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Application of Reverse Time Migration (RTM) for ultrasound tomography problem

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Medical diagnostics

One of the most important problems of medical diagnostics is the early detection of a variety of breast cancer tumors.

Diagnostics Breast Cancer

- ✓ *X-ray tomography (CT)*
- ✓ *Mammography (also called mastography)*
- ✓ *Magnetic resonance tomography (MRT)*

Ultrasound examinations are relatively inexpensive, easy to use and safe diagnostic methods. Ultrasound imaging has great potential for the detection and diagnosis of breast cancer

Nowadays groups of scientists in Russia (MSU, chair of acoustics), in the USA (N. Duric, C. Li, P. Littrup, S. Schmindt, etc.) and in Germany (N. Ruiter, R. Dapp, M. Zapf, R. Jirik, etc) work on creation of models of the ultrasonic tomographs with high resolution and informative.

However, one of the major problems is the development of effective algorithms for measurements processing, i.e. numerical methods with high resolution for acoustics inverse problems.

In order to obtain of the image of the speed of sound, we use RTM procedure (in the version [1]) for the ultrasound data.

[1] D. Rocha, N. Tanushev, P. Sava Acoustic wavefield imaging using the energy norm // 2015 SEG Annual Meeting, 18-23 October, New Orleans, Louisiana, P. 49-68

Forward problem

$$\frac{1}{c(x)^2} p_{tt} - \Delta p = f(t)\delta(x - x_s) \quad in \quad R^2 \times [0,T]$$

$$p|_{t=0} = p_t|_{t=0} = 0$$

c - the speed of sound

$x_s \in \Gamma$

$p(x,t;x_s)$ - the solution of the forward problem

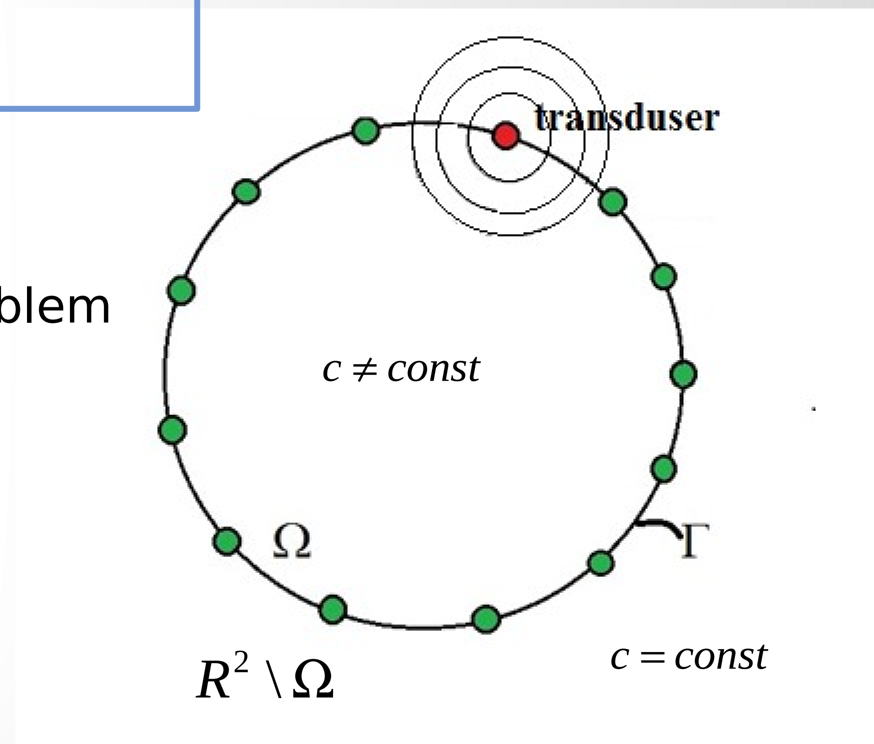
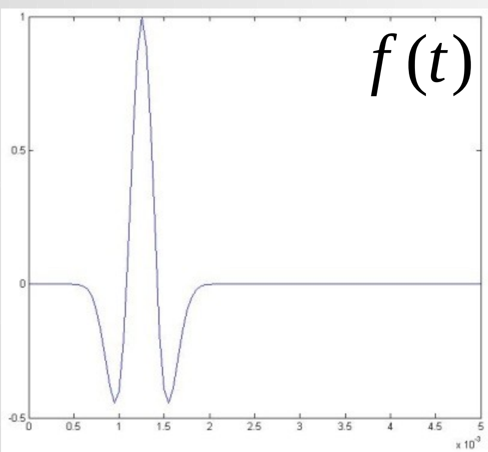


Fig. 1 *The scheme of the experiment*



$$f|_{t \leq 0} = 0$$

Fig. 2 Riker's impulse

Imaging problem

For the given boundary measurements

$$p_0(x,t;x_s) = p(x,t;x_s), \quad x \in \Gamma,\ t \in [0,T]$$

it is required to find the image of the speed of sound in the domain Ω.

Registration time of the waves - T, where

$T > 2T^*,\ T^*$ – «acoustic» radius of domain Ω.

Procedure of RTM

We use a version of the known in geophysics RTM procedure proposed in [1]. RTM procedure consists of the following steps:

Step 1: We calculate the pair $p_t^{for}(x,t;x_s),\ \nabla p^{for}(x,t;x_s)$ for the known acoustical medium (speed of sound c_0)

Step 2: For given $p_0(x,t;x_s)$ we solve reversal time problem:

$$\frac{1}{c_0^2}p_{tt}-\Delta p= f(t)\delta(x-x_s)\ \ in\ \ \ \Omega\times[0,T]$$

$$p\,|_{t=T}= p_t\,|_{t=T}=0, \qquad p\,|_{\Gamma\times[0,T]}= p_0$$

$p^{back}(x,t;x_s)$ - the solution of this problem

Step 3: We use the imaging condition

$$I_E(x)=\sum_s\int_0^T[p_t^{for}p_t^{back}+(\nabla p^{for},\nabla p^{back})]dt$$

[1] D. Rocha, N. Tanushev, P. Sava Acoustic wavefield imaging using the energy norm // 2015 SEG Annual Meeting, 18-23 October, New Orleans, Louisiana, P. 49-68

Numerical experiment

PML – perfectly matched layers

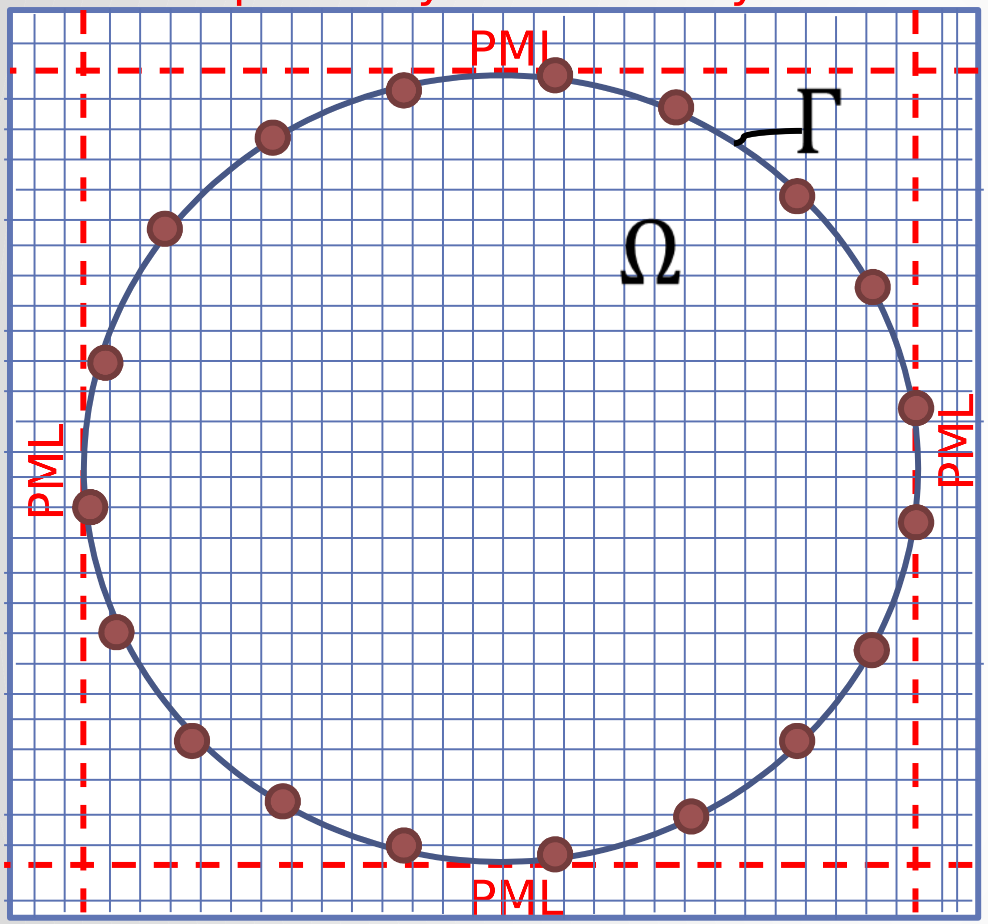


Fig. 3 *Scheme of the experiment*

Parameters of the experiment:

The number of nodes 10 240 000

dx = 0.1 mm

Radius (domain Ω) 15 cm

The number of transducers 256

Radius (domain Ω) 15 cm

The number of transducers 256

Parameters of ultrasound tomography:

The radius of the membrane 15 sm.

The number of transducers 256

The dominant frequency of the impulse 1.3 МГц

Numerical experiment, model

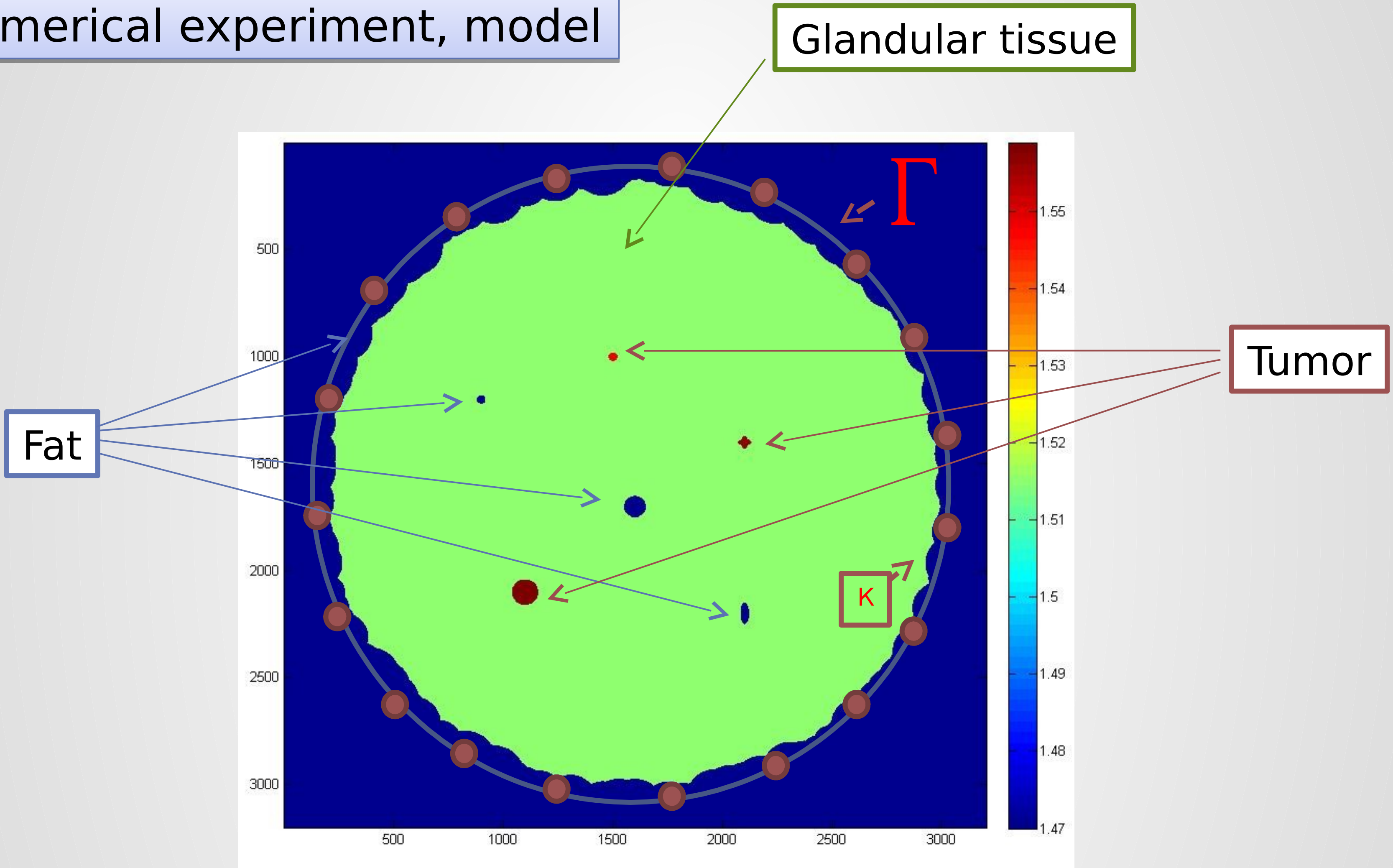


Fig. 4 – *Complicated acoustic model*

Sandhu GY. , Li C., Roy O., Schmidt S., Duric N. Frequency domain ultrasound waveform tomography: breast imaging using a ring transducer // Physics in Medicine & Biology. 2015. 60, P. 5381-5398

Numerical experiment, RTM

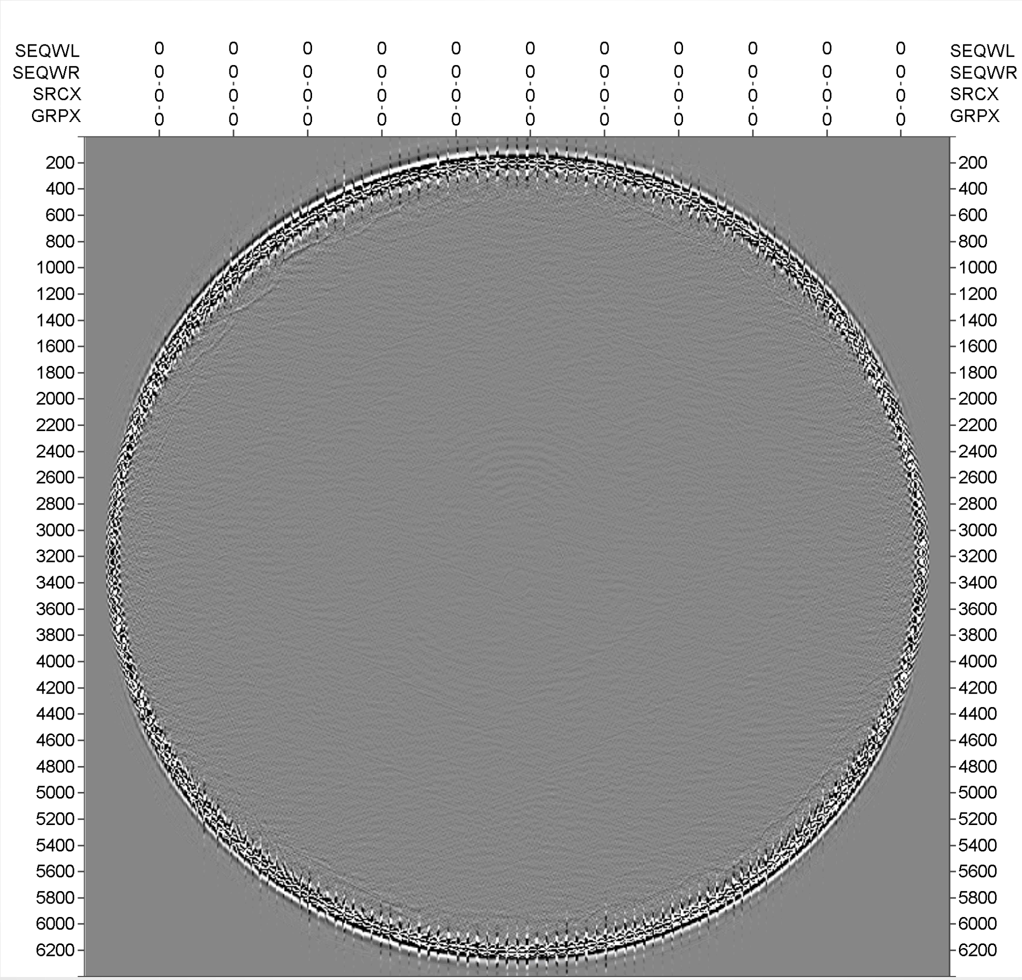


Fig. *5–RTM for complicated acoustic model*

Step 1: We recover the boundary indicated by the letter "K" in Fig. 4. For this we apply RTM procedure for a «small» time T (fig. 6).

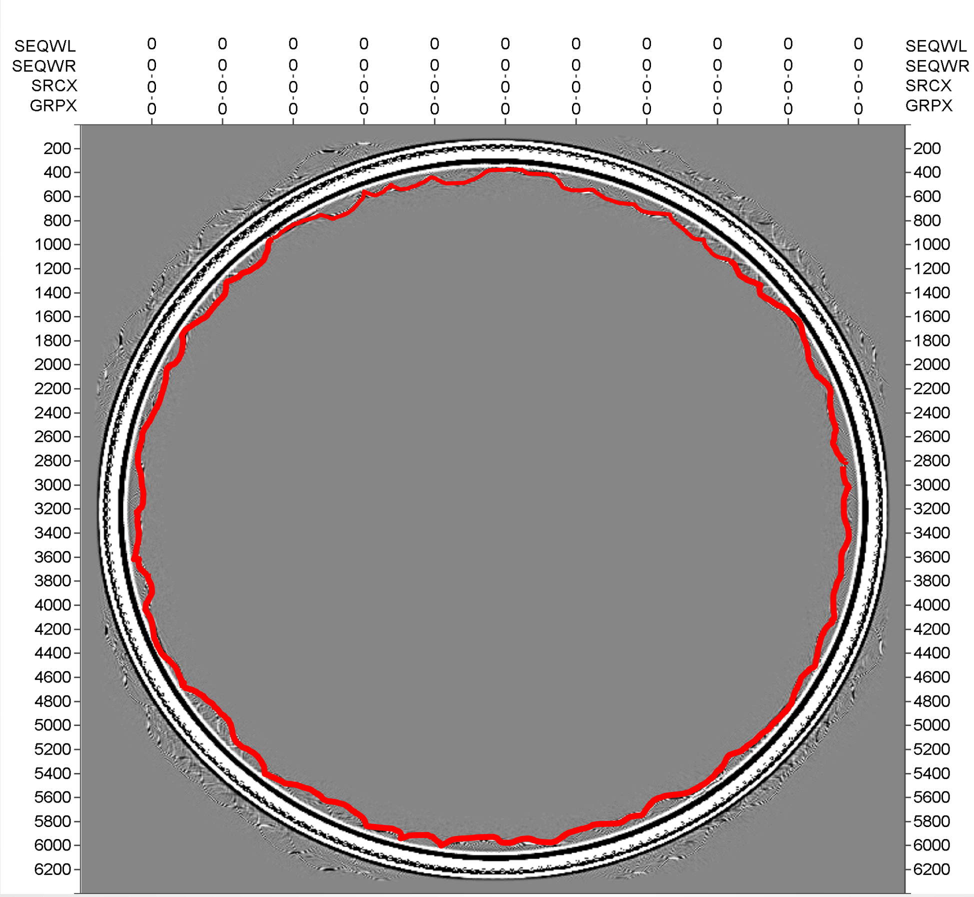


Fig. 6 – *Image of speed of sound for a short time of observation*

Step 2. We continue in reverse time wavefield p_0 and get wavefield denoted by p_1 at the boundary Γ_1 (Fig. 7). So we overcome through the boundary “K”.

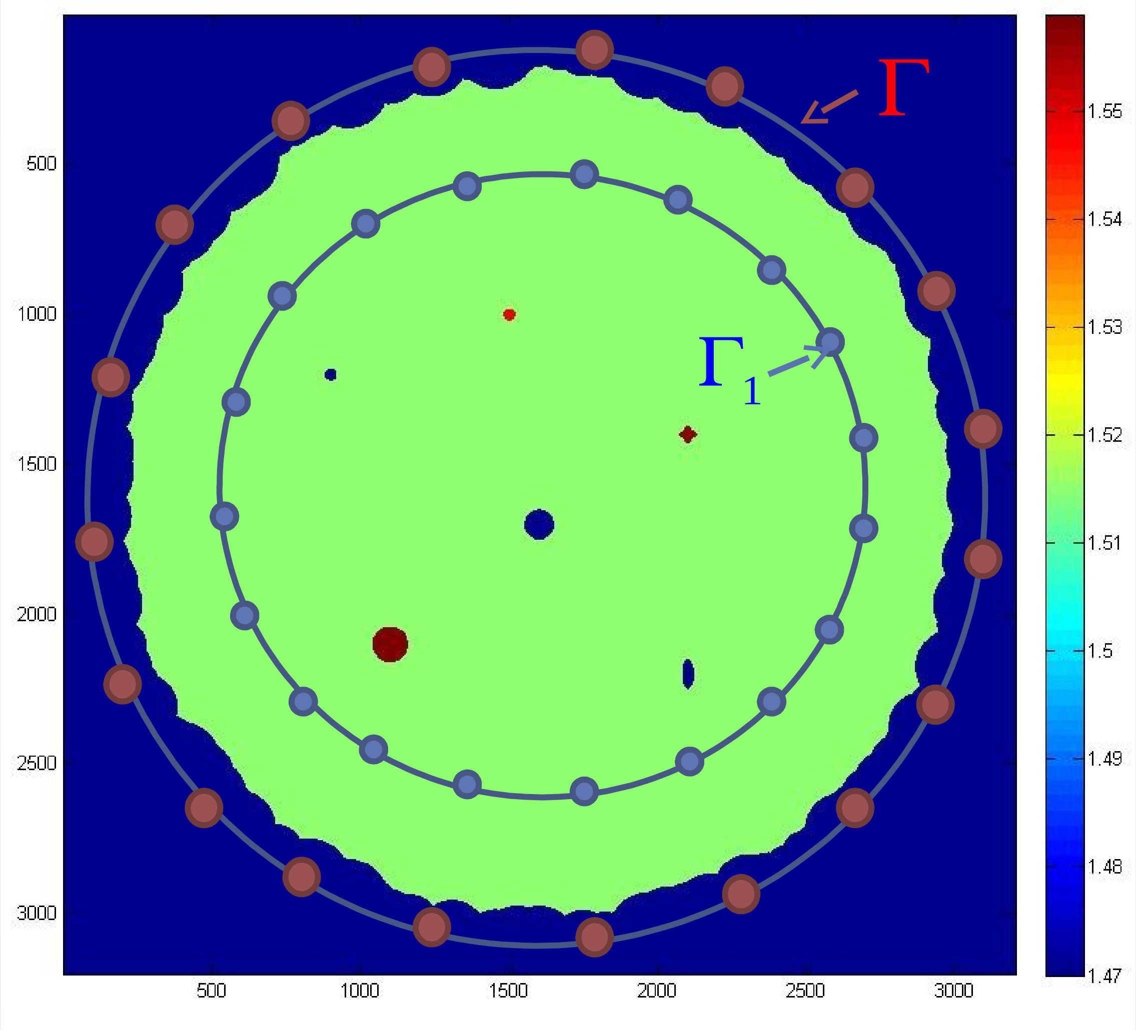


Fig. 7– *Complicated acoustic model, scheme of the boundary* Γ_1 *location.*

Step 3. We apply RTM procedure for data p_1 and get the image of inclusions (Fig. 8)

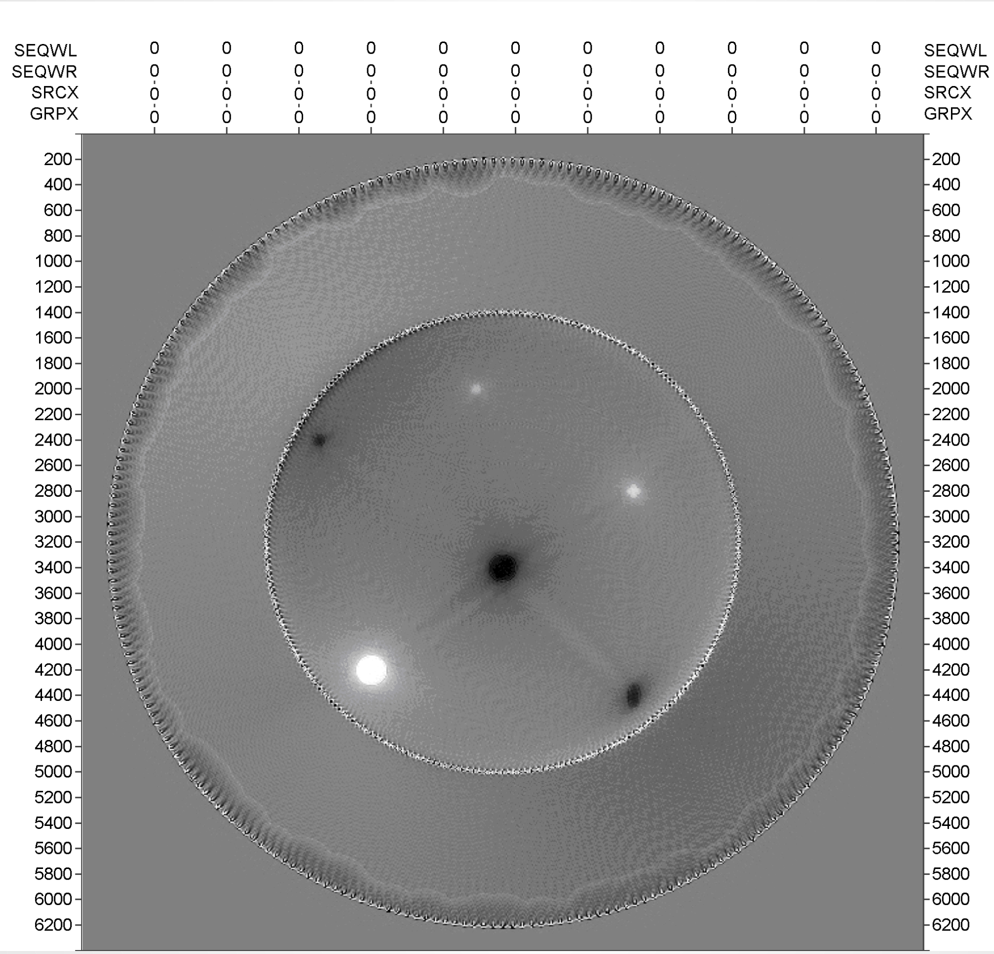


Fig. 8– *The image of the speed of sound.*

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THANK YOU FOR YOUR ATTENTION!