Memory of Gennady Henkin

Complex integral geometry
and around.

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Gena Henkin

Light was radiating from his face -- the brilliance of his intellect behind the soft glow of kindness in his eyes.

His was a powerful mind open to all kind of ideas and problems in mathematics and also, which is unusual even among the best mathematicians, he deeply understood science. And he was a charmingly humble person, the word "arrogance" was evaporating in his presence.

Many will follow the traces he left in mathematics and his friends will keep the warm of his personality in their hearts.

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I am not certain where this might go

privety

misha

[Quoted text hidden]

Gena Henkin was absolutely outstanding mathematician who essentially defined multidimensional complex analysis at ~~the~~ the last quarter of XX century and the beginning of XXI century.

A few remarks about of his style:

– he was ANALIST and explicite formula had an absolute value for him. Formulas for a half of page were typical for his papers.

– he knew the subject through, not just on what he works for a moment (unusual ~~as~~ for a person who migrated from other area). He read a lot preferring originals!). Look at Bibliographies at his paper, especialy expository ones). See his naming of Teorem and Objects (f.e. Abel - Radon - Martineau - Penrose transfor

- He had a lot of ideas and was hungry

on students and collaborators...

- He was skeptical to conceptual facts without "applicatio

His ~~Her~~ Heroes:

~~Deer~~ Leray, Oka, Martineau

Several complex variables

- Crucial difference with one variable:

where
- Isolated zeroes and singularities
- ~~Hol~~ For each domain there are holomorphic functions which live at it.
- Coincidence of holomorphity ~~and~~ and conformity
- A lot of automorphisms
- Non essential singularities on boundaries
- The universal Cauchy integral formula

1890-1910 Great childhood and
Great problems
Hartogs -
- Weierstraß - Poincaré - Cousin - E. Levi -
Verification only on simplest examples
< 30th Problems only accumulated (Oka)

30-40-50e Great epoch
H. Cartan & Oka solved all great problems
- Strong influence A. Weil

Integral formulas!

- < 30th Only direct product of one-dimensional:

  $|z|<1, \ |w|<1$ at $\mathbb{C}^2_{(z,w)}$

  The integral over "edge" $|z|=|w|=1$.

- A. Weyl: polyhedron $|f_1(z,w)|<1$
  $|f_2(z,w)|<1$
  $|f_j(z,w)|<1$

32-34 (Bergman

A "small" step was sufficient for H. Cartan, Oka to solve Cousin problems

- Bochner - Martinelly: ~~Integral~~ Universal kernel
  $B(z,w)$ but not holomorphic.

  gave a natural approach to the Hartogs phenomenon!

$|z_1|^2+|z_2|^2<1$ - complex ball

holomorphic function around boundary extends at the whole ball.

To the beginning of 50-th the building of multidimensional complex analysis was finished. It had a beautiful look thanks approach of coherent ~~al~~ analytic sheafs of H. Cartan following Leray' theory. But integral formulas disappeared...

60-e – 2nd approach through dif. equations ($\bar{\partial}$-equations) of Morrey - Kohn - Hörmander with L^2-estimates.

1956 The Cauchy - Fantappie formula (Leray).

$\mathbb{C}^n_z \qquad z = (z_1, \dots, z_n)$

$\mathbb{C}^n_\xi \qquad \xi = (\xi_1, \dots, \xi_n) \Rightarrow \mathbb{C}P^{n-1}_\xi$

$\langle \xi, z \rangle = \xi_1 \cdot z_1 + \dots + \xi_n z_n$

$dz = dz_1 \wedge \dots \wedge dz_n$

$$\omega(\xi) = \sum_{1 \le j \le n} (-1)^j \xi_j \bigwedge_{k \ne j} d\xi_k$$

$f(z) \in \mathcal{O}(D), \quad D \subset \mathbb{C}^n_z$

C. F. kernel

$$K(w|z) = \frac{f(z)\, dz \wedge \omega(\xi)}{\langle \xi, z - w \rangle^n}$$

T.1. K is closed at ~~D & @~~

$$D \times \left(P\mathbb{C}^{n-1}_\xi \setminus \{\text{sing}\} \right) = M_w$$

T.2. $\displaystyle \int_{\gamma \subset M_w} K(w|z) = c(\gamma) f(w).$

{Different cycles γ} $\Leftrightarrow$ {Different integral formulas}.

Basic construction: Choice of

$\xi(z|w)$, $z \in \partial D$, $w \in D$, s.t. $\langle \xi(z|w), z-w \rangle \neq 0$ [A difficult occupation]

1. Martinelli–Bochner
$$\xi(z|w) = \overline{z} - \overline{w}.$$

2. A. Weil.

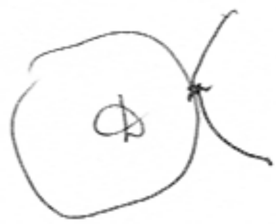
3. D is convex $\quad \rho(z) < 0$

$$\xi(w,z) = \left(\frac{\partial \rho}{\partial z}\right)$$ – independent of w.

(Leray–Martineau)

Henkin-Ramires formula

Stricly pseudoconvex domain D with a smooth boundary



$\rho(z)<0$ E. Levi condition: $\partial^2\rho$ holom. quadratic form is positive.

∀ local quadratic barrier

H.-R. It can be extend globally.

= The kernel isn't explicit but admits very good ~~ex~~ estimate (uniformal).

A big project of Genndy (a competition with a dragon with 2 head

3rd way — Build complet theory only on the base of this formulas

supplying Oka-H. Cartan results by estimates.

It was realized ~~with~~ in a very extende form ($\bar{\partial}$-equation, Cauchy-Green formula, non smooth boundary, Stein menifolds, CR-~~f~~ manifo

G.-Henkin (85). Projective invariant version of

C.-F. formula.

$\Downarrow$

- Other forms of C.-F. formula which Leray suspected but didn't gave their explicit form,
- it included explicit computation of residues for multiple polar singularities.

Holomorphic representations of $\bar\partial$-cohomologies and Penrose transform.

Andreotti's conception: The subject of complex analysis is $H^{(q)}$ - space of $\bar\partial$-cohomology which is ∞-dimensional

(for Stein m., $q=0$).

- q-pseudo-concave domains

(q-linear concave domains at P^n).

Example. $\mathbb{CP}^2$, $z=(z_0,z_1,z_2)$

$$|z_1|^2+|z_2|^2-|z_3|^2>0$$

No holomorphic (non constant functions)

Exterior of the ball

$$\dim H^{(1)}(\mathcal{D}, \mathcal{O}(-2)) = \infty \quad \Big|\ \hat{\mathcal{D}} - \{\text{lines at } D\}.$$

$$\omega = \sum f_i(z)\, d\bar z_j, \quad f_i \in \mathcal{O}(-2)$$

$L(z,w)$ - line through z, w. $(\tau_0 z + \tau_1 w)$

$$P\omega(z,w) = \int_{L(z,w)} \omega \wedge (-\tau_0 d\tau_1 + \tau_1 d\tau_0)$$

$P\omega$ - is holomorphic at $\hat{D}$.

Inversion:

$F = \{z, w\}$ – Stein

$$\sum_i \frac{\partial P_\omega(z,w)}{\partial w_i z_i} dw_i = \varkappa(P\omega)$$

Took $\varkappa$-operation from integr.-geometry

$\varkappa(P(\omega))|_\gamma - \omega - \bar\partial$-exact, where γ- section of $\begin{matrix} F \\ \downarrow \\ D \end{matrix}$.

So we reconstruct the cohomology class of ω.

Interpretation: 1) F is a Stein manifold fibering over D (contractible fibers).

2. On F ("at the sky") we have a holomorphic object & which stops to be holomorphic after the projection on D ("earth").
[cf. with Cauchy - F. formulas and residues]

We can work with $\bar\partial$-cohomology on holomorphic language.