and population size varied in the experiments.

For the adaptive adjustment of the D-IM topology, only the strategy proposed in [8] was used in this work. According to [8], this strategy performed better than that proposed in [7]. It was also observed in [8] that this strategy directs a greater number of solutions to islands with intensifying EAs. This evolutionary behavior may require more intervention as that proposed by the DIV-OP.

### 4.3 Problem Set

For the evaluation of D-IM with DIV-OP, it was used the set of problems proposed in [14] for the competition on bound constrained optimization problems in 2015 CEC edition.

The 15 minimization problems, identified in this work as $Fi$, where $i = 1, 2, \ldots, 15$, were divided into 4 groups in [14]. Problems $F1$ and $F2$ are Unimodal Functions, $F3$, $F4$ and $F5$ are Simple Multimodal Functions, $F6$, $F7$ and $F8$ are Hybrid Functions and the last seven ones are classified as Composition Functions. In this work, it was used $D = 10$, which implies in the computational cost $MFE = 100000$ according to [14], where $MFE$ is the maximum function evaluations.

### 4.4 Evaluation metrics

To analyze the results, among other resources such as the well-known statistical metrics and methods, it was used the technique Performance Profile proposed in [6]. It is applicable in evaluations with a set of algorithms $S$ and a set of problems $P$.

Basically, the Performance Profile indicates the percentage $\rho_s(\tau)$ of problems $p \in P$ that each algorithm $s \in S$ solve under a given value $\tau$ defined according to the performance measure. Additionally, in [4] was pointed that the area under curve $\rho_s(\tau)$ is an indicator of global performance of each $s$. The bigger area indicates the most efficient algorithm in $S$.

Performance Profile requires a performance measure to be defined by the user. In this work, it was used the median of the objective function values of solutions obtained in the independent D-IM runs.

## 5 EXPERIMENTS AND RESULTS

This section presents the two experiments carried out in this work aiming to evaluate the impact of the DIV-OP in the D-IM. The difference between the experiments was the number of islands (EAs) applied in the D-IM topology, consequently, its population size.

### 5.1 Evaluation with 5 islands

Initially it was verified the effect of DIV-OP in D-IM according to some values defined for the parameters $spc$ and $\chi$. In this analysis, it was used a D-IM with 5 islands in which it was applied the following EAs: DE-1, DE-2, DE-3, JADE and SHADE. The population size was defined as 200 (40 solutions for each island initially).

For the migration frequency, two values were considered: $F_{mig} \in \{50, 100\}$, which result respectively in 10 and 5 iterations of EAs between D-IM migrations. In this case, the effect of DIV-OP in D-IM was verified under two different conditions for the convergence of the EAs in the islands, which can impact in their population diversity.

For the DIV-OP parameters, the defined values were $spc \in \{0.1, 0.3\}$ and $\chi \in \{0.25, 0.5, 0.7\}$. So, regarding $spc$, relative fast (higher value) and slow (lower value) interventions in population were evaluated. Regarding $\chi$, at least a quarter of the population was replaced, if necessary according to the $div$ value obtained by (1), and restricted to the worst solutions.

For comparison, it was also included in the analysis a D-IM instance without the DIV-OP. For identification purpose, each D-IM instance was named in the form D-IM_$spc$_$\chi$ in the case of using DIV-OP or just D-IM otherwise. It was defined 30 independent runs for all D-IM instances in solving each problem.

Table 1 presents the objective function value of the best solution obtained for each problem with each D-IM instance considered in this experiment, under $F_{mig} = 50$. The results referring to the D-IM instances with $F_{mig} = 100$ were not presented in this work. It was verified that they were similar to those presented in Table 1. In this case, the reduction in the total iterations of EAs based on DE by increasing $F_{mig}$ did not compromise their convergence between D-IM migrations. In this table, the third column also presents the objective function value of the best solution obtained for each problem by SPS-L-SHADE-EIG when executed individually with the same population size and computational cost adopted in this analysis for D-IM instances. The SPS-L-SHADE-EIG, winner in the competition of CEC, edition 2015, was also identified in [9] as the best algorithm between those used in this work.

Table 1 shows that the D-IM based on DE variants produced slightly better solutions when applying the DIV-OP under different adjustments of its parameters $spc$ and $\chi$. Different D-IM instances with DIV-OP produced better solution than the D-IM without it for some problems as $F4$, $F7$, $F8$, $F11$ and $F12$. For some problems, the D-IM based on DE did not produce the optimal global solution, even under application of DIV-OP. In this case, regardless the portion of the D-IM population that possibly was restarted by DIV-OP, they did not impact in the EAs convergence trend. On the other hand, the possible new solutions did not compromise the solution quality of D-IM.

Regarding the performance of D-IM based on DE variants compared to the individual execution of the respective EAs, in Table 1 note that for all problems, the SPS-L-SHADE-EIG produced the best solutions found by some D-IM instance. In the case of the problems $F4$ and $F8$, SPS-L-SHADE-EIG produced a better solution than any D-IM instance considered in this work, even for problem F8 the difference was relatively small. Note that SPS-L-SHADE-EIG was not applied in D-IM with 5 islands, just in the experiment described in Section 5.2, but it is the best one between all algorithms used in the work, because it, considered in Table 1. Besides, D-IM with 10 islands produced solutions very similar to those in Table 1, so equivalent analysis was not performed in Section 5.2.

Among other characteristics, when applying DE variants in D-IM, their population sizes are changed (reduced), which can impact in their convergence. In this case, according to the results in Table 1, it is possible to say that run different DE variants in parallel in D-IM is beneficial for solving the problem. Even the D-IM has produced a solution similar to that obtained by the best algorithm, in addition to promoting speedup, it automatically decided which variants were most interesting to solve the problem and adjusted the topology.

Tables 2 and 3 show the area under curve $\rho_s(\tau)$ of Performance Profile obtained for each evaluated D-IM, with $F_{mig} = 50$ and $F_{mig} = 100$ respectively, in decreasing order. The metric applied in the Performance Profile in this work represents the variety of solutions obtained by D-IM instances in their runs. Tables 2 and 3 indicate that DIV-OP can contribute positively to the D-IM convergence. Some D-IM instances with DIV-OP outperformed that without it, using both $F_{mig} = 50$ and $F_{mig} = 100$.

Tables 2 and 3 also indicate that it is not interesting to restart more than half of the D-IM population by DIV-OP, mainly under relatively high $F_{mig}$ value. D-IM_0.1_0.7 and D-IM_0.3_0.7, in which $\chi = 0.7$, the highest one evaluated here, were less efficient than D-IM for both $F_{mig} = 50$ and $F_{mig} = 100$. For $F_{mig} = 100$ in particular, according to Table 3, just D-IM_0.1_0.7 and D-IM_0.3_0.7 were less efficient than D-IM. A possible reason for this result is that with high $\chi$, if the DIV-OP replace solutions in islands with not so good new random ones, the next EAs iterations may be insufficient to evolve them from those solutions maintained in the population. Additionally, in a D-IM based on DE variants, given the fast convergence of these EAs, many of the maintained solutions in some islands may be equals. In this case, other possible good solutions/regions have been replaced by random ones through DIV-OP, which may compromise the D-IM convergence. With a moderate value assigned to parameter $\chi$ ($\leq 0.5$ according to this analysis), the D-IM with DIV-OP explores new regions in the space, while keeps a diverse convergence history.

According to Tables 2 and 3, the D-IM_0.3_0.25 was the most efficient D-IM for both $F_{mig} = 50$ and $F_{mig} = 100$. Besides, for $F_{mig} = 100$, the two most efficient D-IMs were those with $spc = 0.3$, except if $\chi = 0.7$, value already commented. In this case, maintaining reasonable coverage of the search space was positive for the D-IM convergence with DIV-OP.

Regarding the main intervention of DIV-OP in the D-IM, Fig. 1 illustrates the variation of the mean value of $div$ over migrations, before and after replacing solutions by new random ones. For each migration, the values in Fig. 1 were calculated considering the set of $div$ values obtained in the 30 runs to solve the 15 problems, which represents the complete experiment. Fig. 1 presents only the data referring to the D-IM instances with $F_{mig} = 100$. The result was similar with $F_{mig} = 50$.

Fig. 1(a) shows that the 5 iterations of EAs between D-IM migrations, value when $F_{mig} = 100$, were enough to converge the population to less than 30% of the search space. The DIV-OP replaced solutions in almost 100% of the D-IM migrations, even under $spc = 0.1$, the lower value evaluated in this work for DIV-OP parameter $spc$.

Fig. 1(b) illustrates that the mean $div$ increased around 2 to 4 times when some solutions were replaced/restarted by DIV-OP, even in the D-IM instances with $\chi = 0.7$, the highest one considered in the experiment. In these D-IM instances the mean $div$ is kept relatively high due to the number of new random solutions to evolve. Even so, they reach the coverage threshold of the search space defined by the $spc$ parameter.

According to the results presented until this point, D-IM_0.3_0.25 is considered the best one between those evaluated here with DIV-OP. So, it was the D-IM instance considered in the following analyses.

Fig. 2 presents the variation of the standard deviation of the objective function values in the population in each island, over D-IM_0.3_0.25 migrations, before and after run iterations of EAs in its islands, to solve problem $F2$. Note that the values after run EAs precede each migration. On the other hand, the values before run EAs reflect the effect of the previous migration in the respective population. Fig. 3 shows equivalent values, however, referring to the resolution of problem $F15$. Problems $F2$ and $F15$ were chosen because they are in different groups in the problem set according to their complexities [14]. Besides, their curves in Fig. 2 and 3 are representative for the obtained results in the experiment with D-IM_0.3_0.25.

Taking the standard deviation as a measure of population diversity from the view point of objective function, comparing each pair of graphs in Fig. 2 and 3, it is possible to observe that independent of the problem complexity, the population diversity in each island was affected by DIV-OP in D-IM_0.3_0.25 migrations. In all cases, the impact was the increase in the respective value.

Comparing Fig. 2(b) with 2(a) and 3(b) with 3(a), it is possible to observe that the standard deviation increased considerably due to the new random solutions created by DIV-OP over D-IM migrations. On the other hand, by comparing the pairs of graphs in Fig. 2 and 3 in the opposite direction, it is possible to verify that the EAs iterations were enough to reduce again the population diversity in islands, even their reduction rate were different. For example, graphs in Fig. 2 and 3 indicate that JADE and SHADE tend to maintain their population more diverse than others used EAs when they are stimulated to do this.

**Table 1: Objective function value of solution obtained for each problem by each D-IM instance with 5 islands, population size = 200 and $F_{mig}$ = 50 and the SPS-L-SHADE-EIG, when executed individually. The best value obtained for each problem is highlighted in boldface. Column $F^*$ presents the optimum objective function value for the respective problem according to [14].**

| | $F^*$ | SPS-L-SHADE-EIG | D-IM | D-IM_0.1_0.25 | D-IM_0.1_0.5 | D-IM_0.1_0.7 | D-IM_0.3_0.25 | D-IM_0.3_0.5 | D-IM_0.3_0.7 |
|---|---|---|---|---|---|---|---|---|---|
| F1 | 100 | **100.0000** | **100.0000** | **100.0000** | **100.0000** | **100.0000** | **100.0000** | **100.0000** | 100.0029 |
| F2 | 200 | **200.0000** | **200.0000** | **200.0000** | **200.0000** | 200.0004 | **200.0000** | **200.0000** | 200.0006 |
| F3 | 300 | **300.0000** | **300.0000** | **300.0000** | **300.0000** | **300.0000** | **300.0000** | **300.0000** | 301.1552 |
| F4 | 400 | **400.0000** | 402.9849 | 401.9899 | 400.9950 | 402.9849 | 401.9899 | 402.9849 | 403.9798 |
| F5 | 500 | **500.0001** | **500.0001** | **500.0001** | **500.0001** | **500.0001** | **500.0001** | **500.0001** | 500.0054 |
| F6 | 600 | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** |
| F7 | 700 | **700.0000** | 700.0291 | 700.0074 | 700.0074 | 700.0099 | 700.0197 | **700.0000** | 700.0075 |
| F8 | 800 | **800.0000** | 800.0004 | 800.0001 | 800.0001 | 800.0001 | 800.0063 | 800.0001 | 800.0001 |
| F9 | 900 | **1516.2448** | **1516.2448** | 1516.7886 | 1516.9261 | 1516.3839 | **1516.2448** | 1517.4152 | 1518.4931 |
| F10 | 1000 | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** |
| F11 | 1100 | **1200.0018** | 1200.0019 | **1200.0018** | 1200.0019 | 1200.0019 | 1200.0019 | 1200.0019 | 1200.0019 |
| F12 | 1200 | **1244.1087** | 1245.6969 | **1244.1087** | **1244.1087** | 1244.1181 | **1244.1087** | **1244.1087** | **1244.1087** |
| F13 | 1300 | **1737.0983** | 1737.1482 | 1737.1482 | 1737.1482 | 1737.1482 | **1737.0983** | 1737.1482 | **1737.0983** |
| F14 | 1400 | **1565.9420** | **1565.9420** | 1649.7911 | 1649.7911 | 1649.7911 | **1565.9420** | 1649.7911 | **1565.9420** |
| F15 | 1500 | **1750.1468** | **1750.1468** | **1750.1468** | 1750.8190 | **1750.1468** | 1751.3871 | 1751.3871 | **1750.1468** |

**Table 2: Areas under $\rho_s(\tau)$ (Performance Profile) for the D-IM variants with 5 islands and $F_{mig}$ = 50.**

| | Area |
|---|---|
| D-IM_0.3_0.25 | 1.000000 |
| D-IM_0.1_0.5 | 0.999422 |
| D-IM | 0.986033 |
| D-IM_0.1_0.7 | 0.946416 |
| D-IM_0.3_0.7 | 0.885776 |
| D-IM_0.1_0.25 | 0.880430 |
| D-IM_0.3_0.5 | 0.853888 |

**Table 3: Areas under $\rho_s(\tau)$ (Performance Profile) for the D-IM variants with 5 islands and $F_{mig}$ = 100.**

| | Area |
|---|---|
| D-IM_0.3_0.25 | 1.000000 |
| D-IM_0.3_0.5 | 0.986578 |
| D-IM_0.1_0.5 | 0.977681 |
| D-IM_0.1_0.25 | 0.970209 |
| D-IM | 0.969727 |
| D-IM_0.1_0.7 | 0.942425 |
| D-IM_0.3_0.7 | 0.910537 |

Fig. 4 presents data equivalent to those in Fig. 3 for problem $F15$, however, related to the D-IM without DIV-OP. Note that, despite the migration, the dispersion of solutions from the view point of objective function values is decreasing over all migrations, even in the islands with JADE and SHADE, the EAs identified as the most exploratory ones. This result reinforce the information that the DIV-OP promoted a significant impact in diversity of the populations in islands when necessary, according to the $div$ value.

Fig. 5 illustrates the variation of the mean population size in each island over D-IM migrations, applying or not the DIV-OP according to Fig. 5(b) and 5(a) respectively. Due to the fact that D-IM presents behavioral tendencies regarding the dynamic adjustment of topology and distribution of solutions [7, 8], for each migration in Fig. 5(b) and 5(a), the value in the vertical axis was calculated considering the values in the 30 runs to solve the 15 problems, which represents the complete experiment.

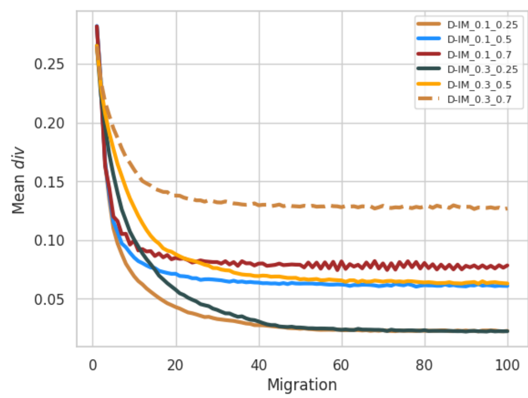
In [8], it was verified that the strategy used in this work to adjust the D-IM topology directs a greater number of solutions to islands with intensifying EAs. Among EAs used in this experiment, JADE and SHADE are the most exploratory ones, as previously mentioned and illustrated in Fig. 2 and 3. Fig. 5, demonstrates that the D-IM based on DE variants also sent more solutions to islands with intensifying EAs, using or not the DIV-OP.

Fig. 5 also indicates that the DIV-OP stimulated the exploratory ability of the EAs in D-IM islands. Comparing Fig. 5(b) with 5(a) it is possible to observe that the D-IM_0.3_0.25 directed less solutions to islands with JADE and SHADE than D-IM. Consequently, the population directed to islands with intensifying EAs was increased in D-IM_0.3_0.25, if compared to the D-IM. This behavior is relevant, because when creating new random solutions by DIV-OP, the D-IM simultaneously establishes condition to evolve them with quality, directing even more solutions to the islands with EAs identified as the best ones. Clarifying, in the context of the dynamic adjustment of the topology in D-IM, the islands with the best EAs are those with increasing population sizes over migrations. According Fig. 5, in this experiment, the islands with DE-1, DE-2 and DE-3.

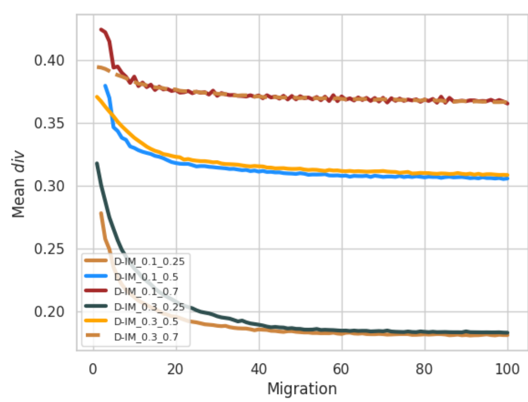
## 5.2 Evaluation with 10 islands

As an additional experiment, the number of islands in D-IM was increased to 10. It aims to verify the D-IM convergence according to the initial population size in its islands, mainly using the DIV-OP. The EAs DE-1, DE-2, JADE, SHADE, SPS-L-SHADE-EIG, DEsPA, L-SHADE, LSHADE-RSP, jSO and CCLSHADE were used in the islands, which maintains the D-IM based on DE and intensification behavior. Like in [9], the EAs which change the population size along their iterations had such operation disabled in D-IM. Their population sizes were controlled just by the D-IM migrations.

In this experiment, the D-IM population size was defined as 300, which results in 30 candidate solutions in each island initially, 25% lower than that in the previous experiment. The DIV-OP were also evaluated with $spc \in \{0.1, 0.3\}$, $\chi \in \{0.25, 0.5, 0.7\}$ and $F_{mig} \in \{50, 100\}$. For comparison purpose, the D-IM was again performed

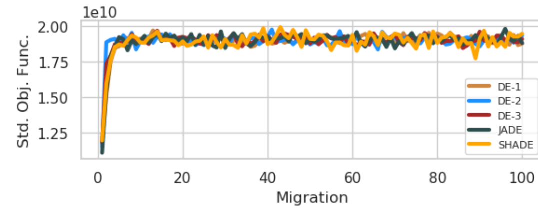



(a) Before New Solutions

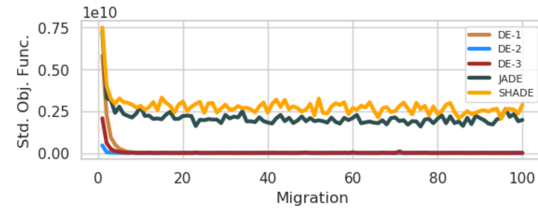



(b) After New Solutions

**Figure 1: Variation of the mean diversity of population of D-IM with 5 islands and $F_{mig}$ = 100, before and after insertion of new random solutions by DIV-OP.**

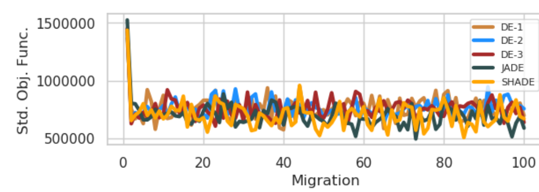



(a) Before Run EAs

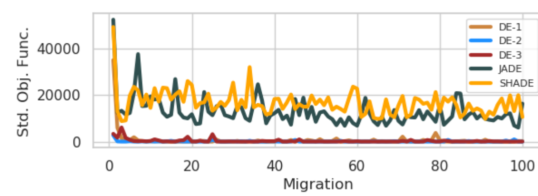



(b) After Run EAs

**Figure 2: Variation of standard deviation of objective function of solutions in each island, over migrations of D-IM_0.3_0.25, before and after executions of iterations of EAs for problem $F2$. In figure, Obj. Func. = Objective Function, Std. = Standard deviation.**




(a) Before Run EAs




(b) After Run EAs

**Figure 3: Variation of standard deviation of objective function of solutions in each island, over migrations of D-IM_0.3_0.25, before and after executions of iterations of EAs for problem $F15$. In figure, Obj. Func. = Objective Function, Std. = Standard deviation.**

without the DIV-OP. The D-IM instances were named in the same scheme adopted in the previous experiment.

Table 4 presents the objective function value of the best solution obtained for each problem with each D-IM instance considered in this experiment, under $F_{mig}$ = 50 and SPS-L-SHADE-EIG, the best EA between those used in this work. The results obtained with D-IM instances with $F_{mig}$ = 100 were similar to those presented in Table 4.

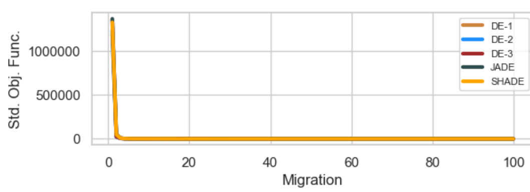
Comparing Table 1 with Table 4, it is possible to observe that the increase in the number of islands in the D-IM caused a variation in the quality of the solution produced by it. However, the change also involves a reduction of 25% in the initial population size of the islands and a greater variety of EAs. For this reason, the difference in results in relation to the best solution is considered small. It is important to consider the set of solutions, verified in the following analysis through the Performance Profile.

Tables 5 and 6 show the area under curve $\rho_s(\tau)$ of Performance Profile obtained for each evaluated D-IM, with $F_{mig}$ = 50 and $F_{mig}$100 respectively, in decreasing order. Those tables indicate that this experiment also pointed that is not interesting to restart more than half of the D-IM population by DIV-OP. As in the previous experiment, instances with $\chi$ = 0.7 performed less than D-IM, for all *spc* values.

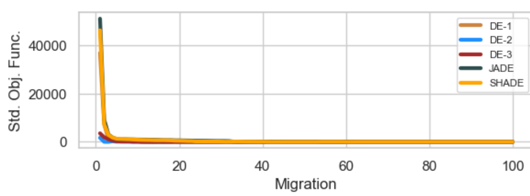
Comparing Tables 5 and 6 with Tables 2 and 3, it is possible to observe that after increasing the number of islands and reducing their initial population sizes, D-IM without DIV-OP performed

**Table 4: Objective function value of solution obtained for each problem by each D-IM instance with 10 islands, population size = 300 and $F_{mig}$ = 50 and the SPS-L-SHADE-EIG, when executed individually. The best value obtained for each problem is highlighted in boldface. Column $F^*$ presents the optimum objective function value for the respective problem according to [14].**

| | $F^*$ | SPS-L-SHADE-EIG | D-IM | D-IM_0.1_0.25 | D-IM_0.1_0.5 | D-IM_0.1_0.7 | D-IM_0.3_0.25 | D-IM_0.3_0.5 | D-IM_0.3_0.7 |
|---|---|---|---|---|---|---|---|---|---|
| F1 | 100 | **100.0000** | **100.0000** | **100.0000** | **100.0000** | 100.0003 | **100.0000** | 100.0128 | 100.1504 |
| F2 | 200 | **200.0000** | **200.0000** | **200.0000** | 200.0172 | 200.2818 | **200.0000** | 200.0261 | 237.8832 |
| F3 | 300 | **300.0000** | 300.0024 | 300.0005 | **300.0000** | 300.0002 | 300.0014 | 301.7289 | 302.3562 |
| F4 | 400 | **400.0000** | 400.9950 | 401.9899 | 401.9899 | 403.9798 | 401.9899 | 401.9899 | 403.9810 |
| F5 | 500 | **500.0001** | **500.0001** | **500.0001** | **500.0001** | **500.0001** | **500.0001** | 500.0038 | 500.0681 |
| F6 | 600 | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** | **600.0000** |
| F7 | 700 | **700.0000** | 700.0094 | 700.0365 | 700.0268 | 700.0292 | 700.0074 | 700.0115 | 700.1434 |
| F8 | 800 | **800.0000** | 800.0003 | 800.0019 | 800.0002 | 800.0477 | 800.0051 | 800.0082 | 800.0315 |
| F9 | 900 | **1516.2448** | **1516.2448** | **1516.2448** | 1516.9261 | 1517.4152 | **1516.2448** | **1516.2448** | 1517.8997 |
| F10 | 1000 | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** | **1.1281e+04** |
| F11 | 1100 | **1200.0018** | **1200.0018** | 1200.0020 | 1200.0019 | 1200.0020 | 1200.0020 | 1200.0020 | 1200.0021 |
| F12 | 1200 | 1244.1087 | 1245.4617 | 1244.1087 | 1244.1087 | 1244.1087 | **1240.8806** | 1244.2053 | 1244.1100 |
| F13 | 1300 | 1737.0983 | 1737.1482 | 1737.1188 | 1737.0983 | 1737.1482 | 1737.0741 | **1737.0634** | 1737.1404 |
| F14 | 1400 | **1565.9420** | **1565.9420** | **1565.9420** | **1565.9420** | **1565.9420** | **1565.9420** | **1565.9420** | 1565.9421 |
| F15 | 1500 | 1750.1468 | **1750.1464** | 1750.1468 | 1750.1468 | 1750.1468 | 1750.1468 | 1750.1466 | 1751.2418 |

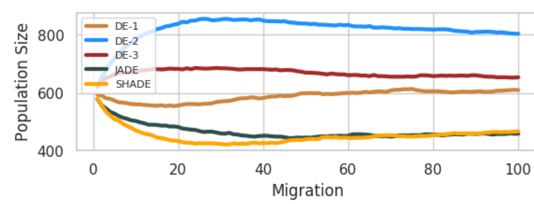



(a) Before Run EAs.

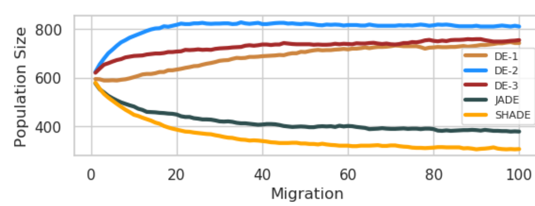



(b) After Run EAs.

**Figure 4: Variation of standard deviation of objective function of solutions in each island, over migrations of D-IM, before and after executions of iterations of EAs for problem $F15$. In figure, Obj. Func. = Objective Function, Std. = Standard deviation.**




(a) D-IM




(b) D-IM_0.3_0.25

**Figure 5: Variation of the mean population size in islands over migrations of D-IM and D-IM_0.3_0.25.**

better than those with it. In this case, from the view point of the convergence of the EAs in the D-IM islands, when applying the DIV-OP, it is important to define a reasonable initial population size to them.

Fig. 6 presents the variation of the mean value of *div* over D-IM migrations in this experiment, before and after replacing solutions by new random ones through DIV-OP. For each migration, the values in Fig. 6 were calculated considering the set of *div* values obtained in the 30 runs to solve the 15 problems, which represents the complete experiment.

Comparing Fig. 6 with Fig. 1, it is possible to observe that the frequency of restarting solutions did not change after increasing the number of islands in D-IM. Besides, the difference between considered D-IM instances was similar in the two experiments for both before and after restarting some solutions by the DIV-OP.

Comparing Fig. 6(a) with 1(a), it is possible to verify that in the second experiment, for most D-IM instances, the reduction in the population diversity was smaller and slower than in the first one. Considering that this difference is observed since initial D-IM migrations, this behavior is not only due to the restart of some solutions by the DIV-OP. It is also due to the increase in the number of islands and consequent reduction in their initial population sizes.

**Table 5: Areas under $\rho_s(\tau)$ (Performance Profile) for the D-IM variants with 10 islands and $F_{mig}$ = 50.**

| | Area |
|---|---|
| D-IM_0.1_0.25 | 1.000000 |
| D-IM | 0.999791 |
| D-IM_0.1_0.5 | 0.998926 |
| D-IM_0.3_0.25 | 0.998808 |
| D-IM_0.3_0.5 | 0.996187 |
| D-IM_0.1_0.7 | 0.994522 |
| D-IM_0.3_0.7 | 0.927784 |

**Table 6: Areas under $\rho_s(\tau)$ (Performance Profile) for the D-IM variants with 10 islands and $F_{mig}$ = 100.**

| | Area |
|---|---|
| D-IM | 1.000000 |
| D-IM_0.1_0.25 | 1.000000 |
| D-IM_0.1_0.5 | 1.000000 |
| D-IM_0.1_0.7 | 0.999989 |
| D-IM_0.3_0.25 | 0.999986 |
| D-IM_0.3_0.5 | 0.999964 |
| D-IM_0.3_0.7 | 0.933306 |

On the other hand, comparing Fig. 1(b) and 6(b), it is possible to observe that the effect in the population diversity promoted by the restart of some solutions through DIV-OP was practically identical in the two experiments according to the respective *spc* and $\chi$ values. In this case, the population diversity promoted by DIV-OP is sensitive to the setting of parameters *spc* and $\chi$, not necessarily to the islands population sizes.

# 6 CONCLUSION

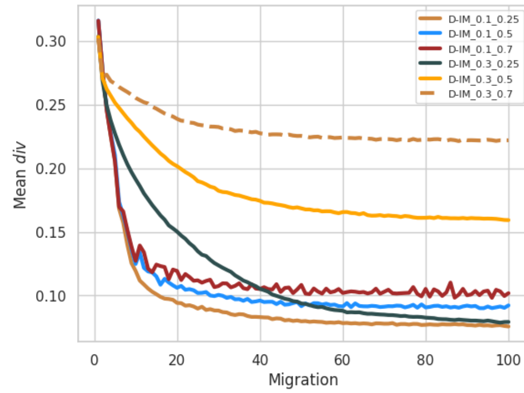
This work proposed the DIV-OP for the D-IM, aiming to promote diversity to the population in islands. By DIV-OP, some solutions are replaced by new random ones if a possible local convergence is identified according to the population diversity, verified by the portion of the search space covered by the candidate solutions.

The D-IM with DIV-OP was evaluated through instances based on DE variants. They were different between themselves according to the values assigned to the DIV-OP parameters *scp* and $\chi$, number of islands and population size. They were also compared to the D-IM without DIV-OP.

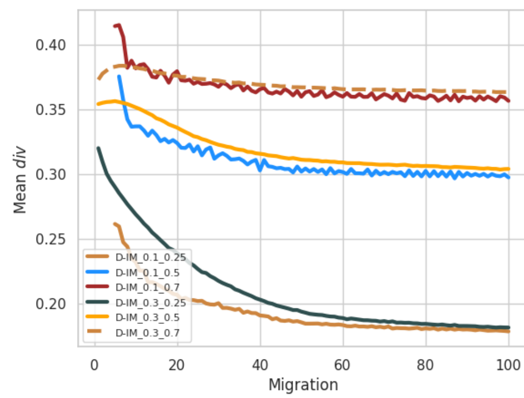
The experiments indicated that the D-IM based on DE variants converge quickly to relative small portions of the search space, which required the replacement of solutions by DIV-OP in almost all migrations, which increased the population diversity.

Regarding the solution quality, despite the use of DIV-OP, the D-IM did not find the optimal global solution for some problems. Besides, different D-IM instances with DIV-OP produced slightly better solution than the D-IM without it for some problems. In this case, even the new random solutions/regions did not contribute to the global convergence, they contributed positively to the evolutionary tendencies of the EAs in the islands. Additionally, D-IM, when applied or not the DIV-OP, produced solutions similar or equal to that produced by the best DE variant among those applied on its islands. In this case, applying different DE variants in parallel in D-IM is a positive strategy to solve the problem. In addition to promoting speedup, the most suitable ones are dynamically identified to be used more intensively.

On the DIV-OP parameters setting, according to the obtained results, this work recommends $\chi \leq 0.5$. On the *spc*, between values evaluated in this work, the results indicated that this parameter did not impact considerably in the DIV-OP. Even so, the best D-IM




(a) Before New Solutions




(b) After New Solutions

**Figure 6: Variation of the mean diversity of population of D-IM with 10 islands and $F_{mig}$ = 50, before and after insertion of new random solutions by DIV-OP.**

instance with DIV-OP was that in which was applied the highest value among those considered for *spc*. In this case, this work recommends *spc* = 0.3.

As future work it is intended to:

- Evaluate the performance of D-IM with the proposed operation using more distinct EAs in its islands.
- Apply other metrics to evaluate the population diversity.
- Evaluate different strategies to produce new solutions in the D-IM population.

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