Applications of the generalized nonparametric method to the analysis of a stock market crash

N. I. Klemashev

Lomonosov Moscow State University

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Plan

1. Chinese stock market crash in Summer 2015.
2. Generalized nonparametric method.
3. Analysis.
4. Conclusions.
5. References.

Chinese stock market crash in 2015

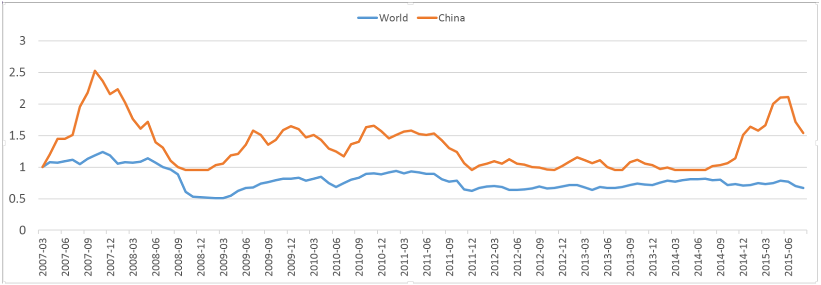

Figure 1: Price indices.

Chinese stock market crash in 2015

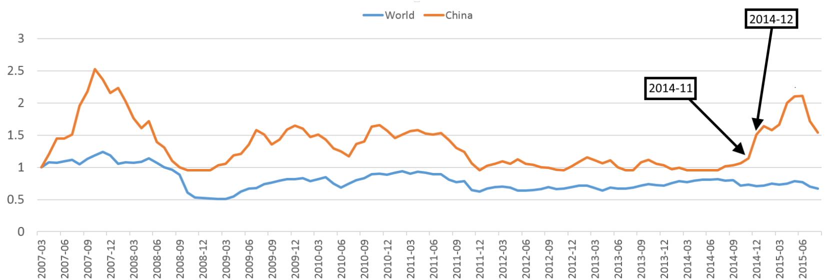

Figure 2: Price indices.

Chinese stock market crash in 2015

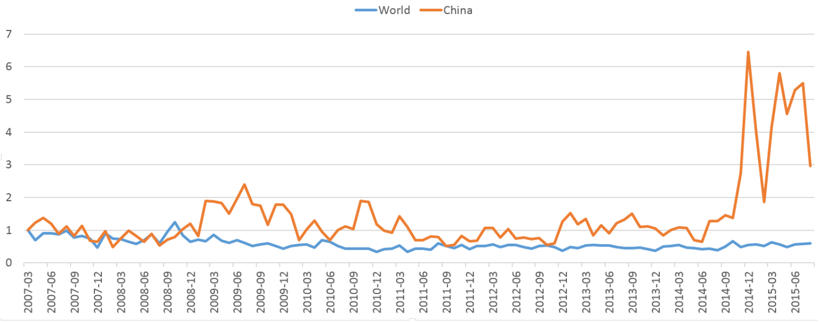

Figure 3: Volume indices.

Chinese stock market crash in 2015

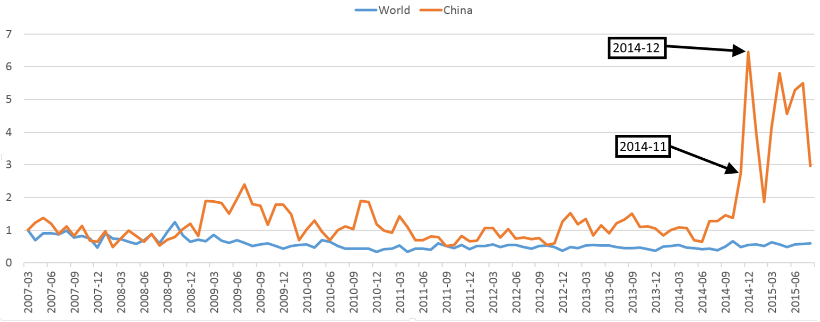

Figure 4: Volume indices.

Structure of Chinese stock market

- Two types of agents;
- Major investors;
  - Long-term strategies;
  - Objective: savings and low-risk profit;
- Minor investors (speculators);
  - Short-term strategies;
  - Objective: fast and risky profit;
- Two types of preferences;
- Two types of utility functions.
- How to reveal minor investors?

Rationalizability of trade statistics

- Trade statistics: $TS = \{(P^t, X^t)\}_{t=1}^{T}$, $P^t \in \mathbb{R}^m_{++}$, $X^t \in \mathbb{R}^m_{+}$.
- Φ_H – set of all functions defined on $\mathbb{R}^m_+$ which are positively homogeneous of degree 1, continuous, concave, non-satiated, and taking nonzero values for all points in $\mathbb{R}^m_+$.
- TS is ratinalizable (in Φ_H) if there exists $F \in \Phi_H$ such that for all $t \in \{1, \ldots, T\}$

$$X^t \in \operatorname{Arg\,max}\{F(X) \mid X \in \mathbb{R}^m_+,\ \langle P^t,\ X\rangle \leqslant \langle P^t,\ X^t\rangle\}. \quad (1)$$

Afriat-Varian theorem on rationalizability in Φ_H

The following statements are equivalent:

1) trade statistics $\{(P^t, X^t)\}_{t=1}^T$ is rationalizable in Φ_H;
2) there exist numbers $\lambda^t > 0$ ($t = 1, \ldots, T$), such that

$$\lambda^t \left\langle P^t, X^\tau \right\rangle \geqslant \lambda^\tau \left\langle P^\tau, X^\tau \right\rangle, \quad \forall t, \tau = 1, \ldots, T; \tag{2}$$

3) trade statistics $\{(P^t, X^t)\}_{t=1}^T$ satisfies Homothetic Axiom of Revealed Preference (HARP), which means that for all subsets of indices $\{t_1, \ldots, t_k\}$ from $\{1, \ldots, T\}$

$$\begin{aligned} &\left\langle P^{t_1}, X^{t_2} \right\rangle \left\langle P^{t_2}, X^{t_3} \right\rangle \ldots \left\langle P^{t_k}, X^{t_1} \right\rangle \geqslant \\ &\qquad \geqslant \left\langle P^{t_1}, X^{t_1} \right\rangle \left\langle P^{t_2}, X^{t_2} \right\rangle \ldots \left\langle P^{t_k}, X^{t_k} \right\rangle \end{aligned} \tag{3}$$

4) the function $u(X) = \min_{\tau \in \{1, \ldots, T\}} \{\lambda^\tau \left\langle P^\tau, X \right\rangle\}$, where $\{\lambda^t\}_{t=1}^T$ satisfy (2) and $\lambda^t > 0$ for all $t \in \{1, \ldots, T\}$, rationalizes trade statistics $\{(P^t, X^t)\}_{t=1}^T$.

Konüs-Divisia indices (nonparametric method)

- trade statistics $\{(P^t, X^t)\}_{t=1}^T$ satisfies HARP;
- $\lambda^t > 0$ $(t = 1, \ldots, T)$ satisfy

$$\lambda^t \left\langle P^t, X^\tau \right\rangle \geqslant \lambda^\tau \left\langle P^\tau, X^\tau \right\rangle, \quad \forall t, \tau = 1, \ldots, T;$$

- Konüs-Divisia consumption index: $F^t = \lambda^t \langle P^t, X^t \rangle$;
- Konüs-Divisia price index: $Q^t = \frac{1}{\lambda^t}$.

Generalized nonparametric method

- trade statistics $TS = \{(P^t, X^t)\}_{t=1}^{T}$ does not satisfy HARP;
- irrationality index – minimum ω such that TS satisfies HARP(ω): for all subsets of indices $\{t_1, \ldots, t_k\}$ from $\{1, \ldots, T\}$

$$\left\langle P^{t_1}, X^{t_2}\right\rangle \left\langle P^{t_2}, X^{t_3}\right\rangle \ldots \left\langle P^{t_k}, X^{t_1}\right\rangle \geqslant \\ \geqslant \frac{1}{\omega^k} \left\langle P^{t_1}, X^{t_1}\right\rangle \left\langle P^{t_2}, X^{t_2}\right\rangle \ldots \left\langle P^{t_k}, X^{t_k}\right\rangle$$

- Afriat-Varian theorem: TS satisfies HARP(ω) iff there exist $\lambda^t > 0$ $(t = \overline{1, T})$ such that

$$\lambda^t \left\langle P^t, X^t\right\rangle \leqslant \omega\lambda^\tau \left\langle P^\tau, X^t\right\rangle. \quad \forall t, \tau = \overline{1, T}$$

Forecasting

Forecasting set:
$K(P; TS, \omega) = \{X \in \mathbb{R}_+^m \mid TS \cup \{(P, X)\}$ satisfies HARP$(\omega)\}$

Theorem [Grebennikov, Shananin, 2008]
Assume that the trade statistics $TS = \{(P^t, X^t)\}_{t=1}^T$ satisfies HARP(ω) with $\omega \geqslant 1$ and P is not equal to one of P^ts.
Then

$$K(P; TS, \omega) = \left\{X \in \mathbb{R}_+^m \mid \gamma_s(P, \omega)\langle P^s, X\rangle \geqslant \langle P, X\rangle \ \forall s \in \{1, \ldots, T\}\right\}$$

Forecasting

where

$$C^*_{ts}(\omega) = \max\left\{\omega^{-k-1} C_{tt_1} C_{t_1 t_2} \dots C_{t_{k-1} t_k} C_{t_k s} \;\middle|\; \{t_1, \dots, t_k\} \subset \{1, \dots, T\},\ k \geqslant 0\right\},$$

$$\gamma_s(P, \omega) = \min_{t \in \{1, \dots, T\}} \left\{ \frac{\omega^2}{C^*_{ts}(\omega)} \frac{\langle P,\ X^t \rangle}{\langle P^t,\ X^t \rangle} \right\},$$

and

$$C_{t\tau} = \frac{\langle P^\tau,\ X^\tau \rangle}{\langle P^t,\ X^\tau \rangle}. \tag{4}$$

Analysis – irrationality indices MW

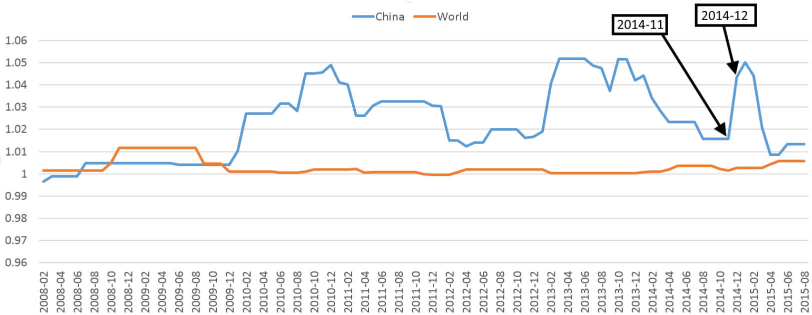

Figure 5: 12-months moving window for irrationality indices.

Analysis – acceptable level of irrationality

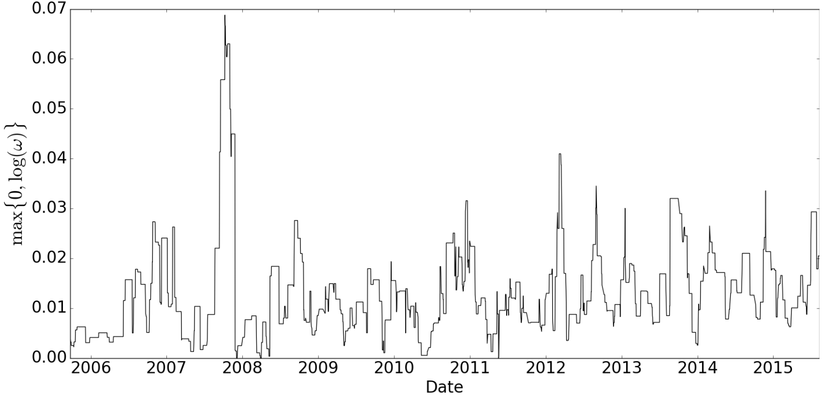

Figure 6: 31-days moving window for log-irrationality index.

Analysis – Trade statistics

- 100 stocks;
- Daily data aggregated to monthly data.
- January 2014 – August 2015.
- Acceptable level of log-irrationality: 0.035.
- Actual log-irrationality: 0.049.
- **Objective:** Find stocks most responsible for the increased irrationality.

Analysis – irrationality index

- log-Paasche indices: $c_{t\tau} = \log\left(\frac{\langle P^\tau, X^\tau\rangle}{\langle P^t, X^\tau\rangle}\right)$.
-

$$\log(\omega) \to \min_{\omega,\lambda^t}, \tag{5}$$

$$\log(\omega) + \log(\lambda^t) - \log(\lambda^\tau) \geqslant c_{t\tau}, \quad (t, \tau = \overline{1, T}, t \neq \tau) \tag{6}$$

$$\log(\omega) \geqslant 0 \tag{7}$$

Analysis – irrationality indices for pairs of periods

$$\sum_{t=1}^{T}\sum_{\substack{\tau=1\\ \tau\neq t}}^{T}\log(\omega_{t\tau}) \to \min_{\omega_{t\tau},\lambda^t}, \tag{8}$$

$$\log(\omega_{t\tau}) + \log(\lambda^t) - \log(\lambda^\tau) \geqslant c_{t\tau}, \quad (t,\tau=\overline{1,T}, t\neq\tau) \tag{9}$$

$$\log(\omega_{t\tau}) \geqslant 0 \qquad (t,\tau=\overline{1,T}, t\neq\tau) \tag{10}$$

Analysis – selecting periods

t	τ	$\log(\omega_{t\tau})$
2015–01	2014–01	0.076
2015–01	2014–02	0.073
2015–01	2014–03	0.065
2015–04	2014–02	0.063
2015–04	2014–03	0.058
2015–04	2014–01	0.056
2014–12	2014–01	0.051
2015–03	2014–03	0.051
2014–12	2014–03	0.051
2015–03	2014–02	0.049
2015–03	2014–01	0.049
2014–12	2014–02	0.041
2015–02	2014–02	0.040
2015–02	2014–01	0.039
2014–02	2014–12	0.038
2015–02	2014–03	0.037

Analysis – selecting periods

Selected periods:

- 2014–12;
- 2015–01;
- 2015–02;
- 2015–03;
- 2015–04.

Analysis – volume projection

Given

- Trade statistics satisfying HARP(ω),
- An observation (P, X),

find the projection of X on $K(P; TS, \omega)$:

$$\|X - Y\|^2 \to \min_{Y \in \mathbb{R}^m}, \tag{11}$$

$$Y \in K(P; TS, \omega) \tag{12}$$

Selecting stocks

Let us split the set $\{1, \ldots, T\}$ into two subsets with empty intersection:

$$\{1, \ldots, T\} = V^* \cup (\{1, \ldots, T\} \setminus V^*), \tag{13}$$

where V^* is the set of periods selected on the previous step. We order elements of V^* in ascending order, project the observed volumes X^t for $t \in V^*$ with consequent adding the projected volumes to the trade statistics and collect the differences between the observed volumes and the projected ones.

Checking procedure – partial projection

Fix a set of stocks I.

$$\|X - Y\|^2 \to \min_{Y \in \mathbb{R}^m}, \tag{14}$$

$$Y \in K(P; TS, \omega) \tag{15}$$

$$Y_i = X_i \qquad (i \notin I) \tag{16}$$

The resulting set of stocks

- CITIC Securities Co Ltd (ticker 600030).

Conclusions

- New method for analysis of stock marker crises.
- It allows an analyst to select only few stocks for further analysis.
- The method is computationally efficient.
- We managed to reduce the number of stocks for detailed analysis from one hundred to just one.

References

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