A flat approximation of inverse MEG-problems

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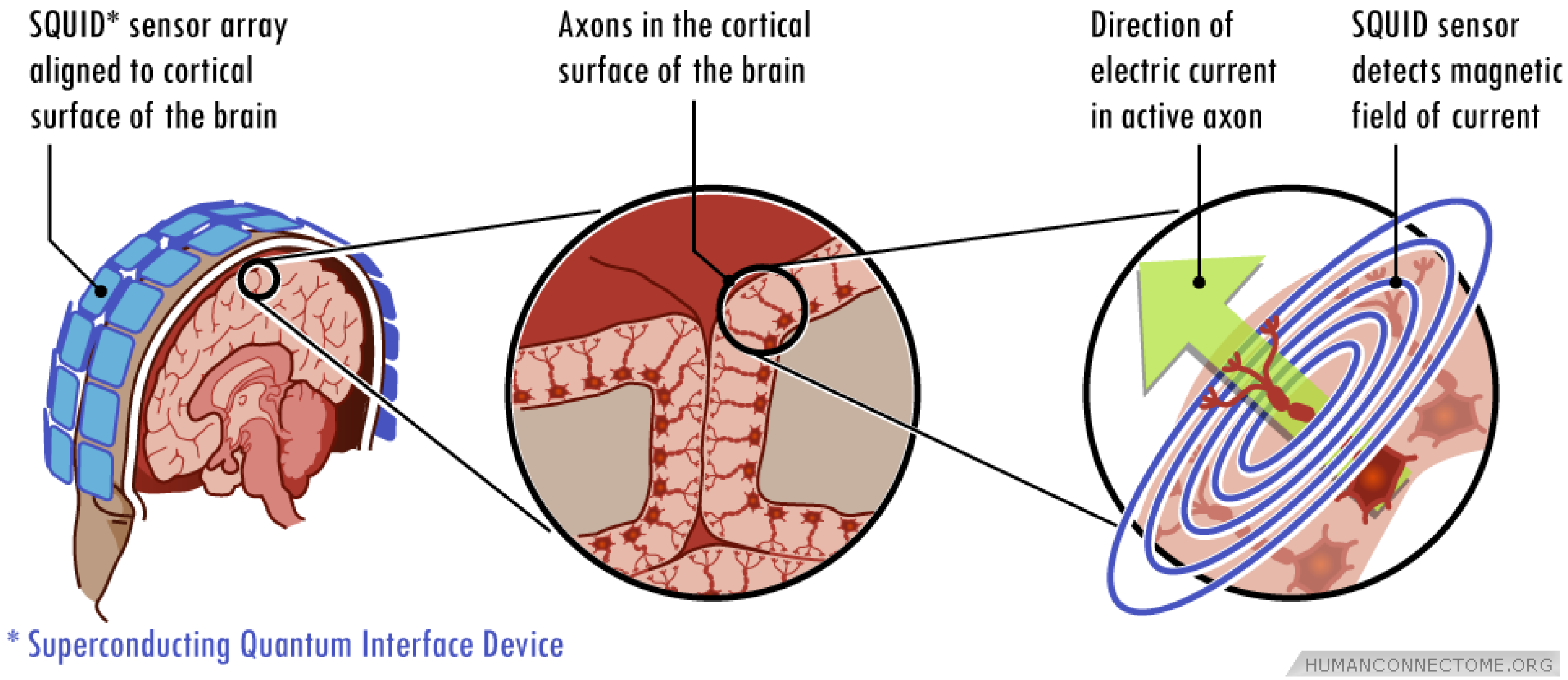
Plan

- What is magnetoencephalography (MEG)?
- Inverse problem
- The reason of our interest in this problem
- Steps of solution
- Subsequent goals

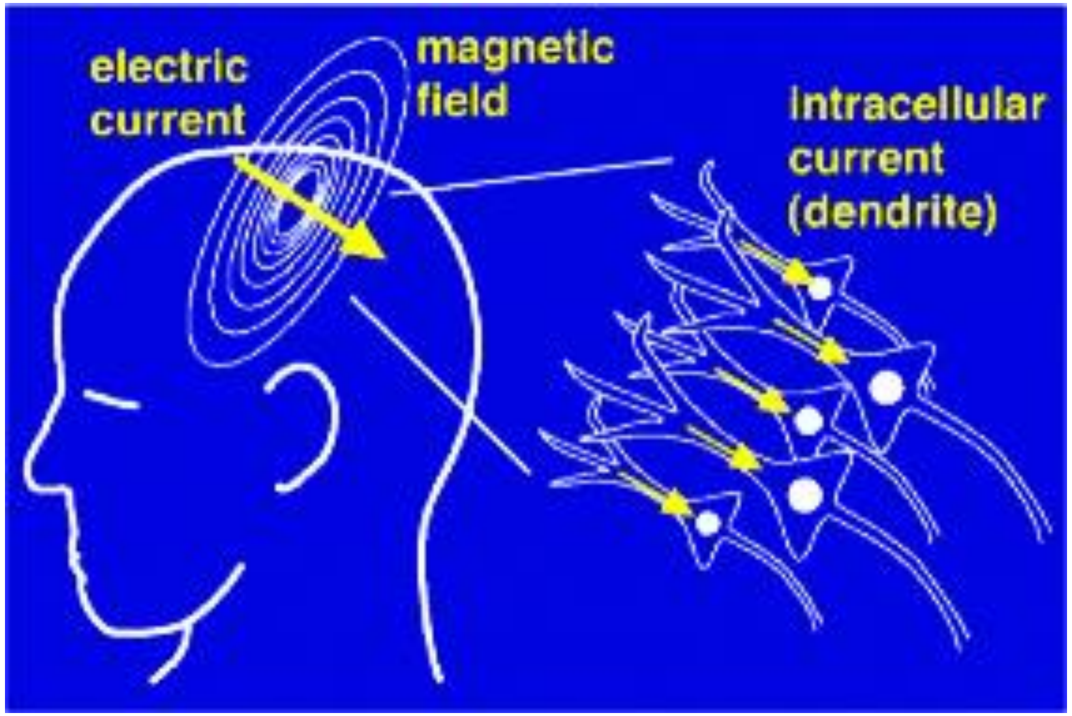
What is MEG?



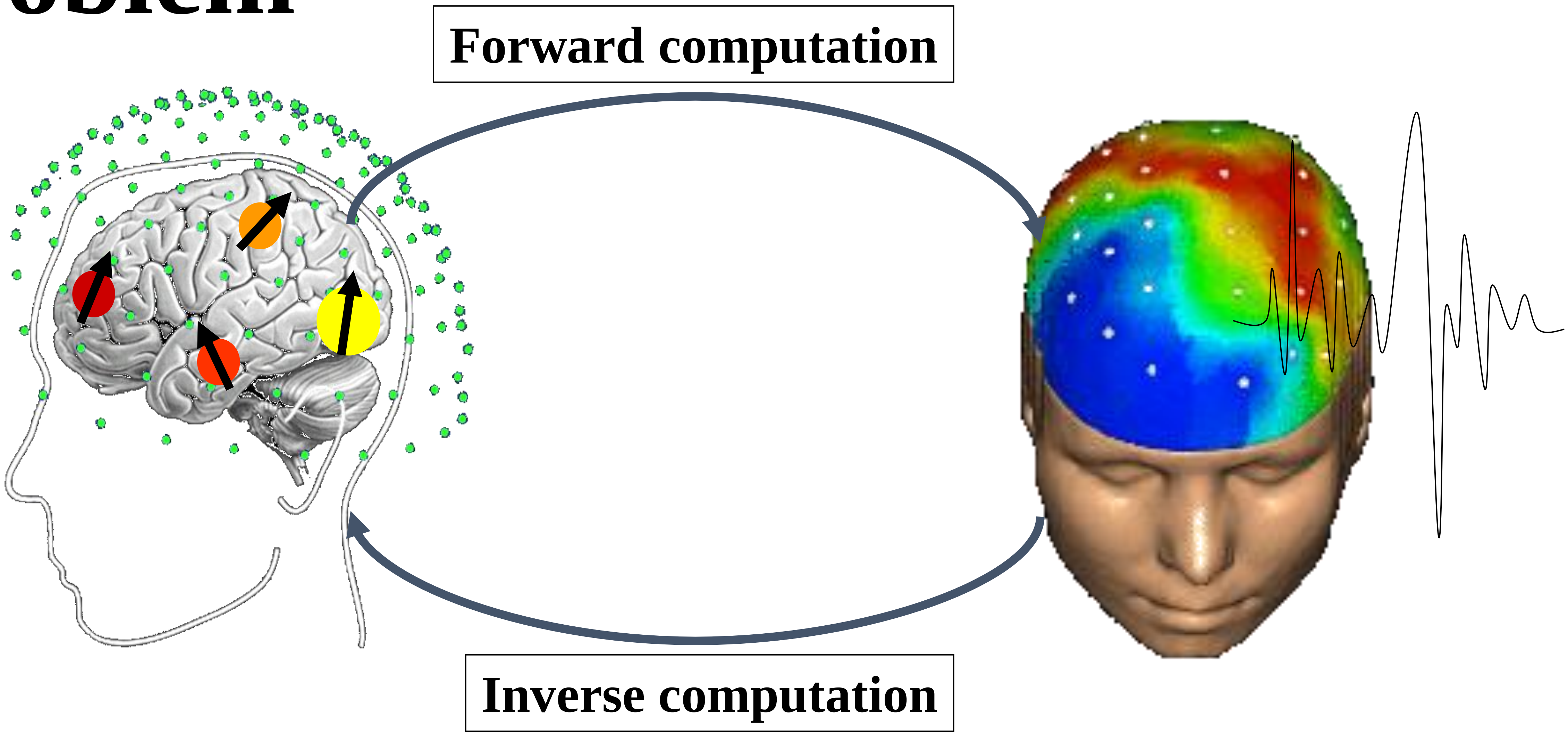
Magnetoencephalography is a noninvasive technique for investigating neuronal activity in the living human brain.

Inverse problem

- a problem of finding electrical impulses' distribution in some area Y (associated with cortex), that based on data of its induced magnetic field in another place X that we obtain by MEG system.



The reason of our interest in this problem



According to Biot–Savart law

$$B(x) = \int_Y K(x,y)\, Q(y)\, dy \quad , \quad x = (x_1, x_2, x_3) \in X$$

$$K(x,y) = \frac{\mu}{4\pi} \begin{pmatrix} 0 & K_{12}(x,y) & -K_{31}(x,y) \\ -K_{12}(x,y) & 0 & K_{23}(x,y) \\ K_{31}(x,y) & -K_{23}(x,y) & 0 \end{pmatrix}$$

$$K_{12}(x,y) = \frac{x_3 - y_3}{|x-y|^3}, \qquad K_{31}(x,y) = \frac{x_2 - y_2}{|x-y|^3}, \qquad K_{23}(x,y) = \frac{x_1 - y_1}{|x-y|^3}$$

At first we observe a following model:

$$X = \mathbb{R}_2 \ni x = (x_1, x_2), |x_k| < \infty$$

$$(\mathbb{R}_3 \supset Y) \ni y = (y_1, y_2, -\varepsilon): |y_k| < \infty$$

ε = 1

The equation assumes the following form:

$$\sum_{m=1}^{3} \int_{Y} K_{lm}(x-y) Q_m(y) dy = B_l(x), \qquad l = 1,2,3$$

Lemma 1

$$\widetilde{K}_{12}(\xi) = E(\xi), \text{where } E(\xi) = 2\pi e^{-2\pi|\xi|} \text{ :}$$

$$\xi = (\xi_1, \xi_2), |\xi| = \sqrt{\xi_1^2 + \xi_2^2}, \text{ and}$$

$$\widetilde{K}_{23}(\xi) = -i\,\frac{\xi_1}{|\xi|} E(\xi), \widetilde{K}_{31}(\xi) = -i\,\frac{\xi_2}{|\xi|} E(\xi),$$

where $\widetilde{K}(\xi) = \mathcal{F}_{s\to\xi}\, K(s)$

A consequence of Lemma 1

$$(1)\, Op\left(\widetilde{K}(\xi)\right) Q(z) = B(x),$$ **where**

$$Op\left(\widetilde{K}(\xi)\right) = \mathcal{F}^{-1}_{\xi\to x}\widetilde{K}(\xi)\mathcal{F}_{z\to\xi}\,,$$

$$\widetilde{K}(\xi)=\mathcal{F}_{s\to\xi}\,K(s)$$

$\widetilde{K}(\xi)$- **is a symbol of pseudodifferential operator**

$$(2)\ \widetilde{K}(\xi)\tilde{Q}(\xi) = \tilde{B}(\xi), \xi = (\xi_1, \xi_2)$$

Lemma 2

Equation form	Basis	Details
$\widetilde{\boldsymbol{K}}(\boldsymbol{\xi})\widetilde{\boldsymbol{Q}}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{B}}(\boldsymbol{\xi})$	$e_1 = (1,0,0)$, $e_2 = (0,1,0)$ $e_3 = (0,0,1)$	$\tilde{Q}(\xi)=(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3)$ $\tilde{B}(\xi)=(\tilde{B}_1, \tilde{B}_2, \tilde{B}_3)$
$\boldsymbol{\sigma}(\boldsymbol{\xi})\widetilde{\boldsymbol{u}}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{g}}(\boldsymbol{\xi})$ $\boldsymbol{\sigma}(\boldsymbol{\xi}) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$	$e_1'=(-\frac{i\xi_1}{\lvert\xi\rvert}, -\frac{i\xi_2}{\lvert\xi\rvert}, 1)$ $e_2'=(\frac{i\xi_2}{\lvert\xi\rvert}, -\frac{i\xi_1}{\lvert\xi\rvert}, 0)$ $e_3'=(\frac{i\xi_1}{\lvert\xi\rvert}, \frac{i\xi_2}{\lvert\xi\rvert}, 0)$	$\tilde{u} = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$ $\tilde{g} = (\tilde{g}_1, \tilde{g}_2, 0)$ $\sigma = (S^t)^{-1}\widetilde{K}(\xi)S^t$, where S -amplication matrix

Lemma 3

$$\sigma(\xi)\tilde{u}(\xi) = \tilde{g}(\xi) \;\Rightarrow\; \begin{cases} \tilde{u}_2(\xi) = \dfrac{\tilde{g}_1(\xi)e^{2\pi|\xi|}}{2\pi} \\ \tilde{u}_3(\xi) = \dfrac{\tilde{g}_2(\xi)e^{2\pi|\xi|}}{2\pi} \end{cases}$$

The algorithm of obtaining u_1

1st step	
Transition from the basis $\boldsymbol{e}'$ to $\boldsymbol{e}$	$e' = Se;\ \beta_e = S^t\alpha_{e'};\ \tilde{u}(\xi) \to \tilde{Q}(\xi)$

2nd step	
Making the inverse Fourier transform	$Q(z, u_1) = \mathcal{F}^{-1}_{\xi\to z}\left(\tilde{Q}(\xi)\right)$

The algorithm of obtaining u_1

3rd step	
According to Biot–Savart law, calculate all components of the magnetic field	$\sum_{m=1}^{3} \int K_{lm}(x-y) Q_m(y) dy = B_l(x), l = 1,2,3$

The algorithm of obtaining u_1

4th step	
Finding the magnitude of the vector B	$\|B\|^2 = B_1^2 + B_2^2 + B_3^2$

The algorithm of obtaining u_1

5th step	
Getting the final integral equation for u_1	$\|B\|^2 = \sum_{l=1}^{3} \left(\sum_{m=1}^{3} \int_{Y} K_{lm}(x-y)(G_m(y) + Op\left(-i\frac{\xi_m}{\|\xi\|}\right)u_1(y))dy \right)^2$

Lemma 4

Suppose that $y_1 = r\cos 2\pi\Theta$, $y_2 = r\sin 2\pi\Theta$.
$G(r,\Theta) = g(y_1, y_2) = \sum_{m\in\mathbb{Z}} G_m(r)e^{i2\pi m\Theta}$,
where $G_m(r) \in \mathbb{C}$. **Then**

$$\tilde{G}(|\xi|,\omega) = \sum_{n\in\mathbb{Z}} e^{i2\pi\left(\omega-\frac{1}{4}\right)n} \int_0^\infty rG_n(r)J_n(2\pi|\xi|r)dr,$$

where $\tilde{G}(|\xi|,\omega) = \mathcal{F}_{\,y\to\xi}\, g(y)$,

$$\xi_1 = |\xi|\cos 2\pi\omega\,, \xi_2 = |\xi|\sin 2\pi\omega$$

Subsequent goals

1. To get the computational solution of our equation for $\boldsymbol{u_1}$
2. To make the model suitable for the human brain topology
3. To true up our calculations corresponding to MEG data

Thank You for Your attention!