High-order schemes in numeriical problems of seismic exploration in the Arctic

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- Numerical method
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Aims of Study

- Modeling of wave propagation in elastic media by grid-charactersitic method.
- Correct definition and calculation of boundary and interface conditions

Mathematical model
Elastic medium

Components of vector of velocity and components of stress tension describing the state of linear-elastic medium are the solutions of the following system of equations:

$$\rho \partial_t \vec{v} = \left(\nabla \cdot \boldsymbol{\sigma} \right)^{\mathrm{T}}$$

$$\partial_t \boldsymbol{\sigma} = \lambda \left(\nabla \cdot \vec{v} \right) \mathbf{I} + \mu \left(\nabla \otimes \vec{v} + \left(\nabla \otimes \vec{v} \right)^{\mathrm{T}} \right)$$

Mathematical model
Acoustic medium

For numerical modeling of sea water we use the prefect fluid approximation, solve acoustic wave equation and find components of vector of velocity and pressure.

$$\rho \frac{\partial}{\partial t} \vec{v} = -\nabla p$$

$$\frac{\partial}{\partial t} p = -c^2 \rho (\nabla \cdot \vec{v})$$

Grid-charactristic method

Method for solving hypergolic systems of equations. We use it for solving both acoustic and elastic wave equations. In 2D-case these systems could be written in the following form

$$\frac{\partial \vec{q}^{\,2e}}{\partial t} + \mathbf{A}_1^{2e} \frac{\partial \vec{q}^{\,2e}}{\partial x_1} + \mathbf{A}_2^{2e} \frac{\partial \vec{q}^{\,2e}}{\partial x_2} = 0$$

$\vec{q}^{\,2e}$ - vector of unknown fields

Grid-characteristic method

We use splitting on spatial directions and obtain 2 systems of equations

$$\frac{\partial \vec{q}^{\,2e}}{\partial t} = \mathbf{A}_j^{2e} \frac{\partial \vec{q}^{\,2e}}{\partial x_j}$$

Grid-characteristic method

Both of these systems:

- is hyperbolic
- obtains 5 real eigenvalues
- So we can write it in the following form

$$\frac{\partial \vec{q}^{\,2e}}{\partial t} = \left(\mathbf{\Omega}_j^{2e}\right)^{-1} \mathbf{\Lambda}_j^{2e} \mathbf{\Omega}_j^{2e} \frac{\partial \vec{q}^{\,2e}}{\partial x_j}$$

Grid-characteristic method

Change of unknown fields:

$$\vec{p}^{2e} = \mathbf{\Omega}^{2e}\vec{q}^{2e}$$

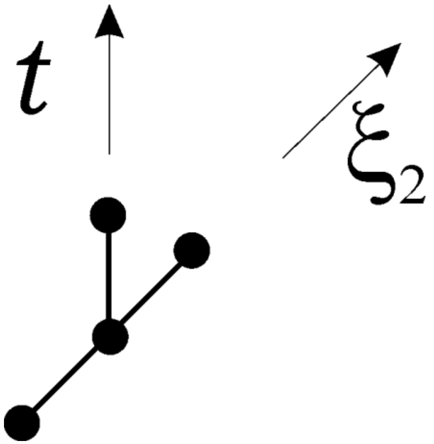
All of obtained systems becomes the system of 5 independant transport equations:

$$\frac{\partial \vec{\mathbf{p}}^{2e}}{\partial t} + \mathbf{\Lambda}^{2e}\frac{\partial \vec{\mathbf{p}}^{2e}}{\partial x} = 0$$

Grid-characteristic method

Then one can find the solution of the given system of equations:

$$\vec{q}^{\,2e}\left(x_1, x_2, t+\tau\right) = \left(\mathbf{\Omega}^{2e}\right)^{-1} \vec{p}^{\,2e}\left(x_1, x_2, t+\tau\right)$$



$$\frac{\partial \vec{u}}{\partial t} + \mathbf{A}_1 \frac{\partial \vec{u}}{\partial \xi_1} = 0$$

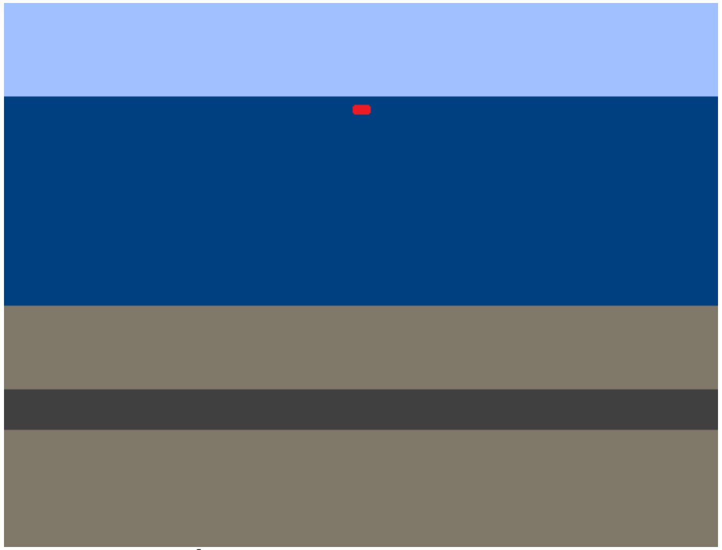
$$\vec{u}' = \vec{u}^n - \tau \mathbf{A}_1 \Delta_1 \vec{u}^n$$

2D Model

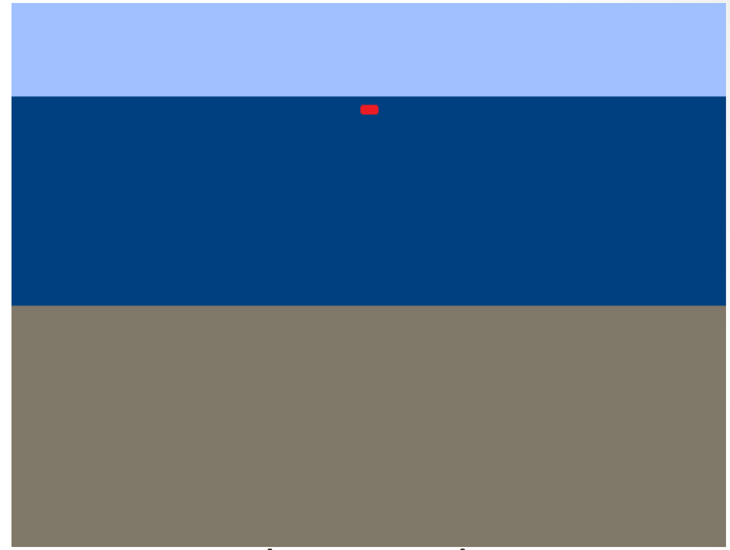
- Spatial step 0.2 *м*
- Time step $3 \cdot 10^{-5}\,c$
- 15 000 time steps.
- Region for integration *1200 x 600 м*
- System “ice-water-ground-carbon reservoir-ground
- Absorbing conditions at the sides and at the bottom of the region
- Free boundary condition on the top side of the region

1. Sources in the water and at the seabed, the case without ice

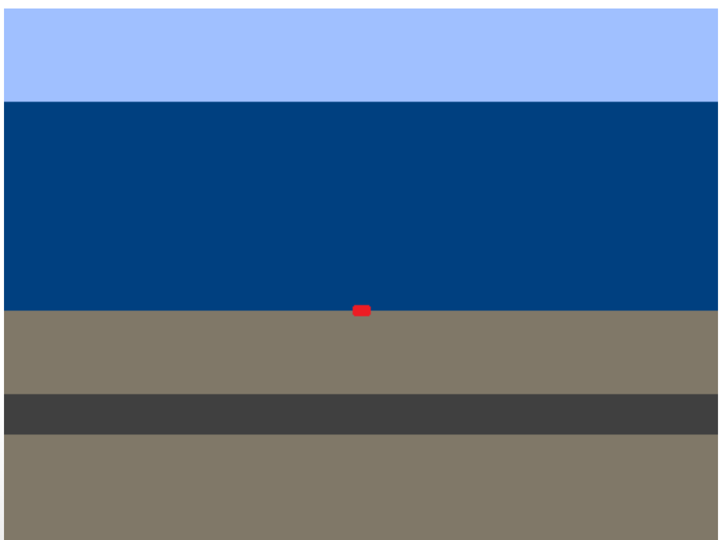
Problem definitions



Source in the water



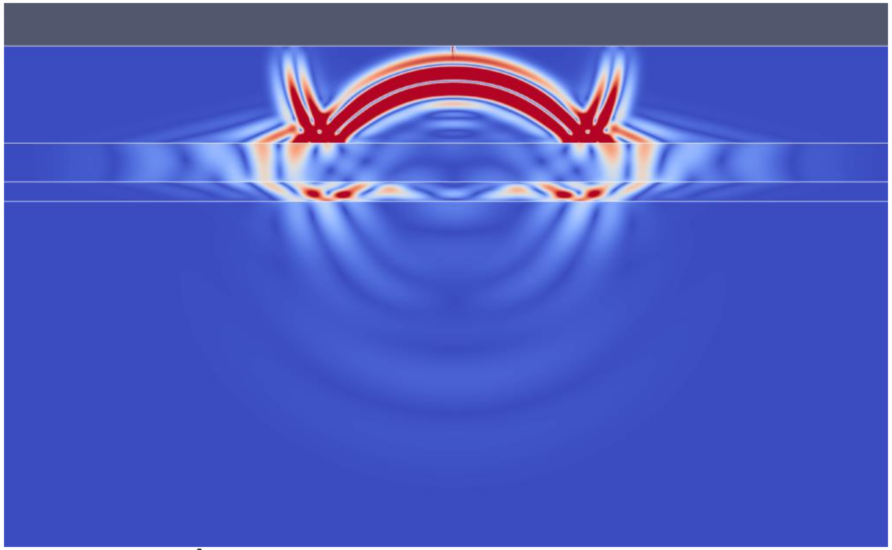
Source in the water, without carbon reservoir



Source at the seabed

Source at the seabed, without carbon reservoir

Wave patterns



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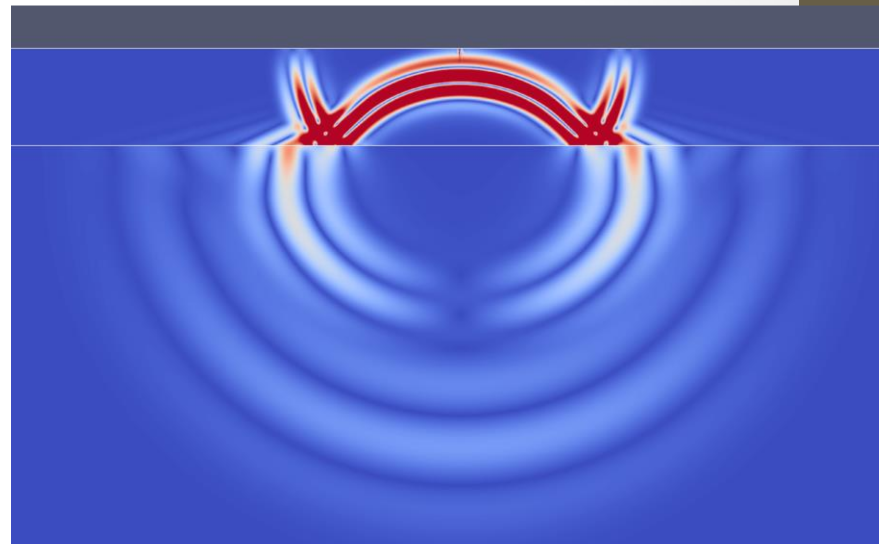
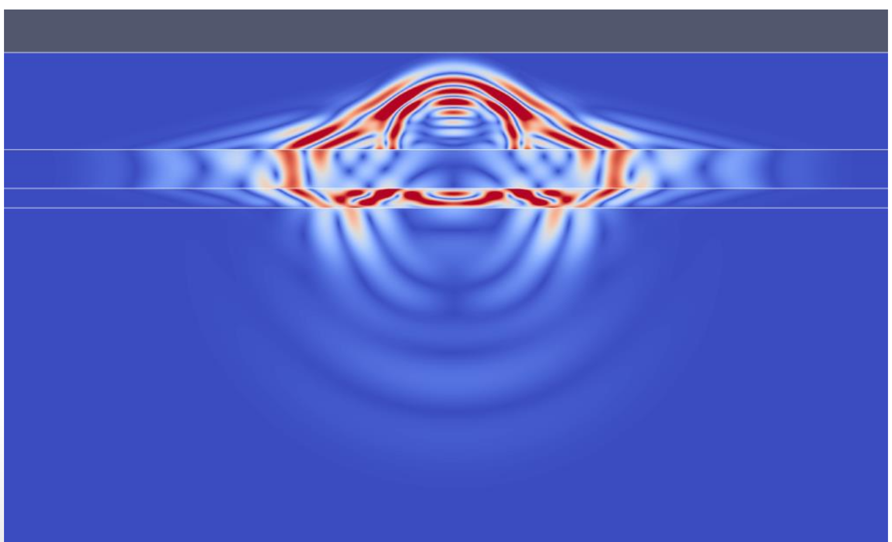
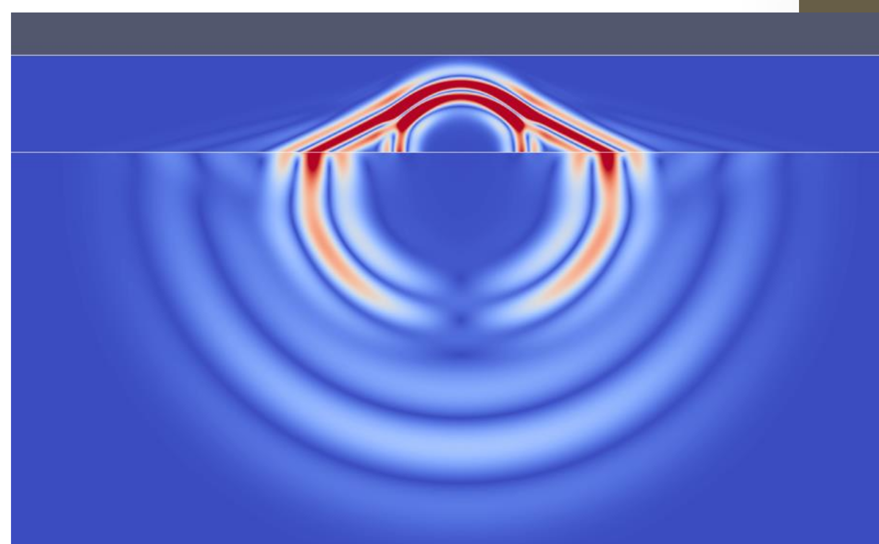


Source in the water, without carbon reservoir

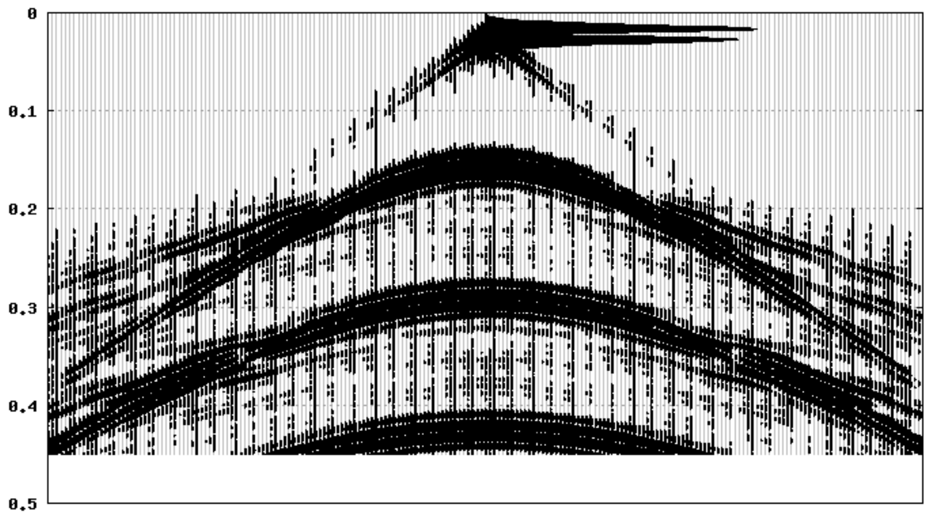


Source at the seabed



Source at the seabed, without carbon reservoir

Seismograms, receivers in the water, V



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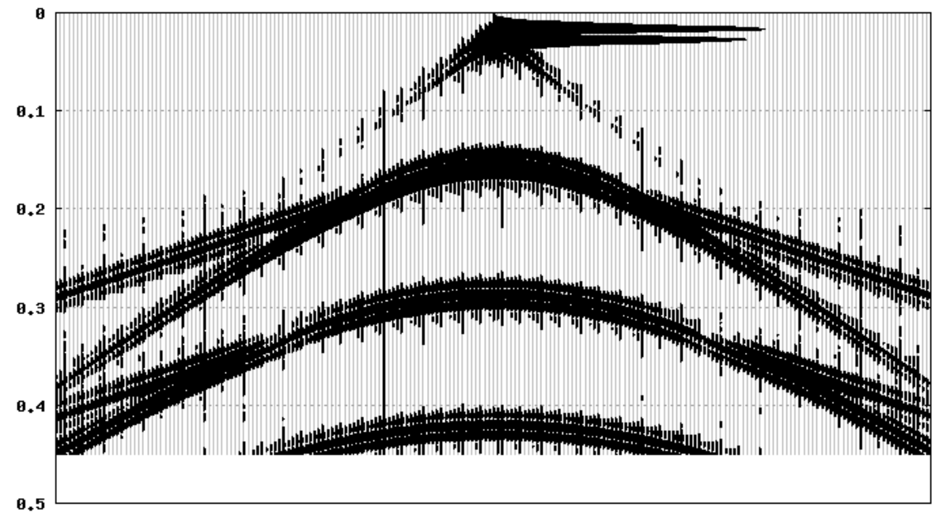
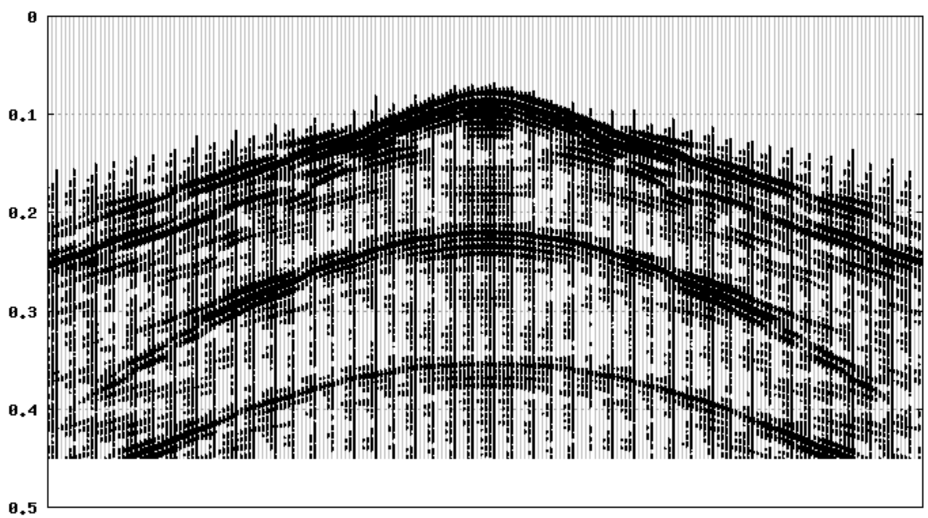


Source in the water, without carbon reservoir



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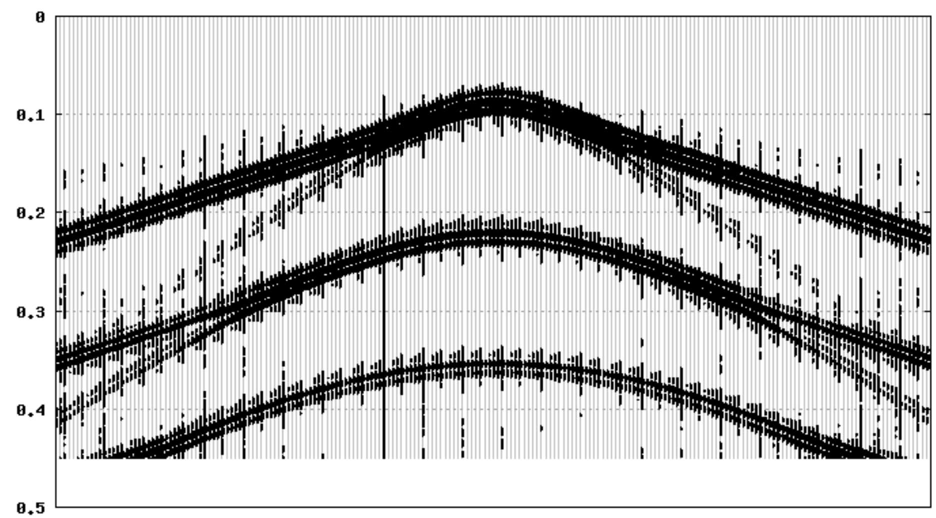
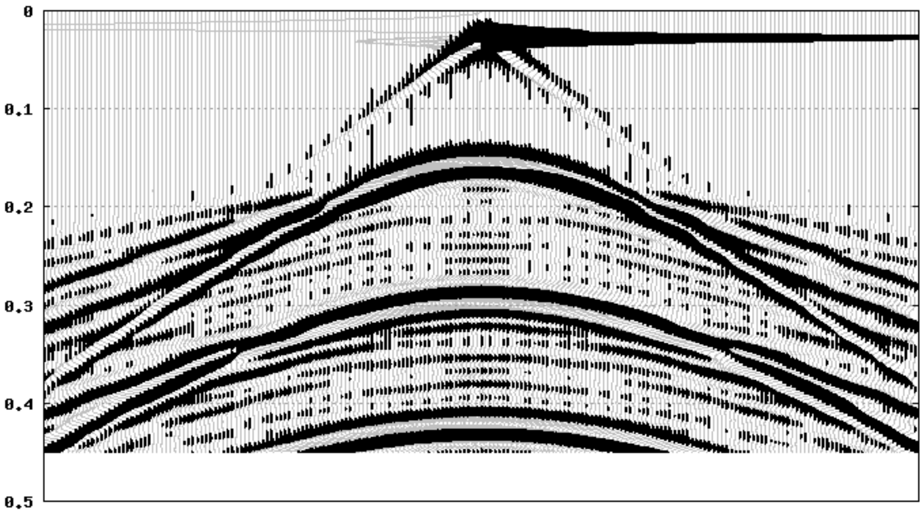


Source at the seabed, without carbon reservoir

Seismograms, receivers in the water, Vy



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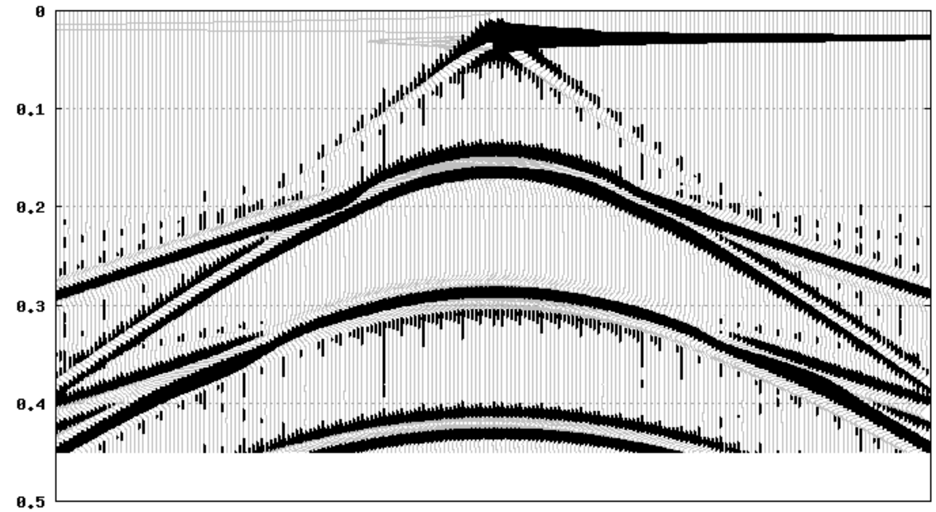

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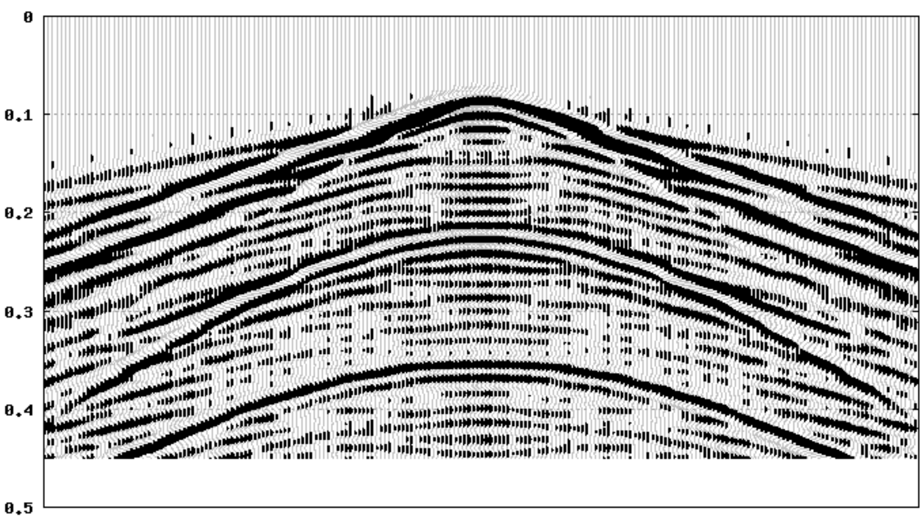

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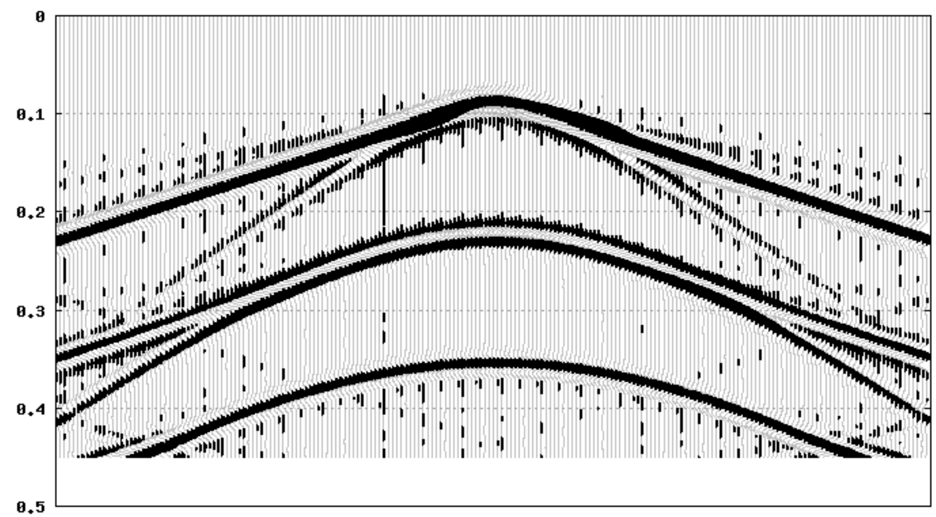
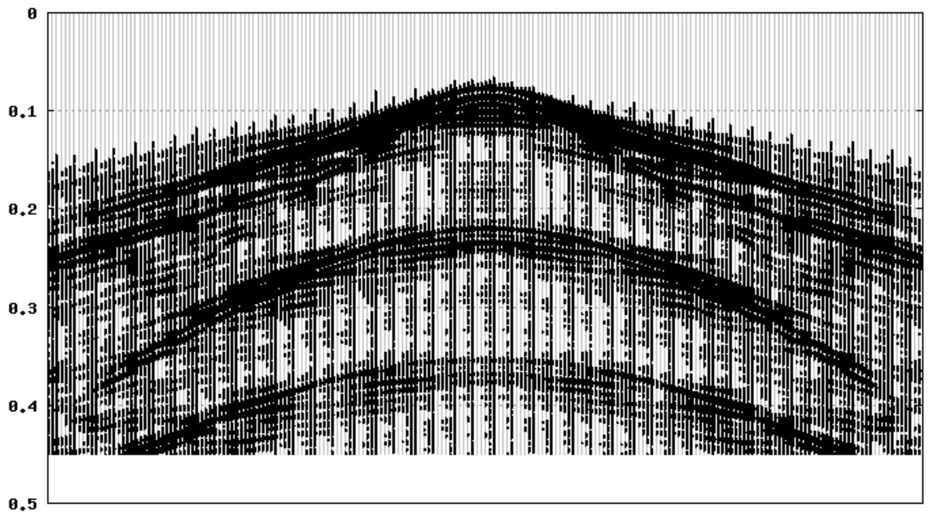


Source at the seabed, without carbon reservoir

Seismograms, receivers at the seabed, V



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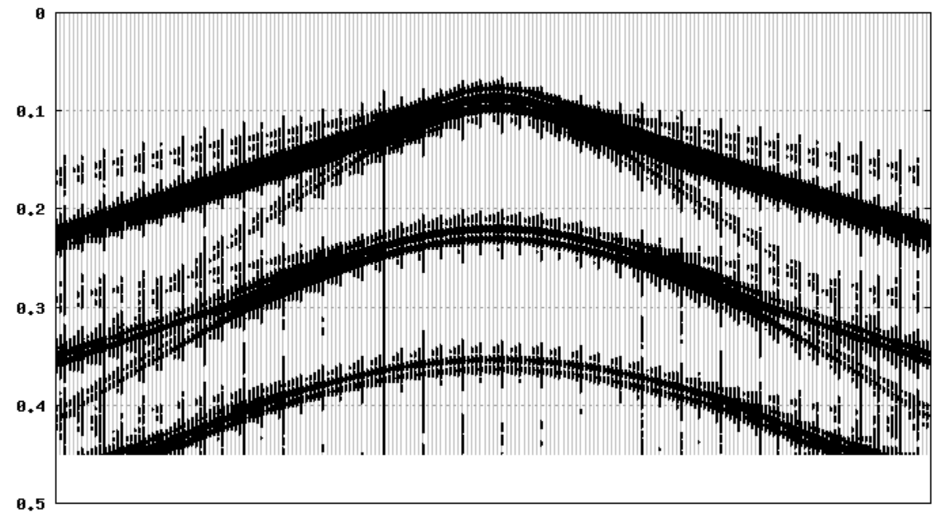
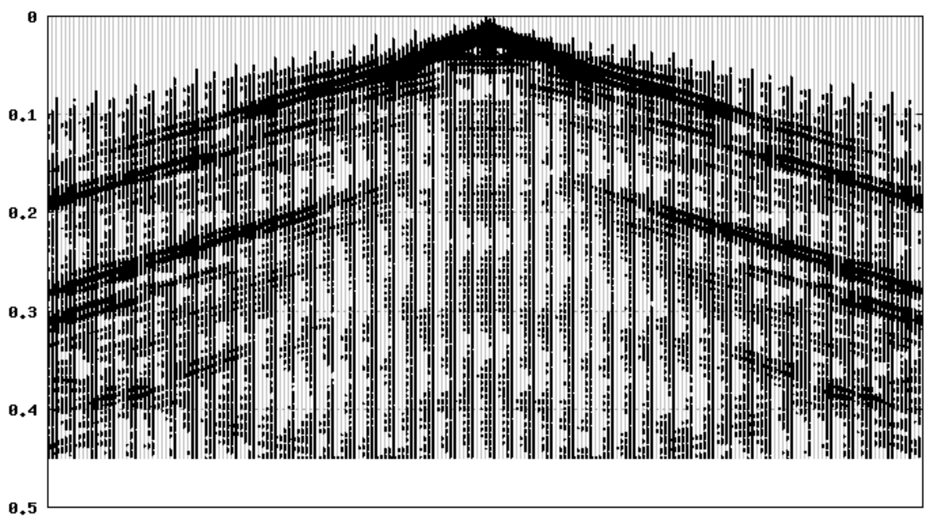


Source in the water, without carbon reservoir



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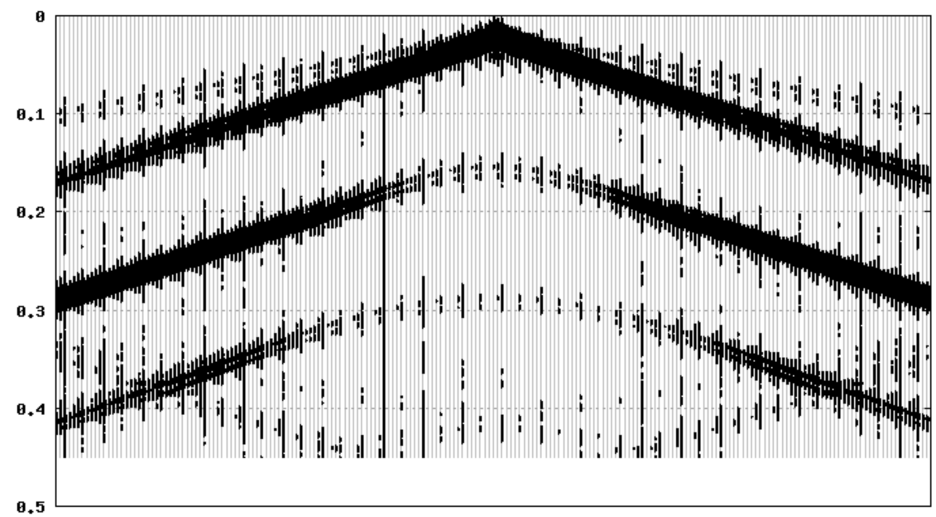
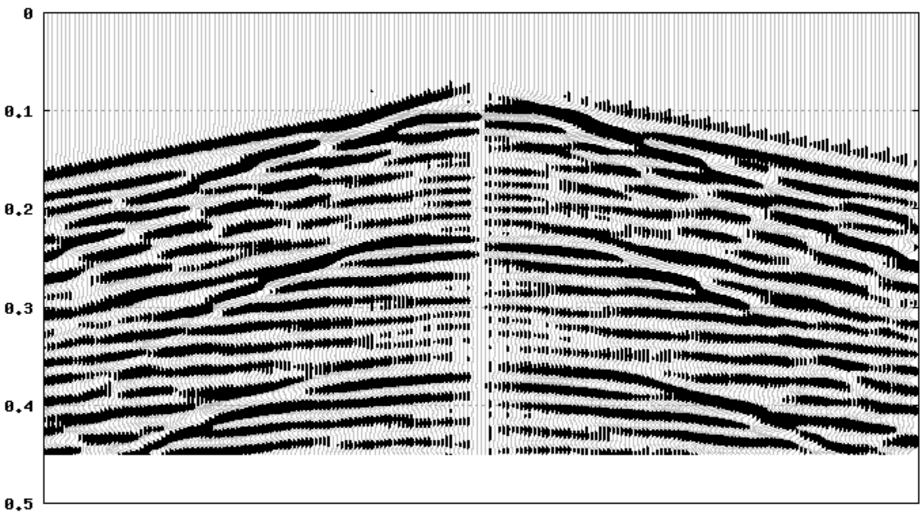


Source at the seabed, without carbon reservoir

Seismograms, receivers at the seabed, Vx



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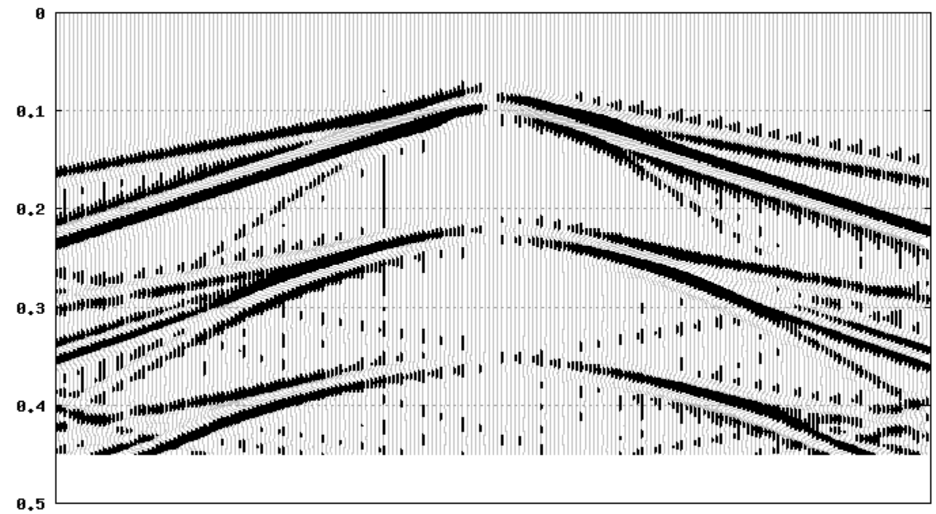

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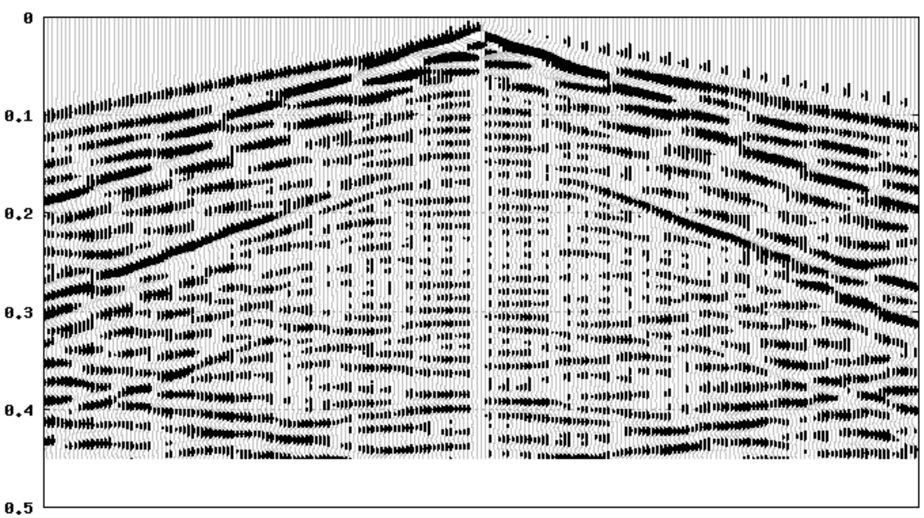

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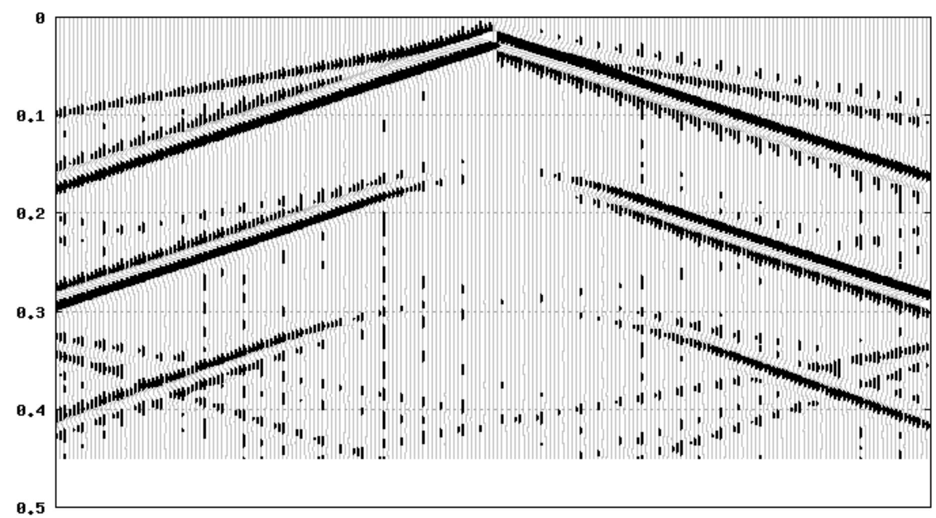
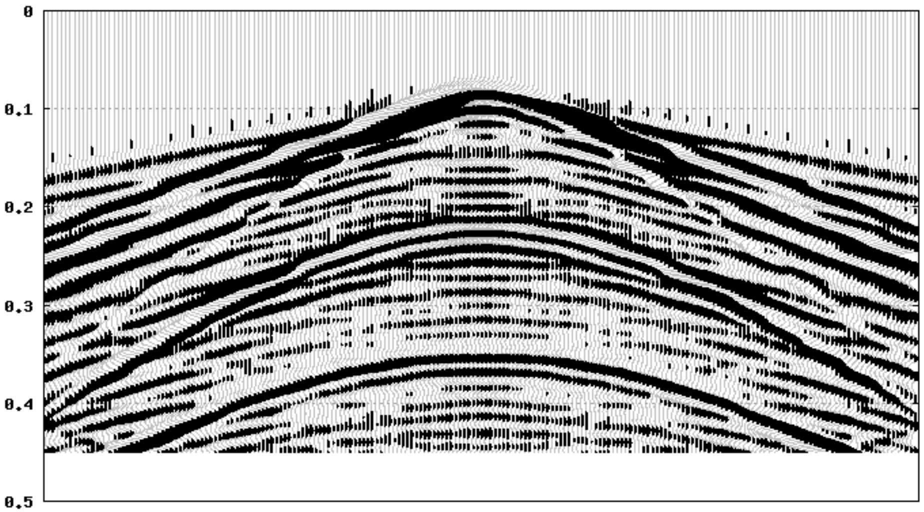


Source at the seabed, without carbon reservoir

Seismograms, receivers at the seabed, Vy



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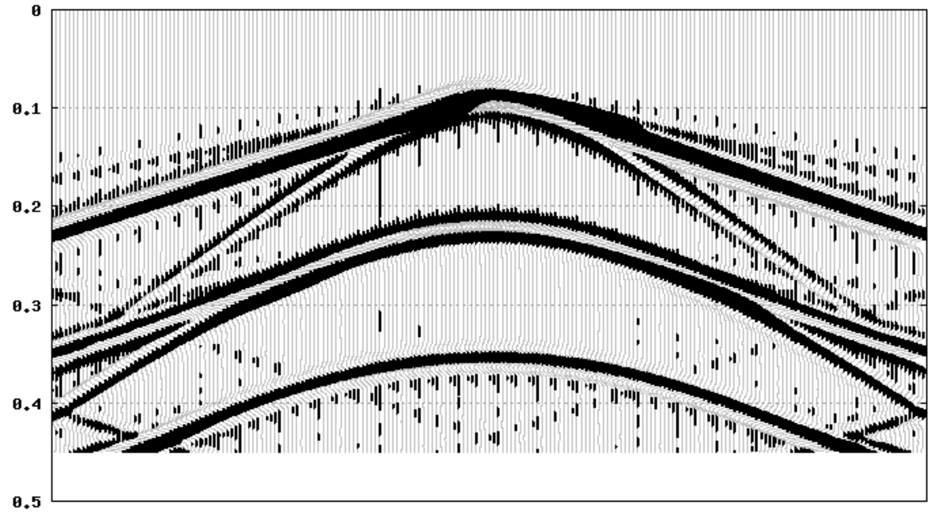

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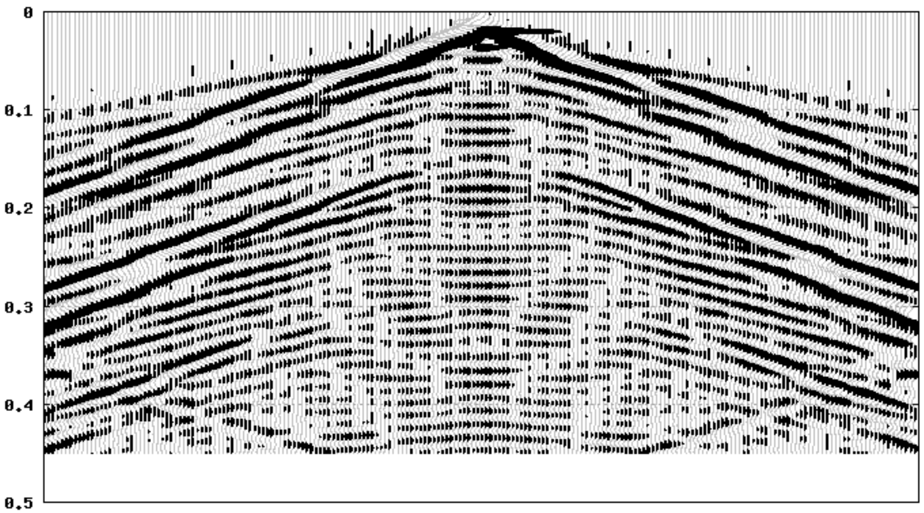

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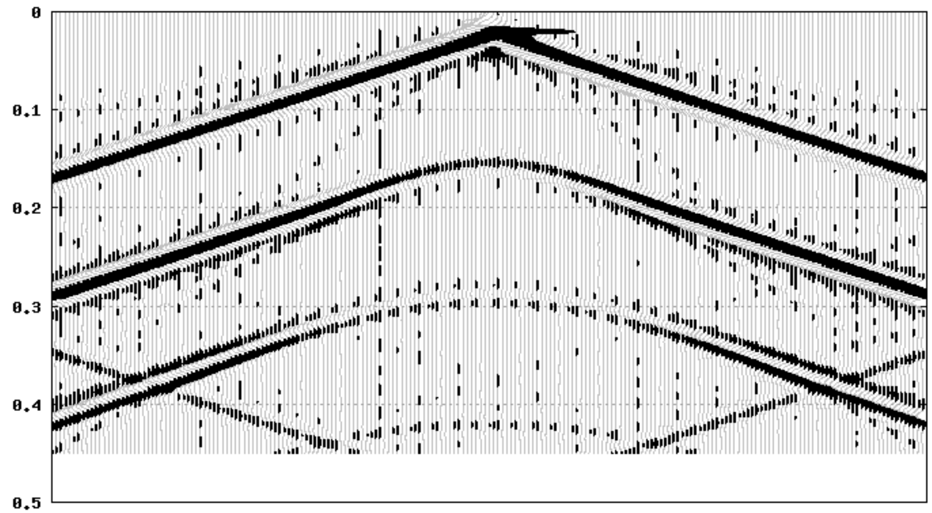
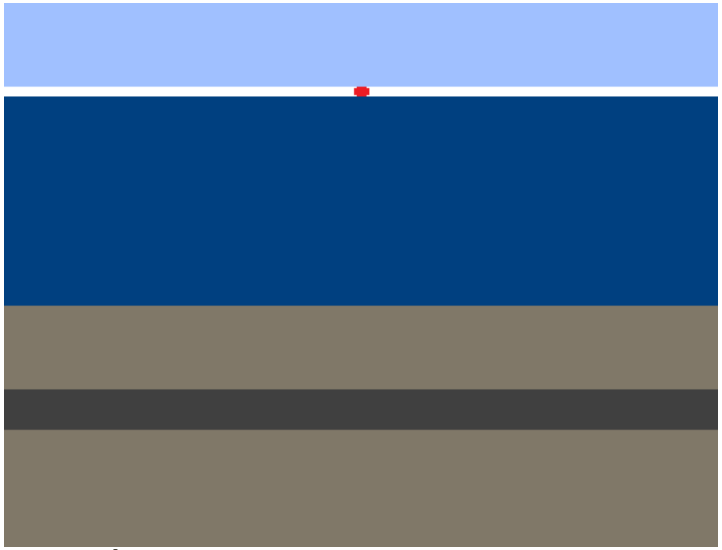


Source at the seabed, without carbon reservoir

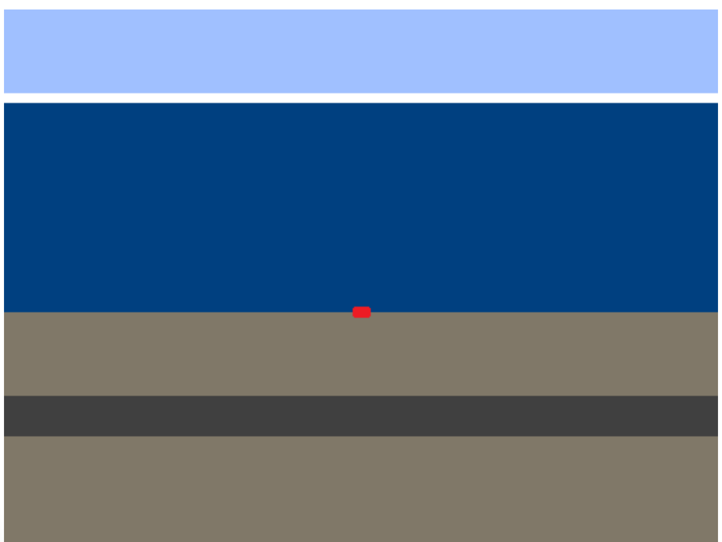
2. Sources in the ice and at the seabed, the case with ice

Problem definitions



Source in the ice

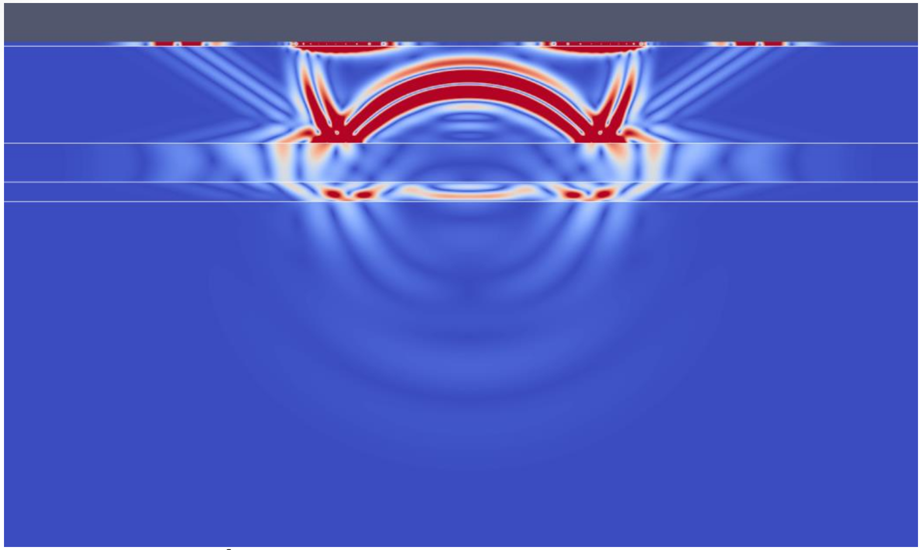
Source in the ice, without carbon reservoir



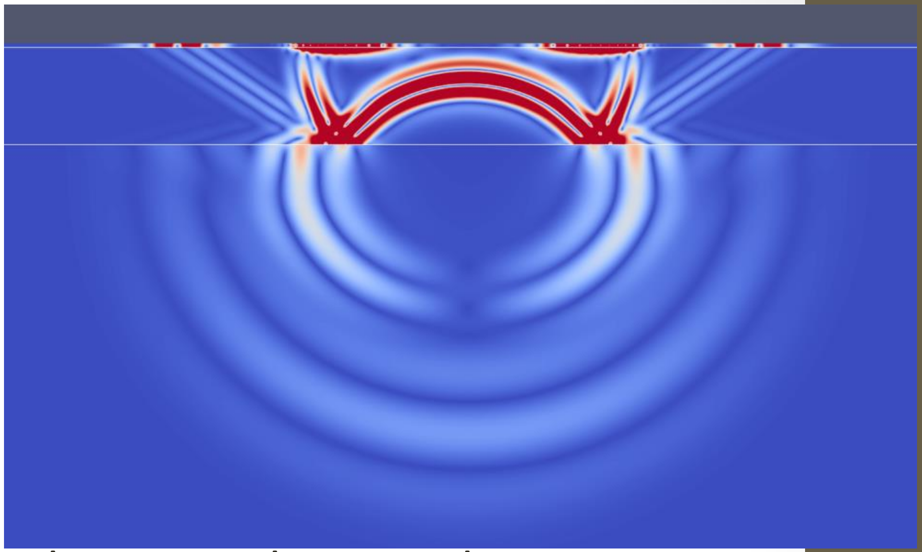
Source at the seabed

Source at the seabed, without carbon reservoir

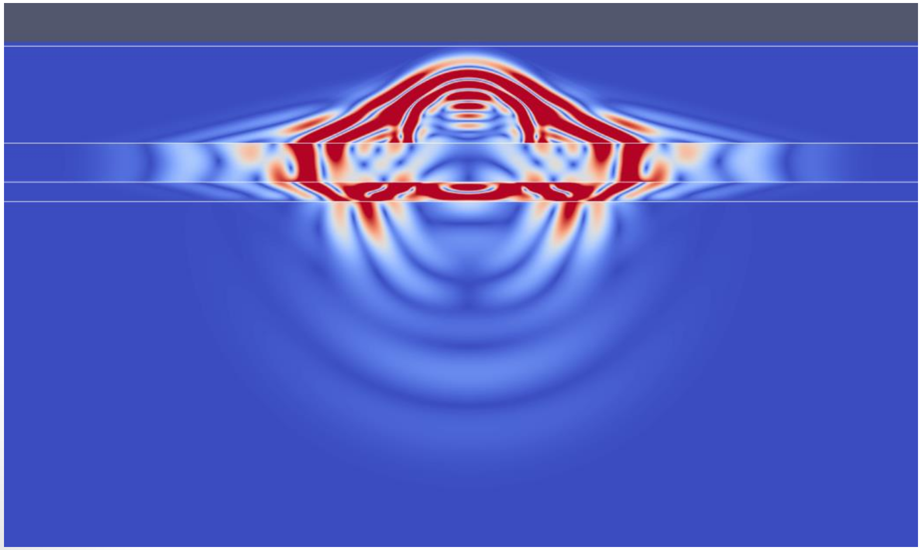
Wave patterns



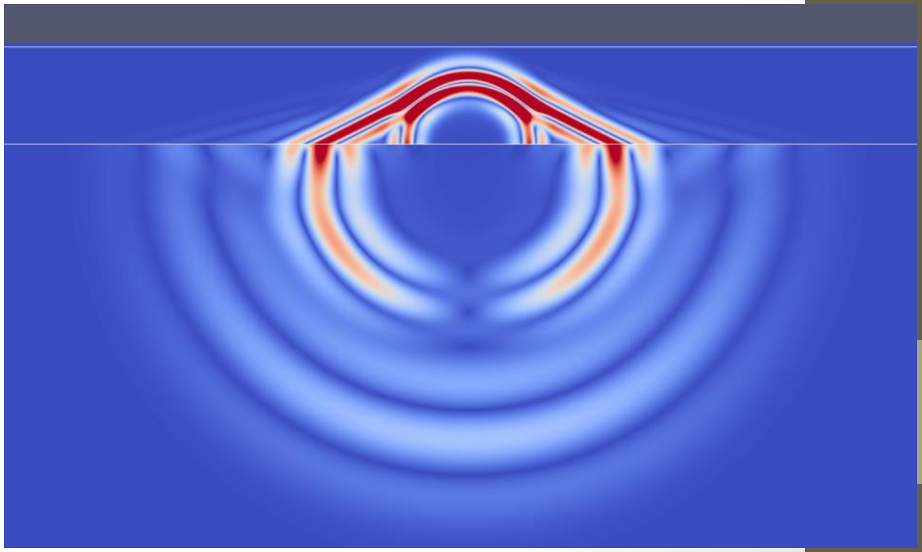
Source in the ice



Source in the ice, without carbon reservoir

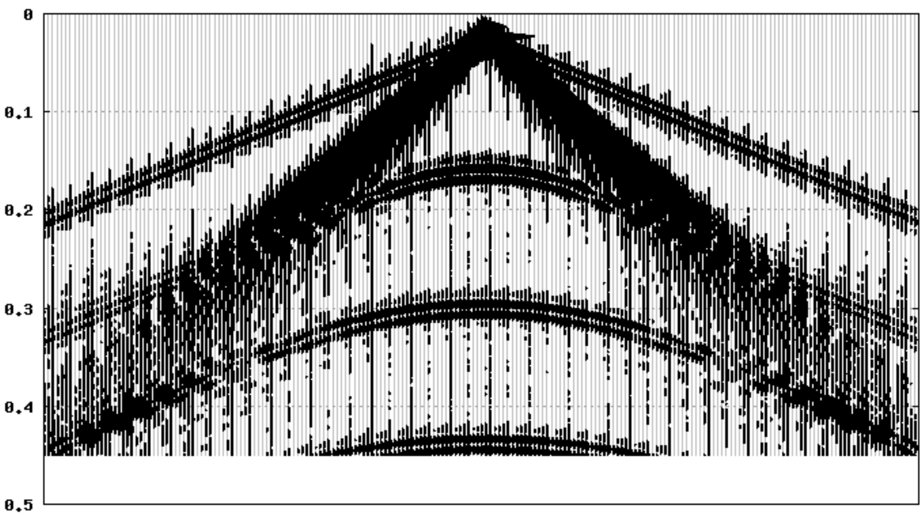


Source at the seabed

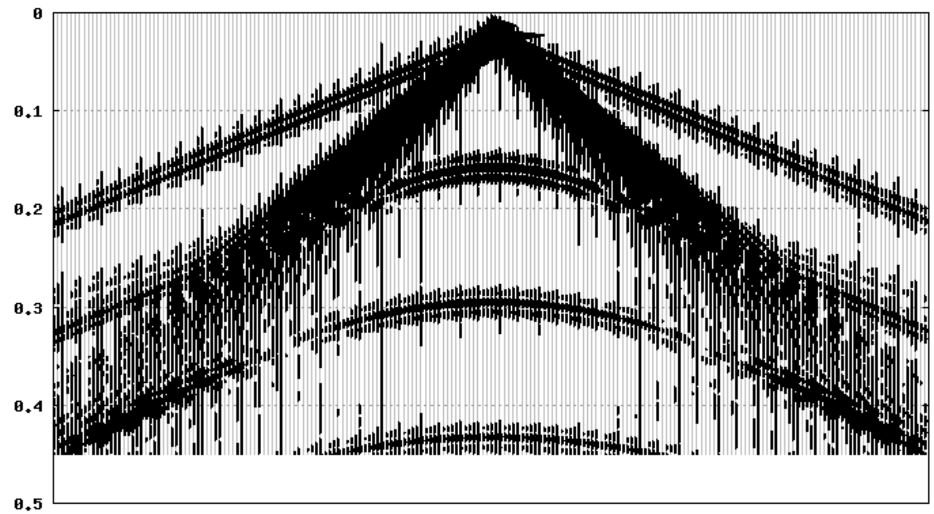


Source at the seabed, without carbon reservoir

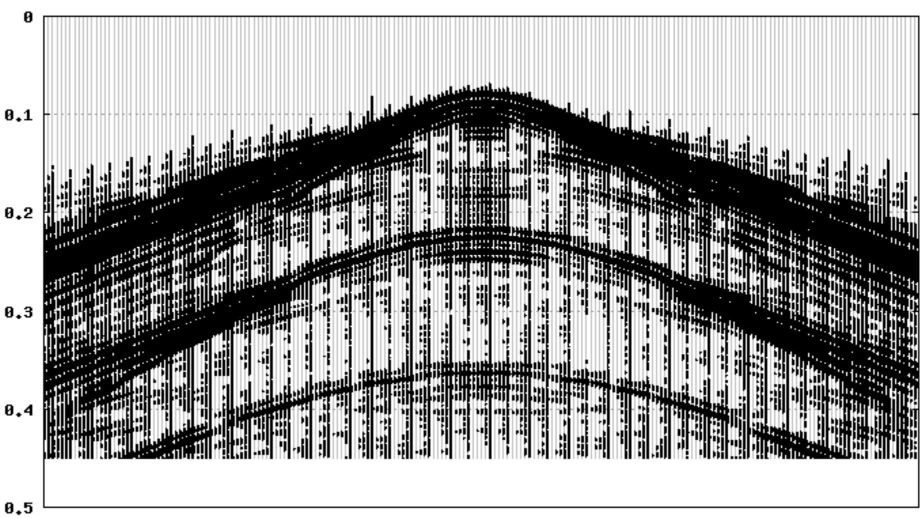
Seismograms, receivers in the ice, V



Source in the ice



Source in the ice, without carbon reservoir



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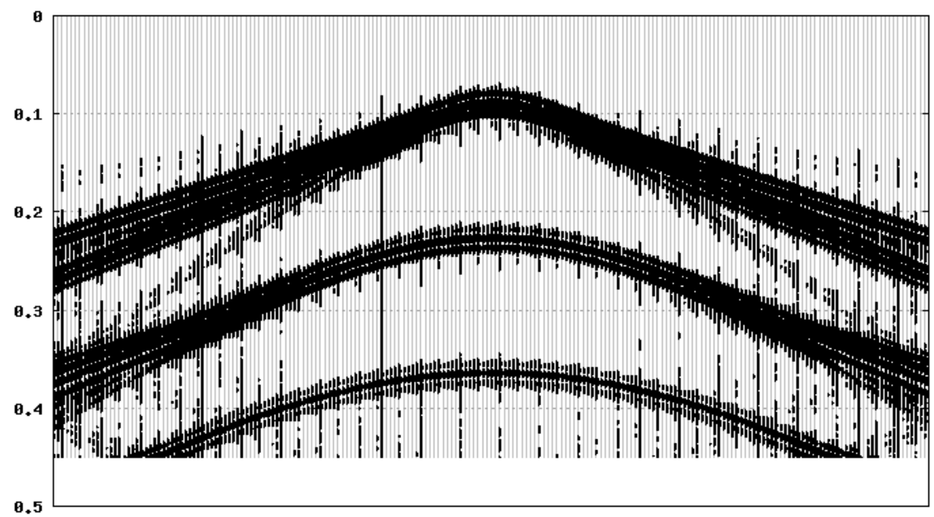
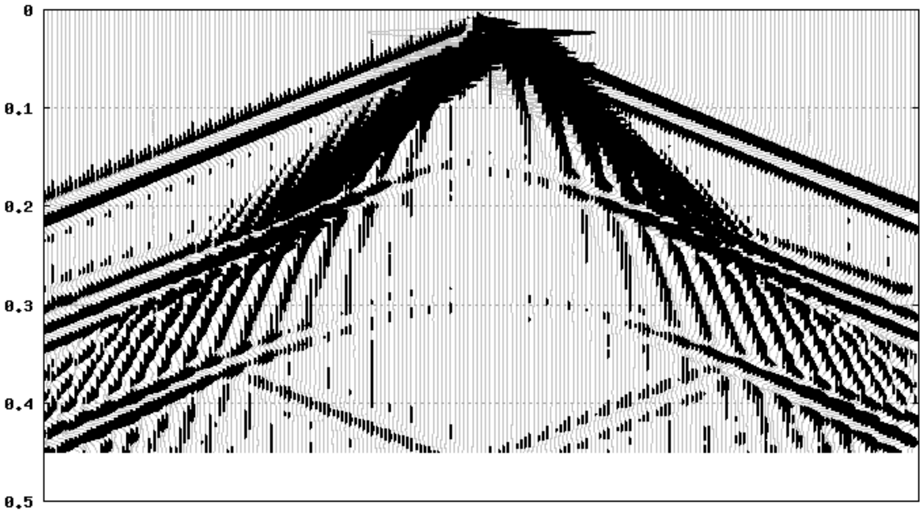


Source at the seabed, without carbon reservoir

Seismograms, receivers in the ice, Vx



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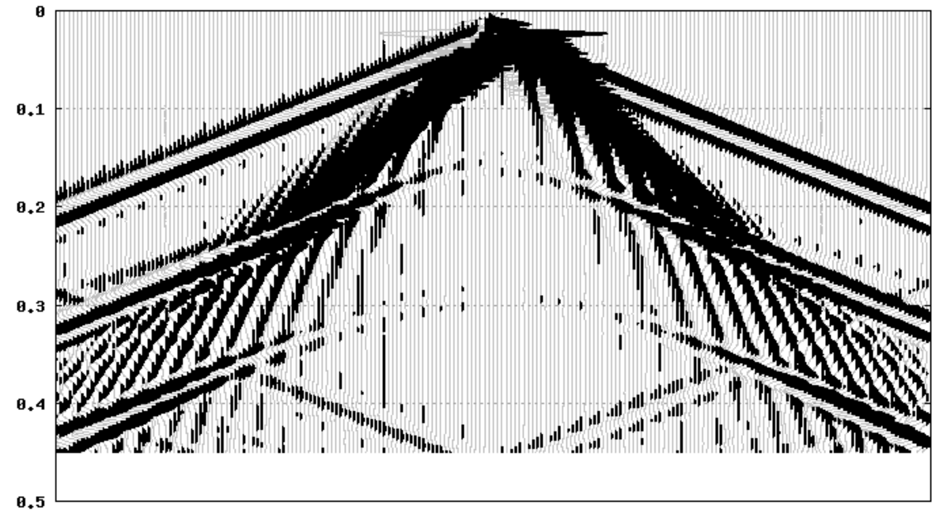

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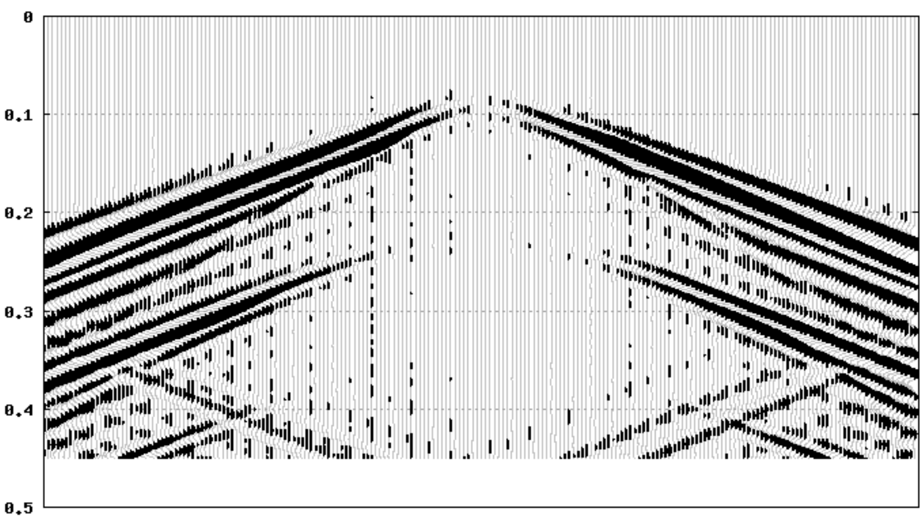

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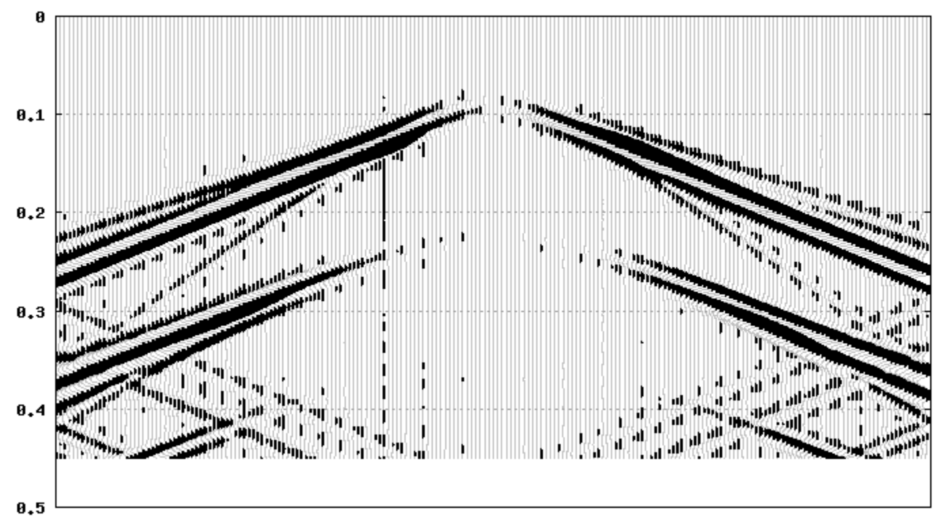
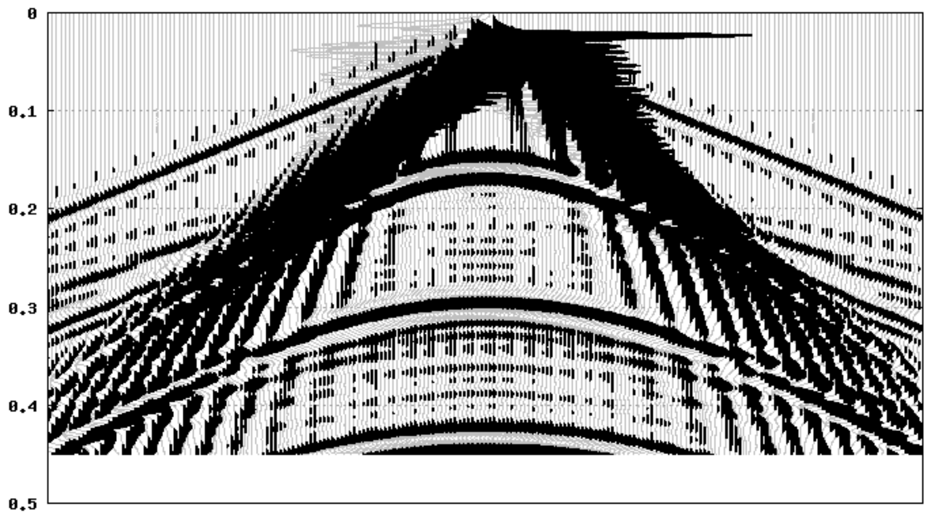


Source at the seabed, without carbon reservoir

Seismograms, receivers in the ice, Vy



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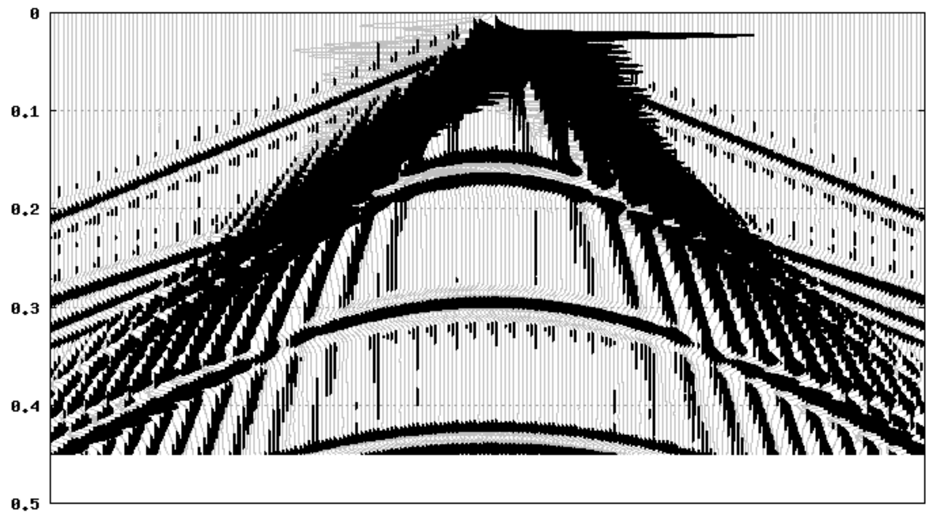

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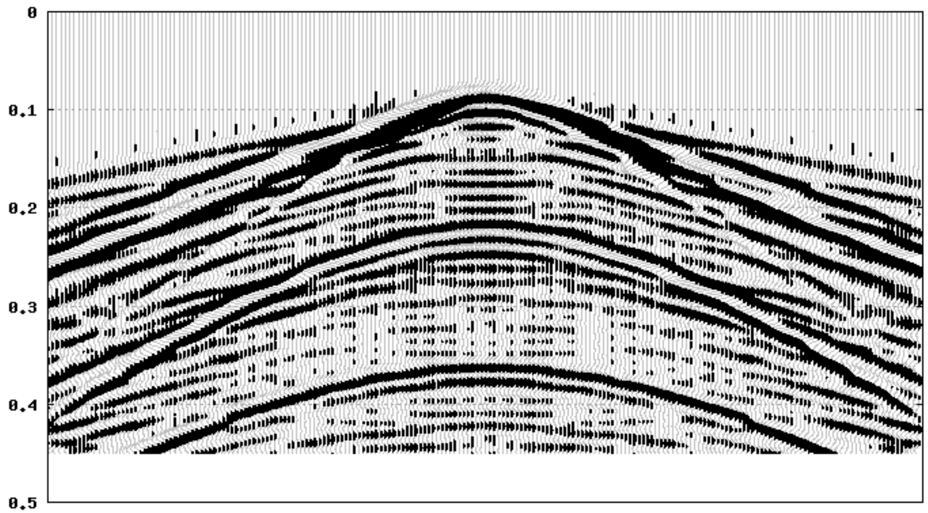

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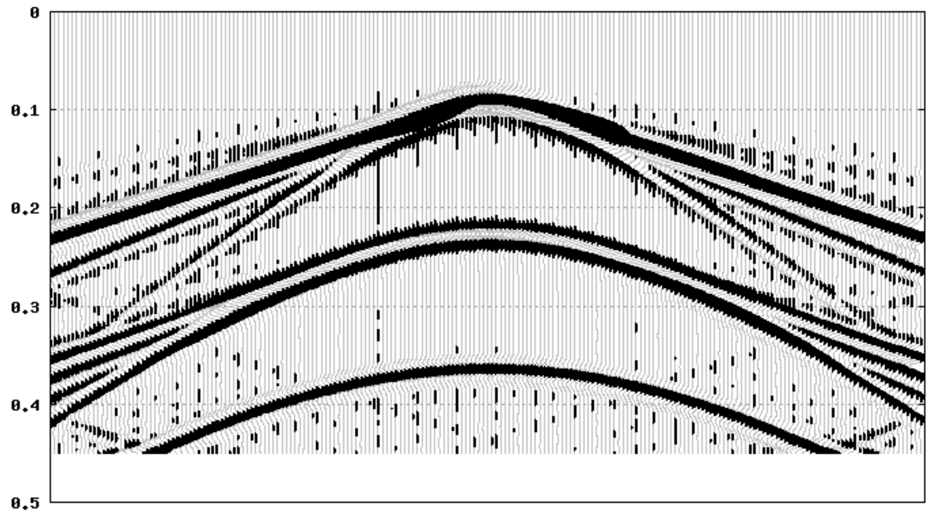
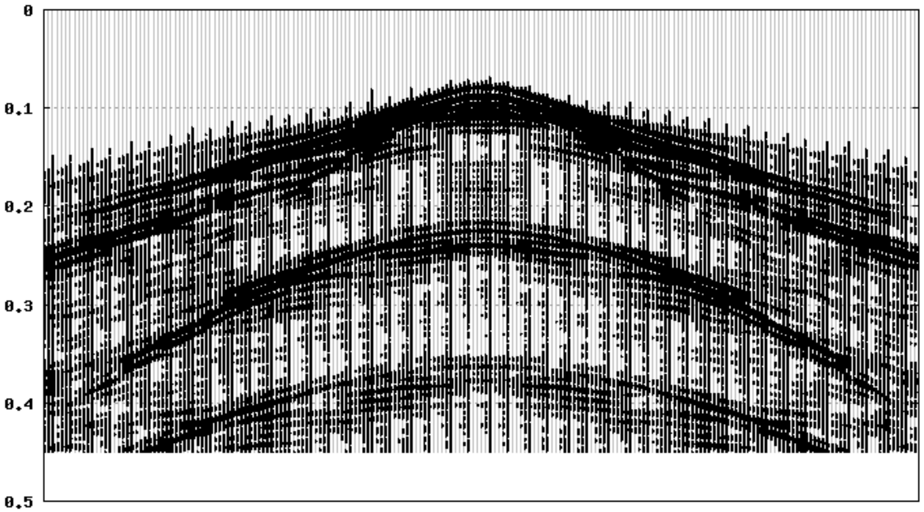


Source at the seabed, without carbon reservoir

Seismograms, receivers at the seabed, V



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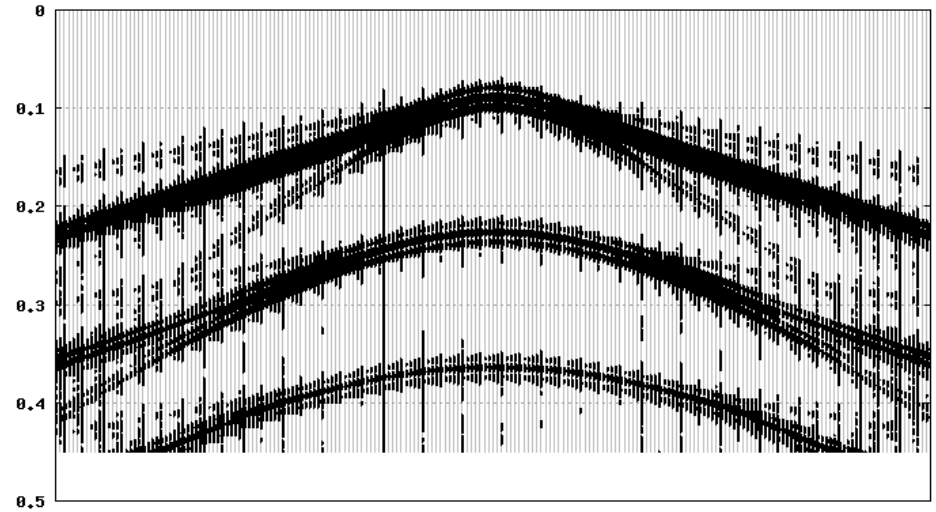

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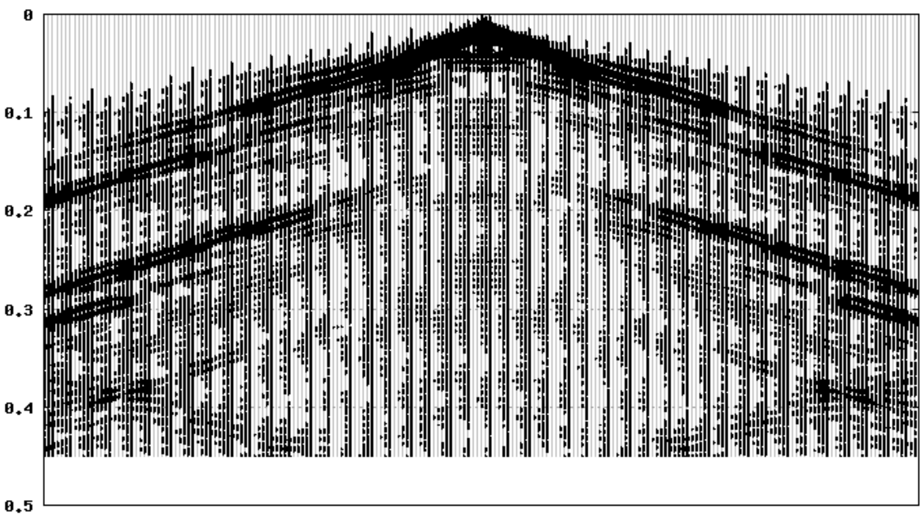
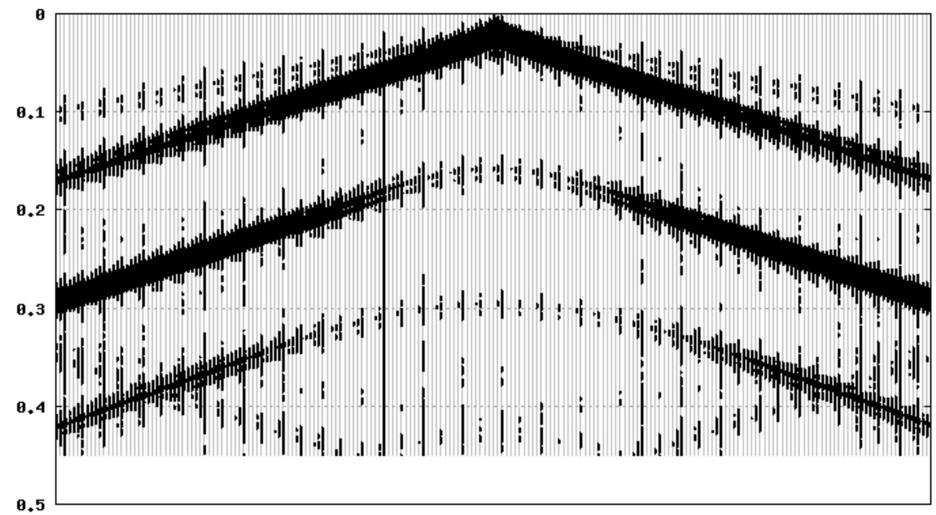
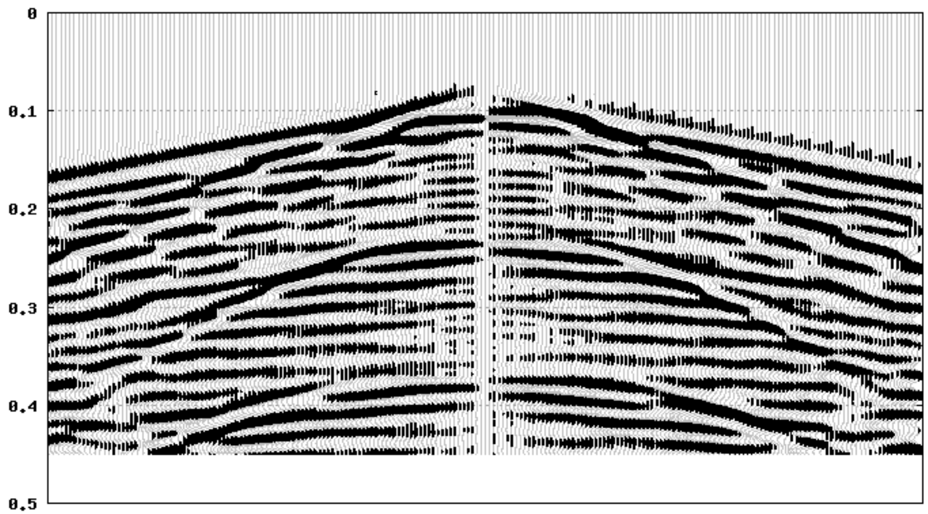


Source at the seabed



Source at the seabed, without carbon reservoir

Seismograms, receivers at the seabed, Vx



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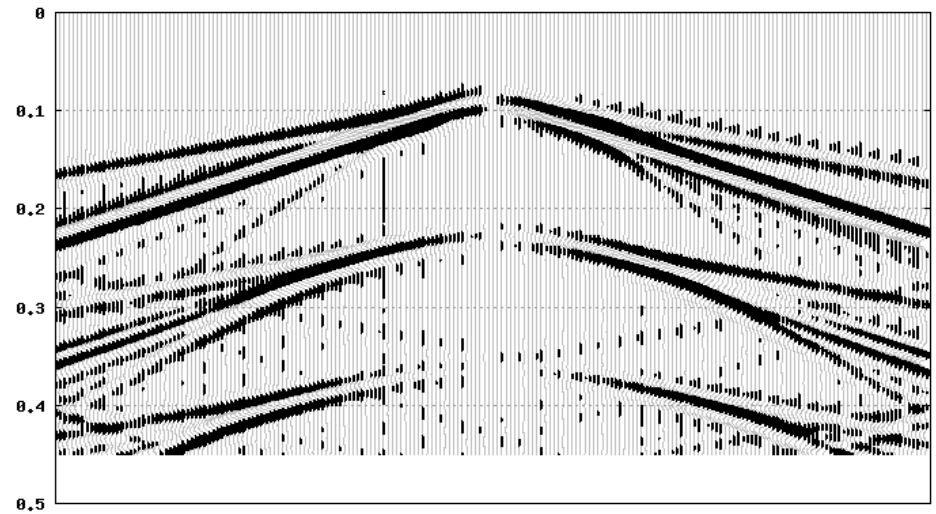
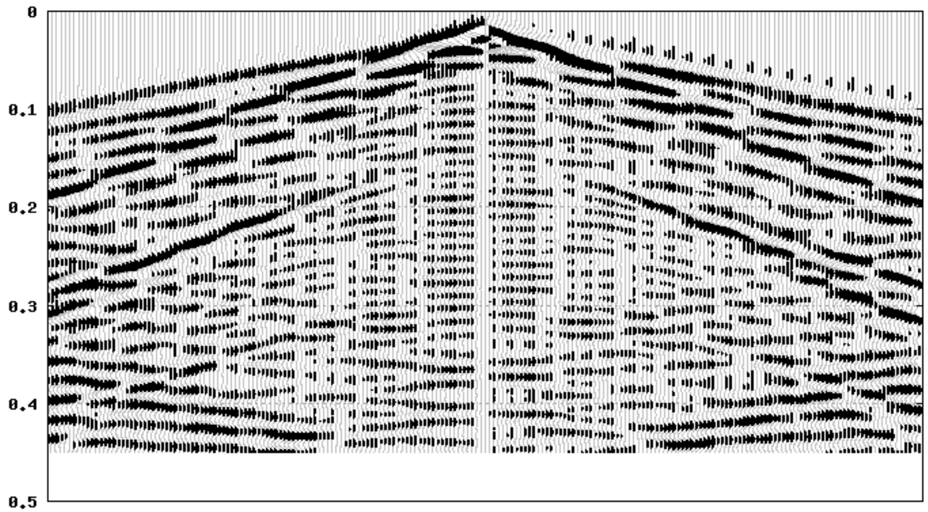


Source in the ice, without carbon reservoir



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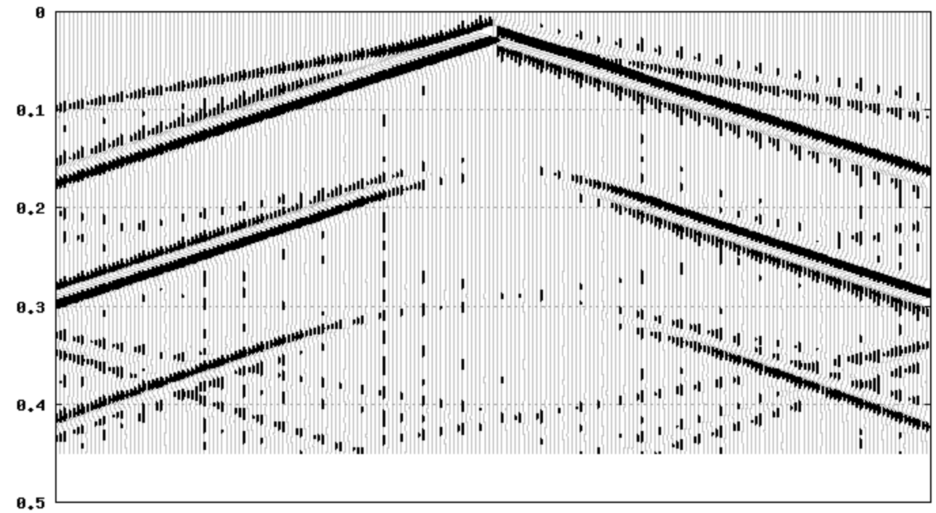
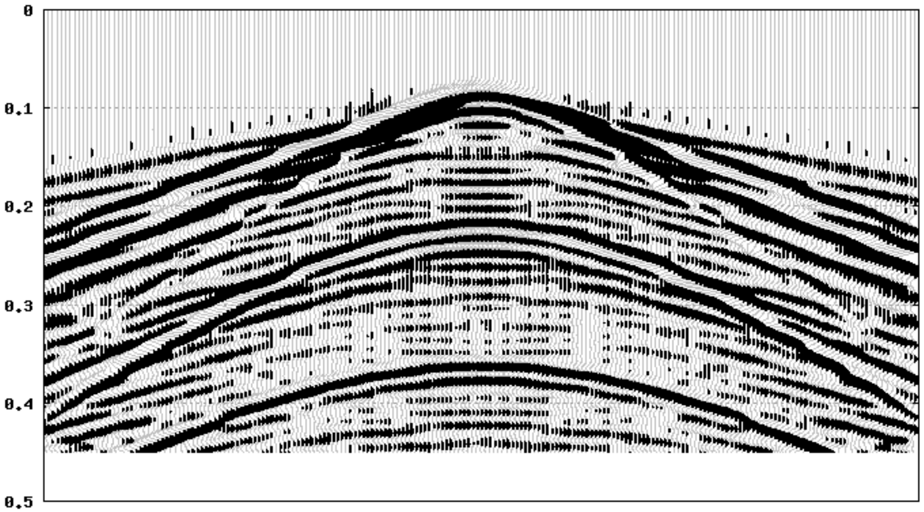
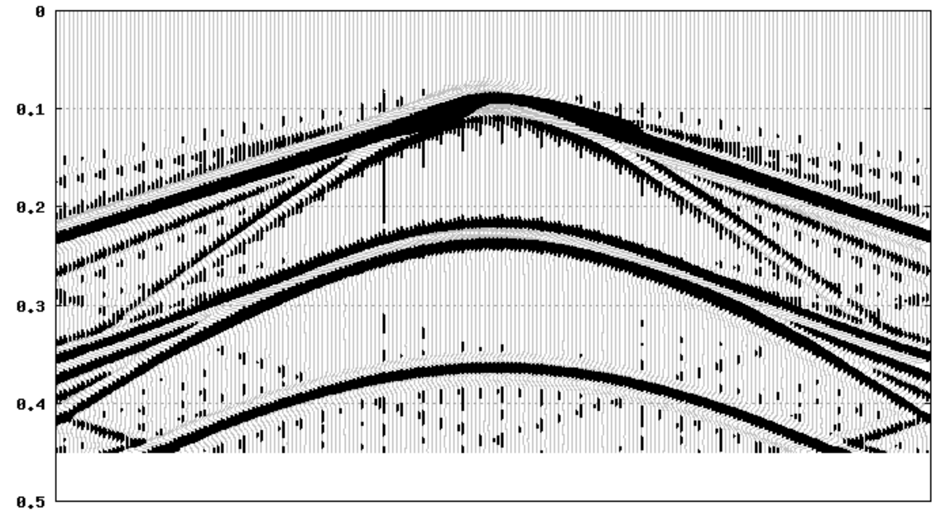


Source at the seabed, without carbon reservoir

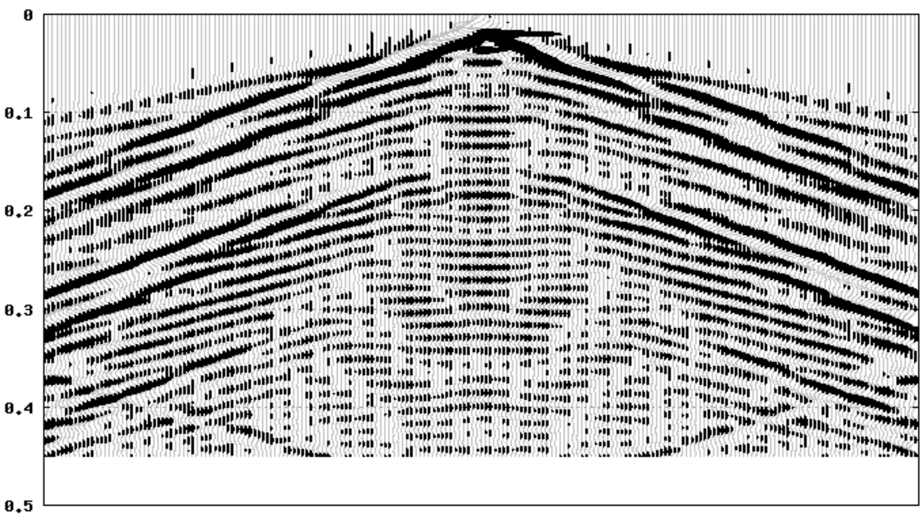
Seismograms, receivers at the seabed, Vy



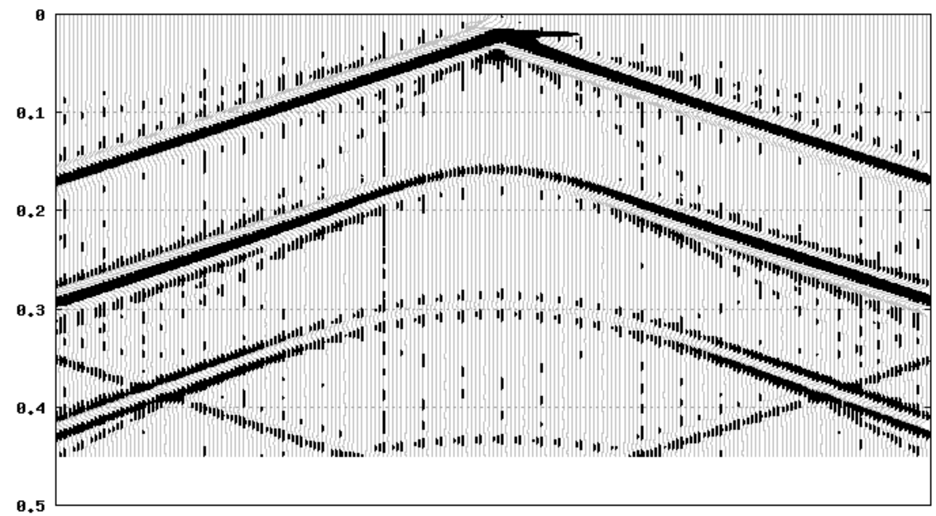
Source in the ice



Source in the ice, without carbon reservoir



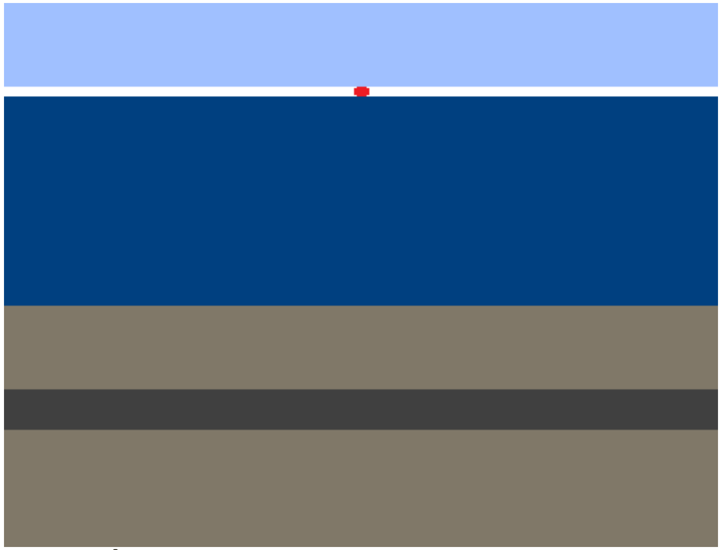
Source at the seabed



Source at the seabed, without carbon reservoir

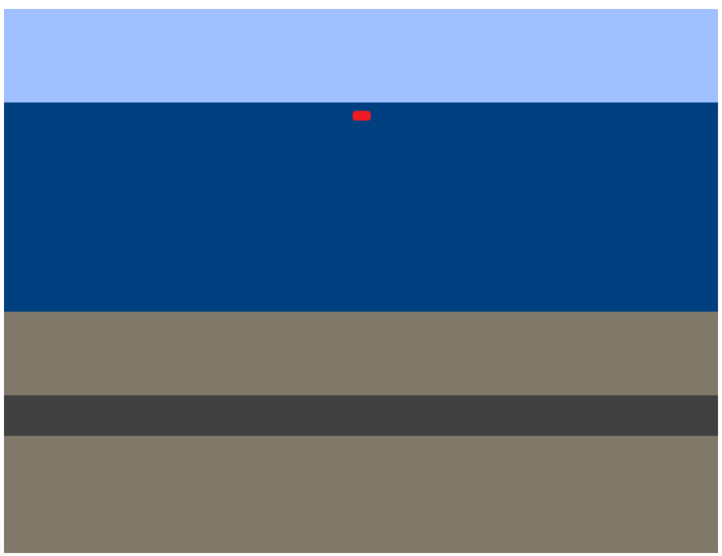
3. Influence of ice. Sources in the ice and in the water.

Problem definitions

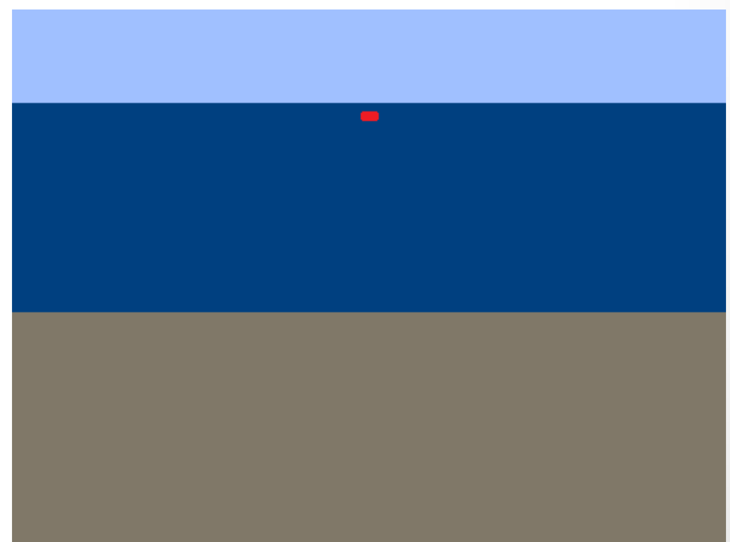


Source in the ice

Source in the ice, without carbon reservoir

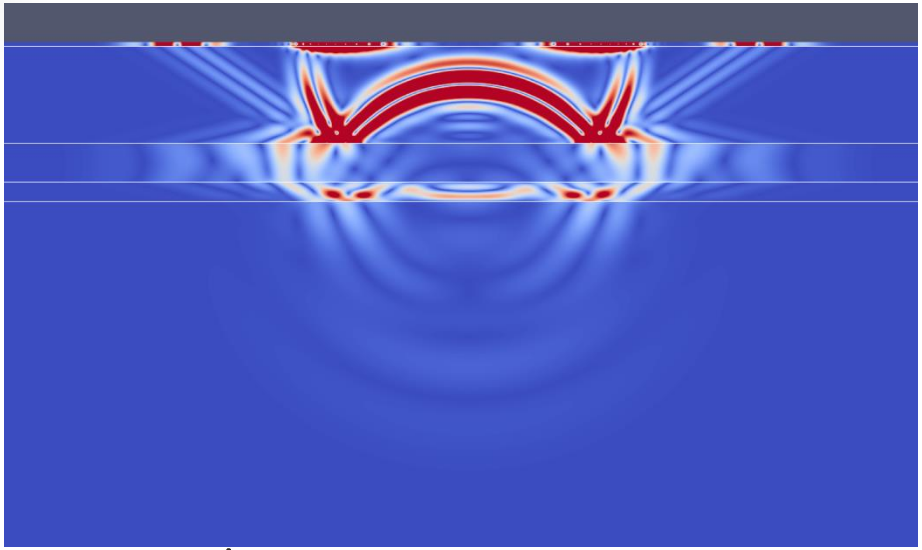


Source in the water

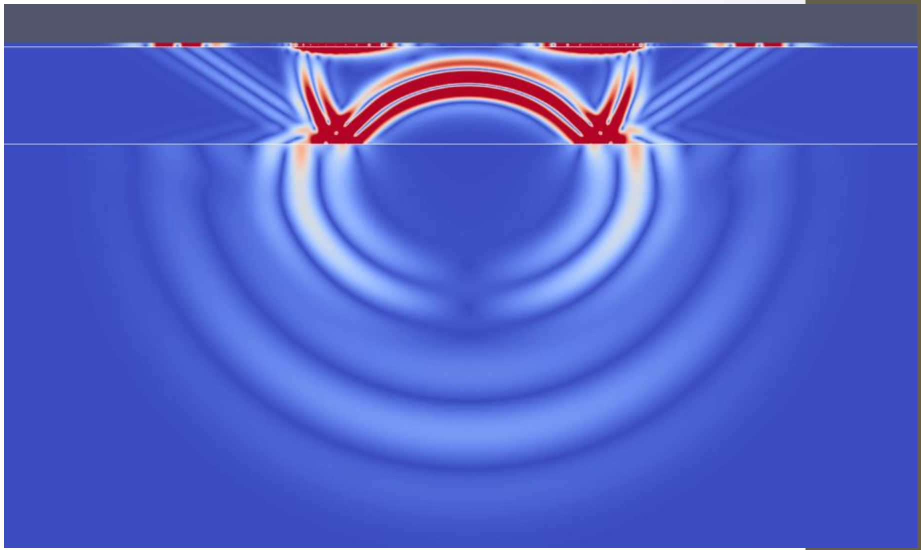


Source in the water, without carbon reservoir

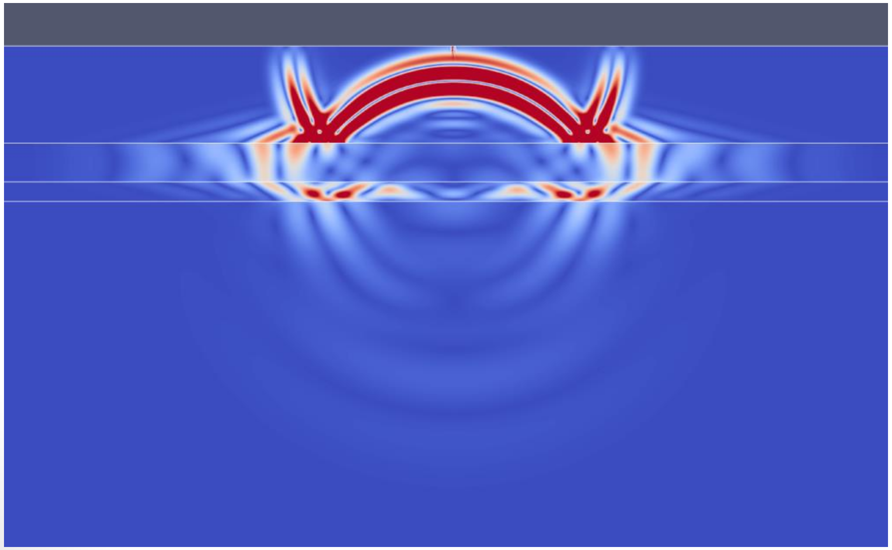
Wave patterns



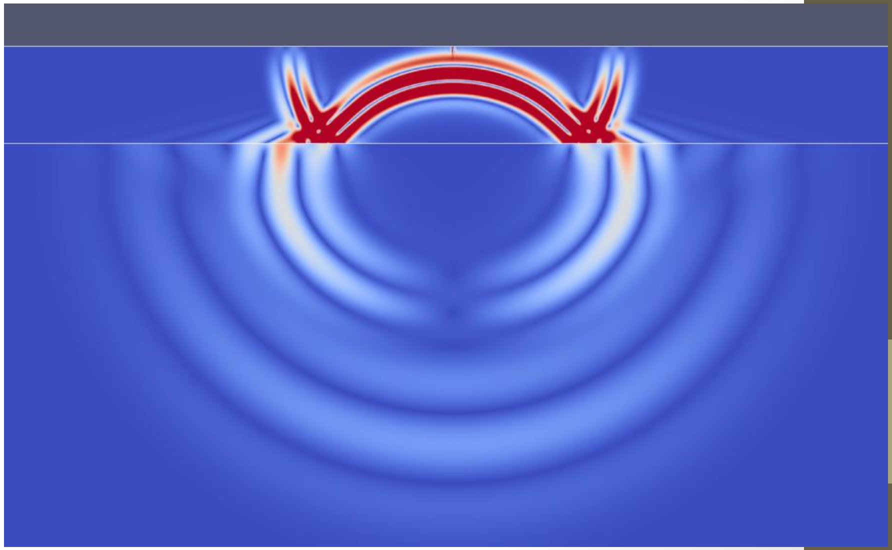
Source in the ice



Source in the ice, without carbon reservoir

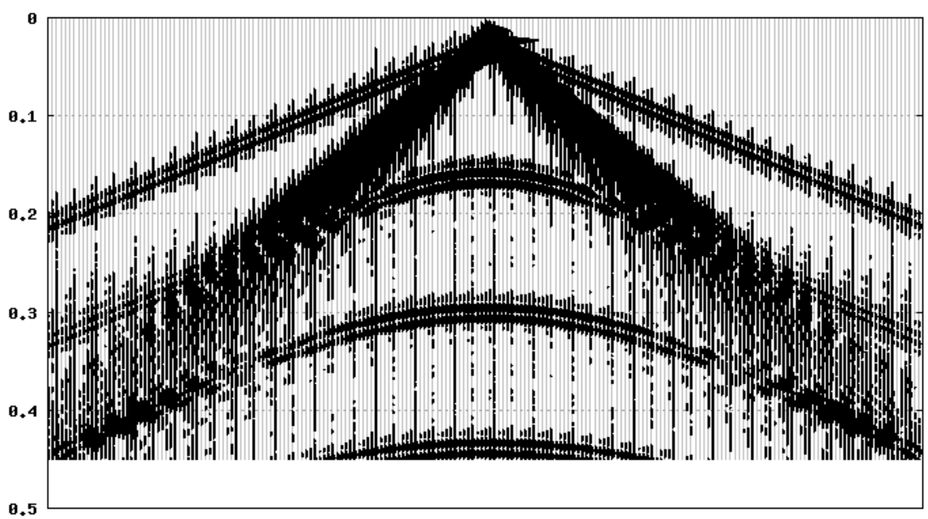


Source in the water

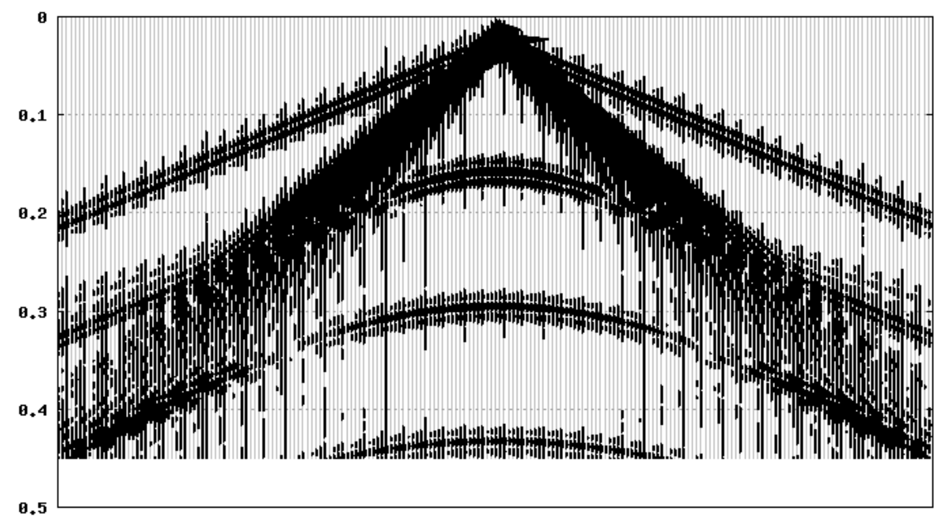


Source in the water, without carbon reservoir

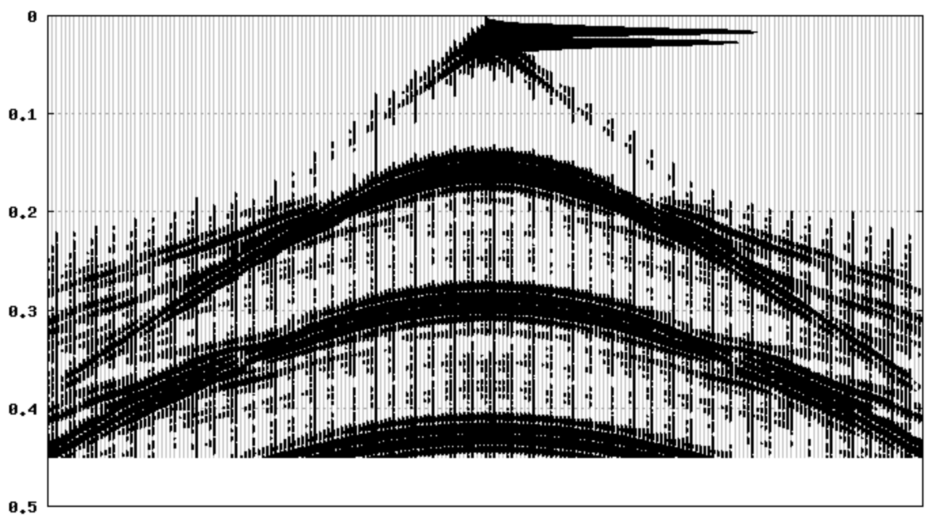
Seismograms, receivers in the water/ice, V



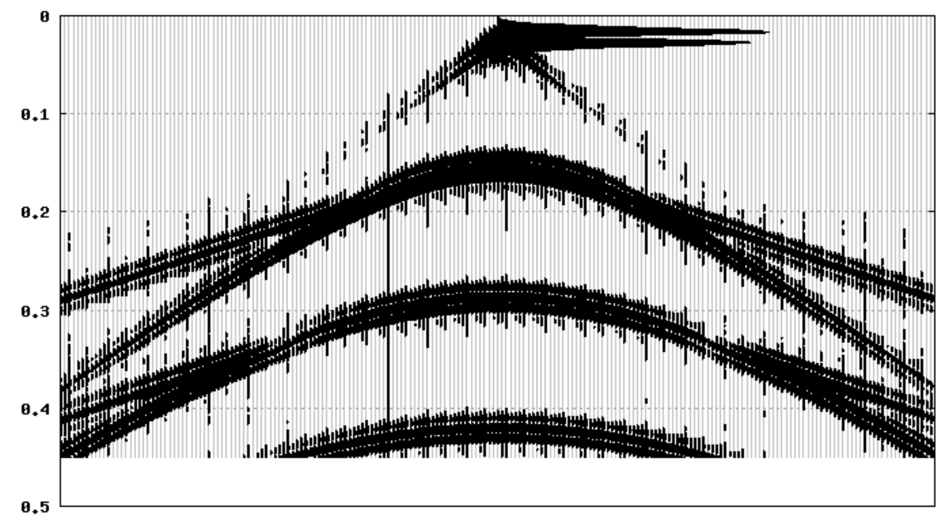
Source in the ice



Source in the ice, without carbon reservoir

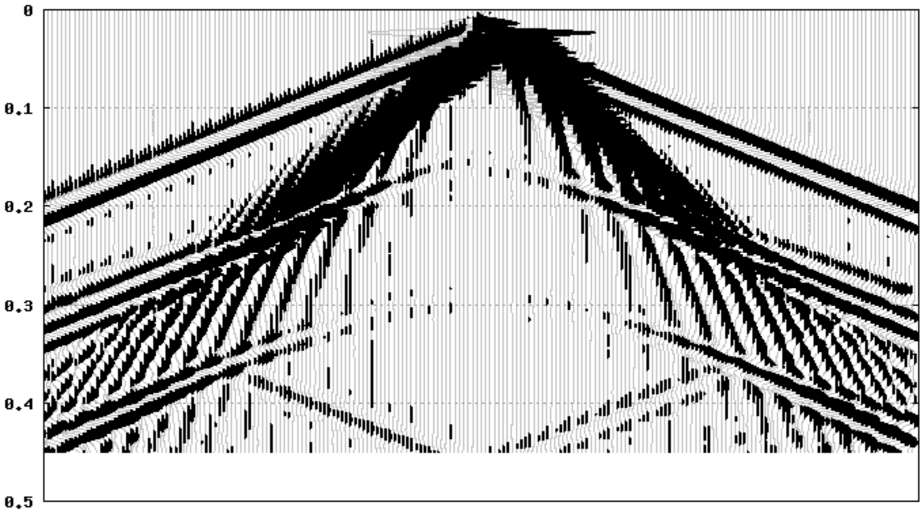


Source in the water

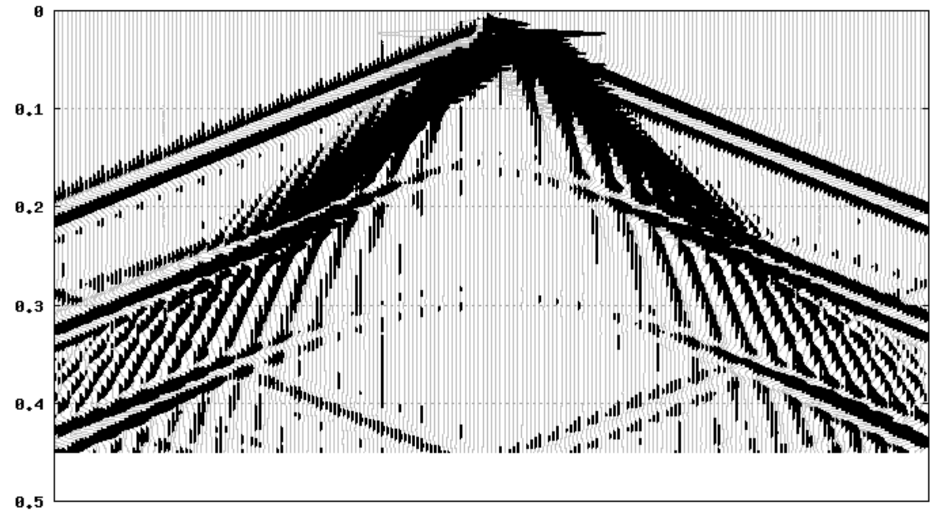


Source in the water, without carbon reservoir

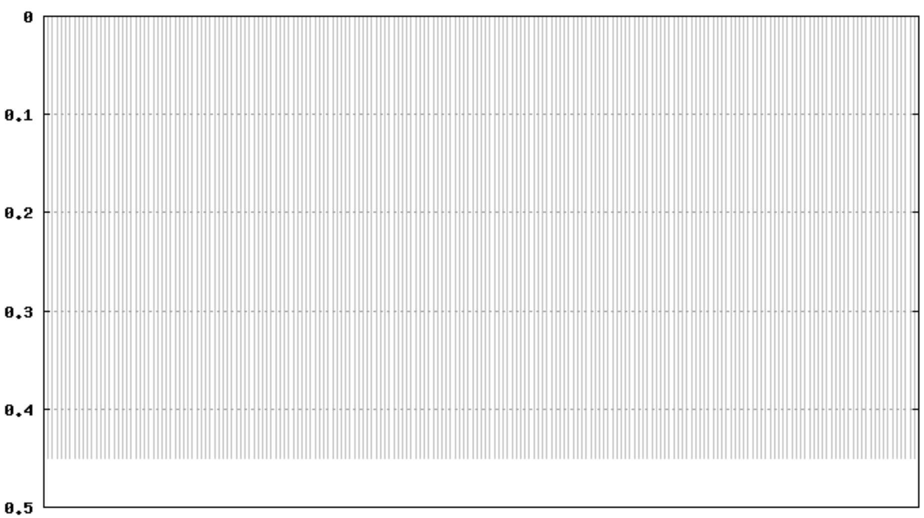
Seismograms, receivers in the water/ice, Vx



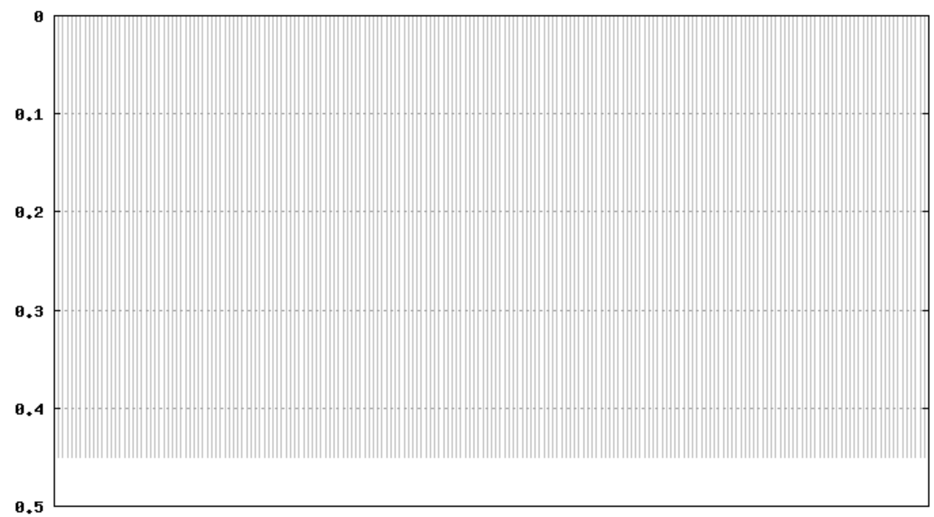
Source in the ice



Source in the ice, without carbon reservoir

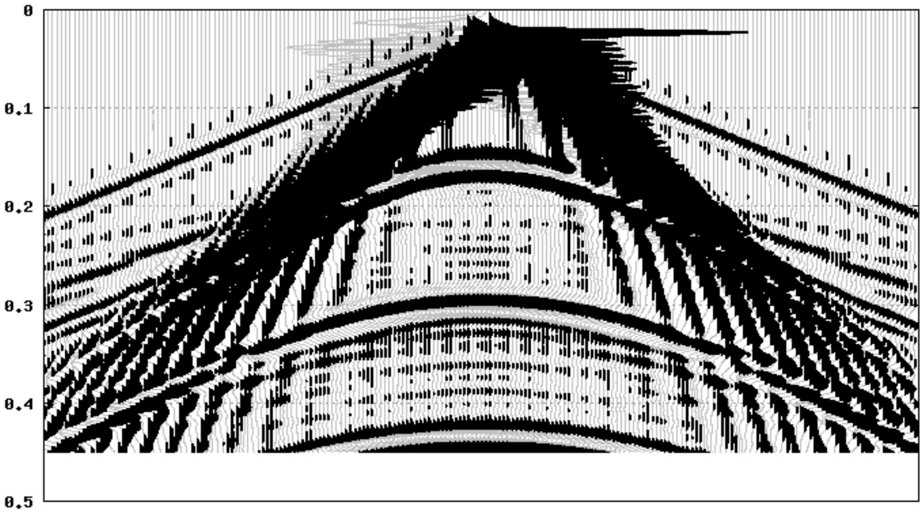


Source in the water

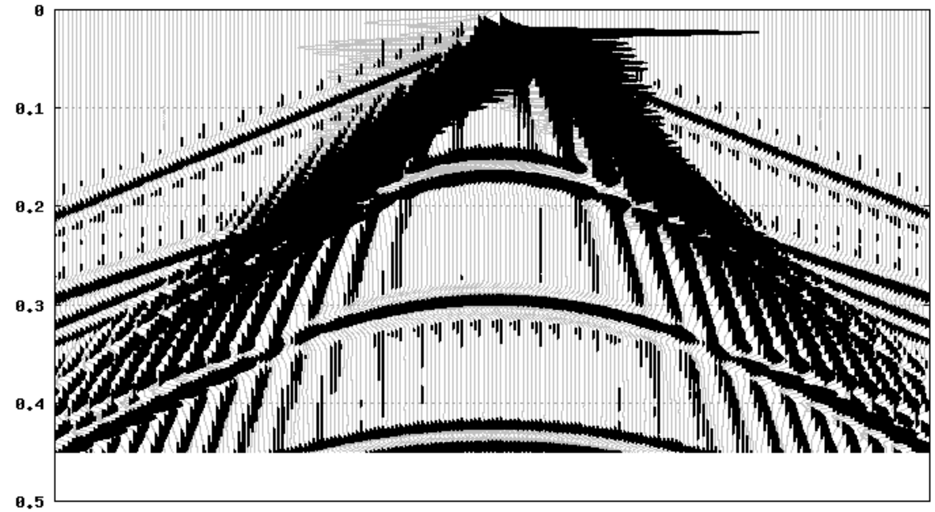


Source in the water, without carbon reservoir

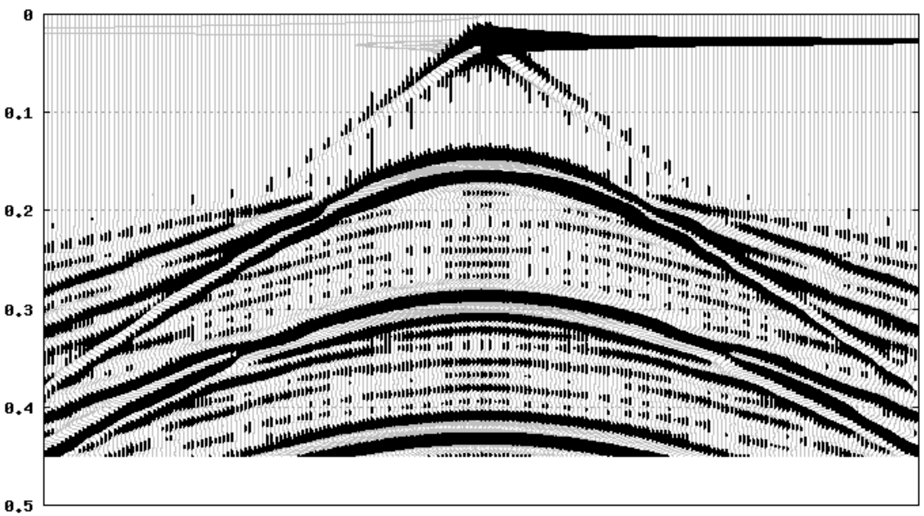
Seismograms, receivers in the water/ice, Vy



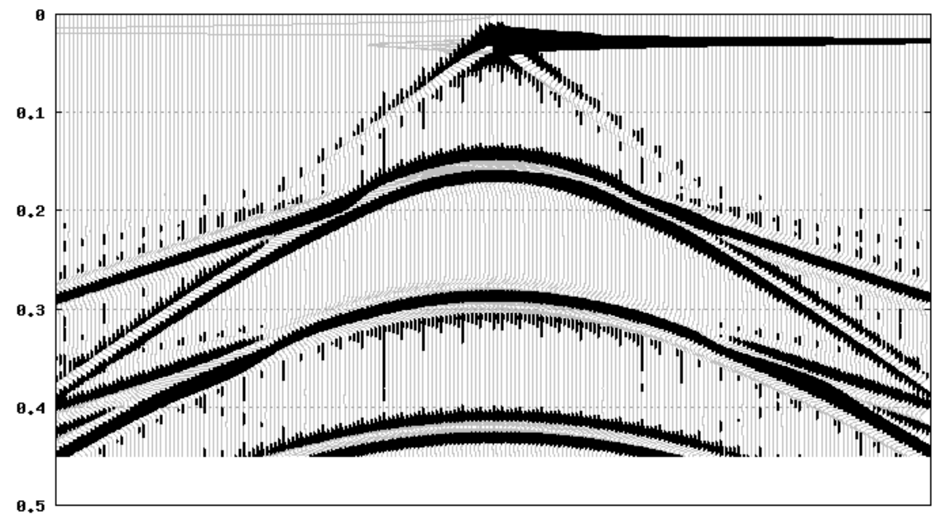
Source in the ice



Source in the ice, without carbon reservoir

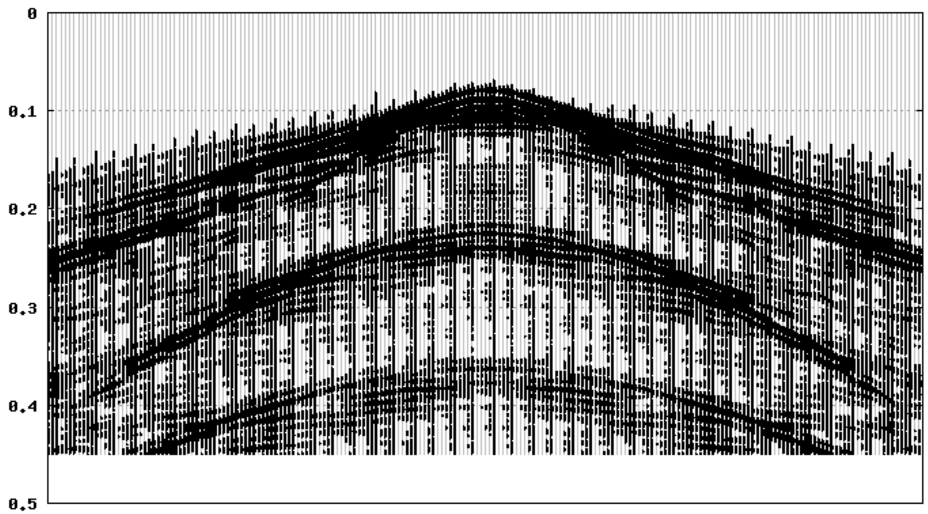


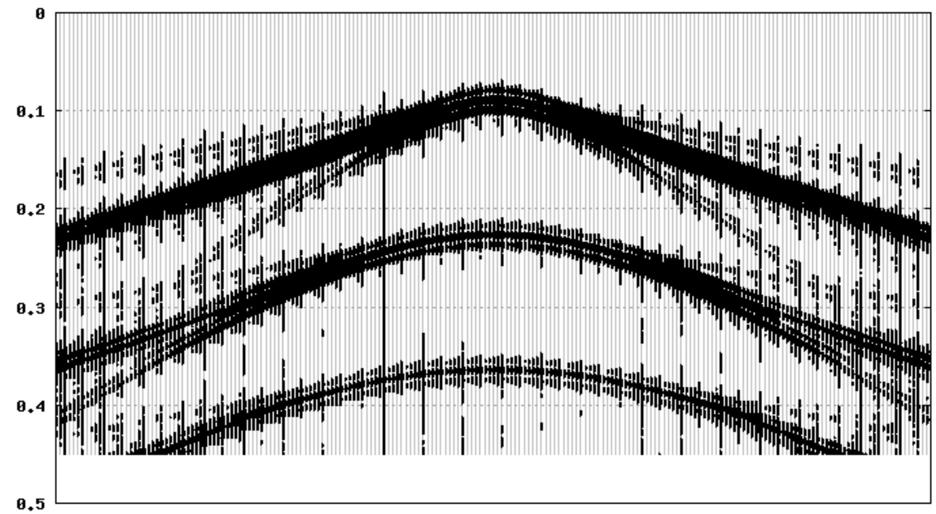
Source in the water



Source in the water, without carbon reservoir

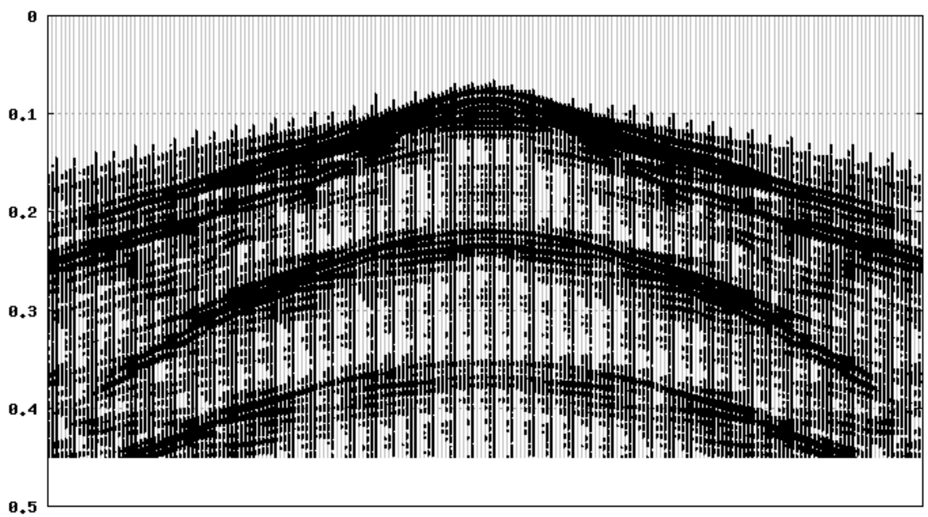
Seismograms, receivers at the seabed, V

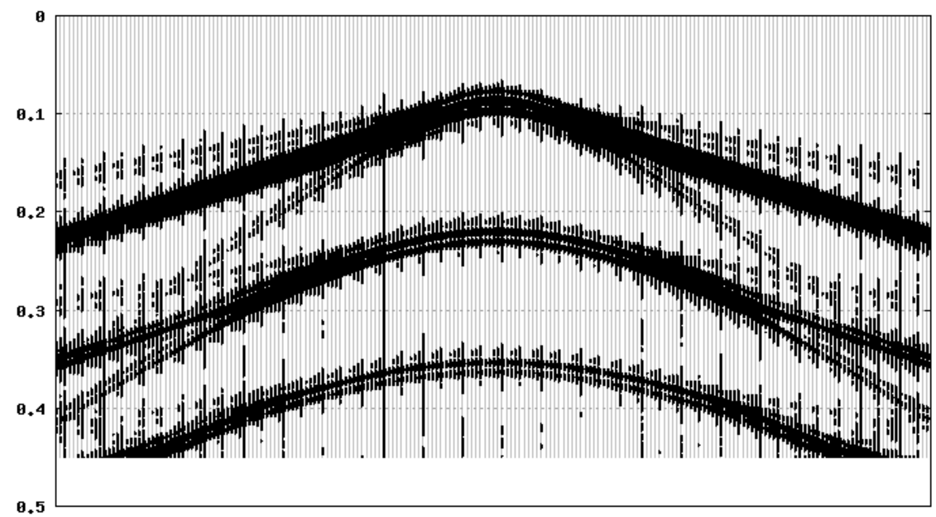




Source in the ice

Source in the ice, without carbon reservoir

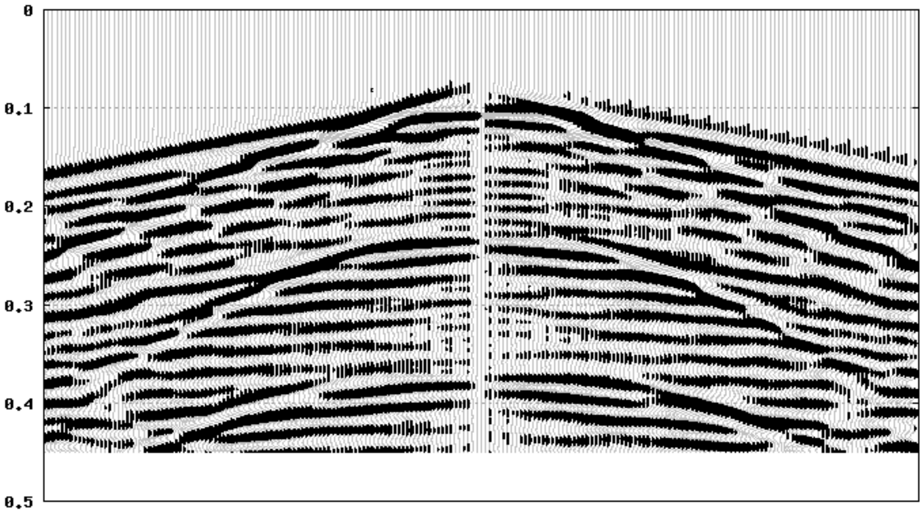




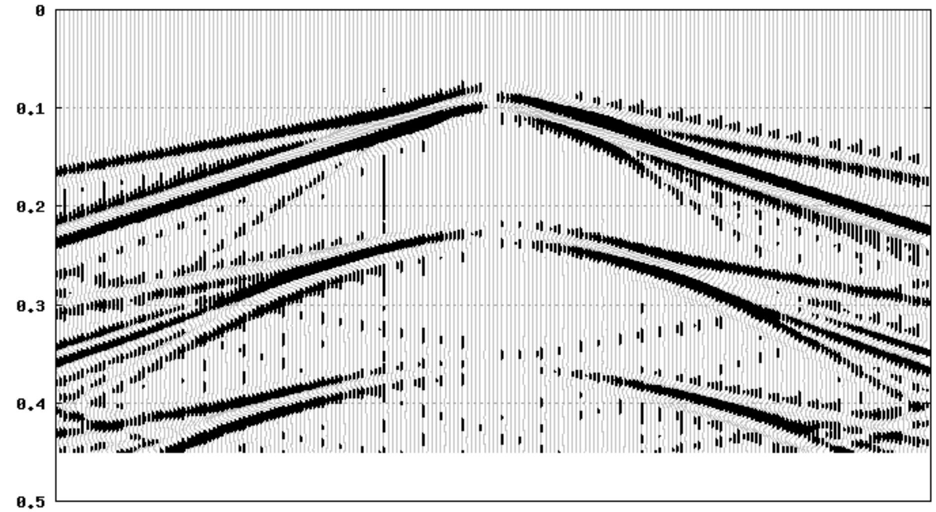
Source in the water

Source in the water, without carbon reservoir

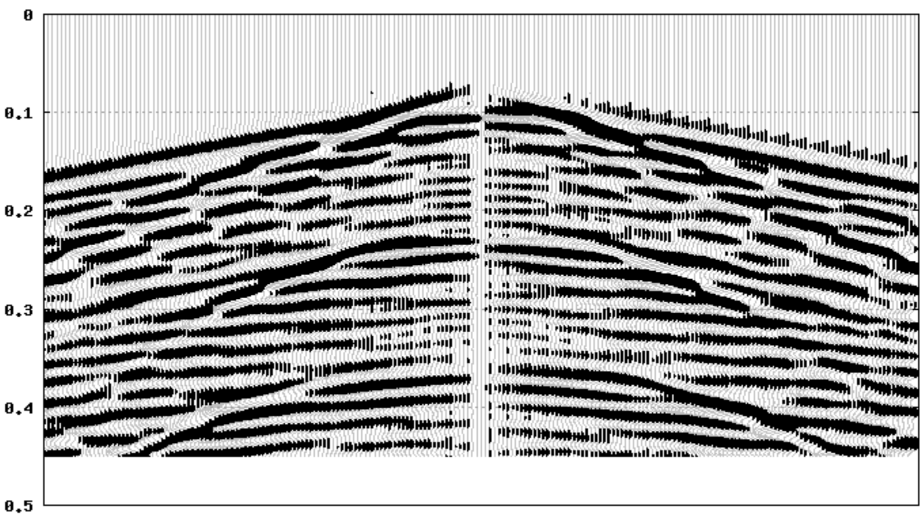
Seismograms, receivers at the seabed, Vx



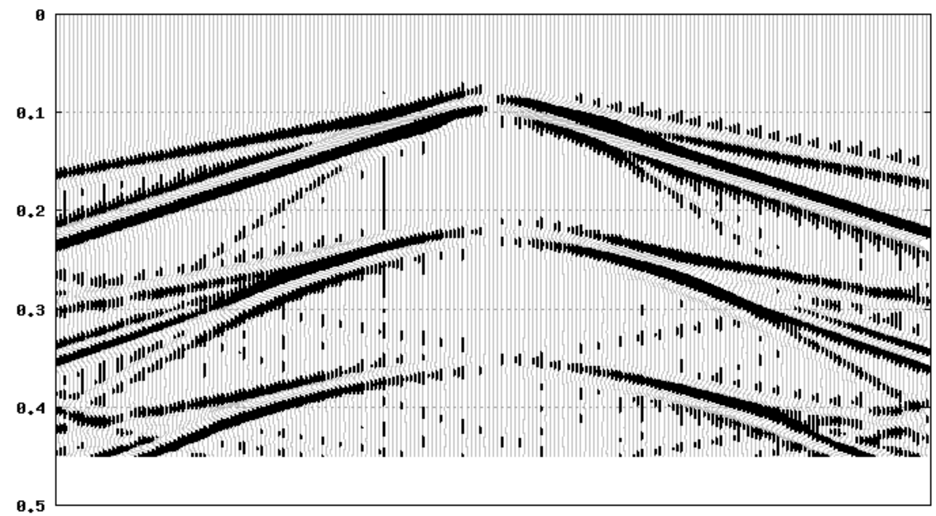
Source in the ice



Source in the ice, without carbon reservoir

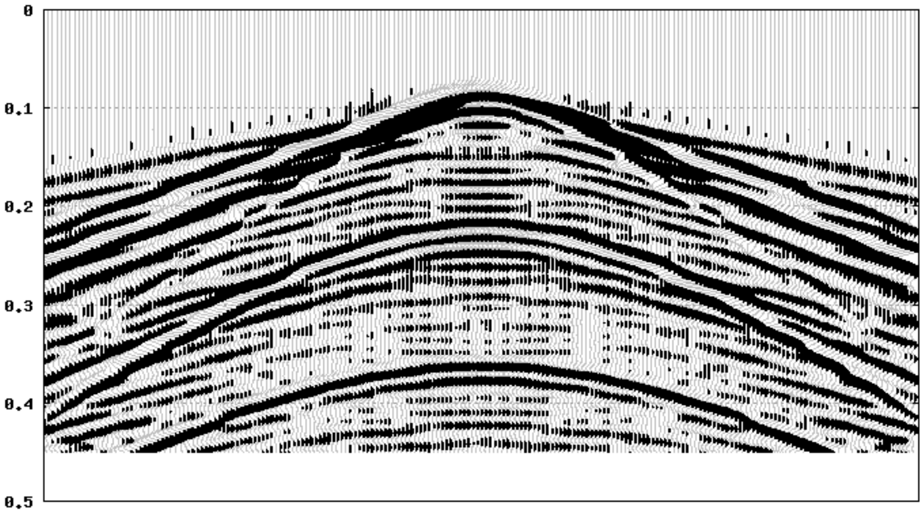


Source in the water

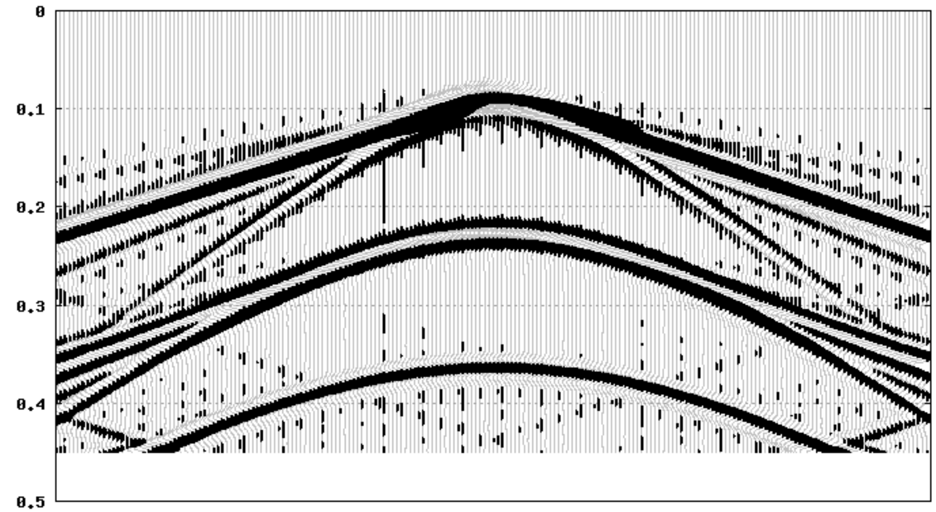


Source in the water, without carbon reservoir

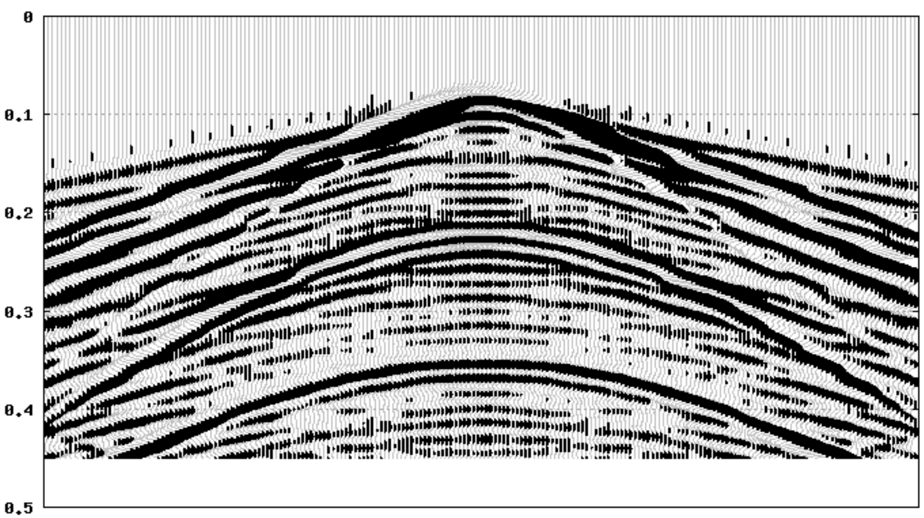
Seismograms, receivers at the seabed, Vy



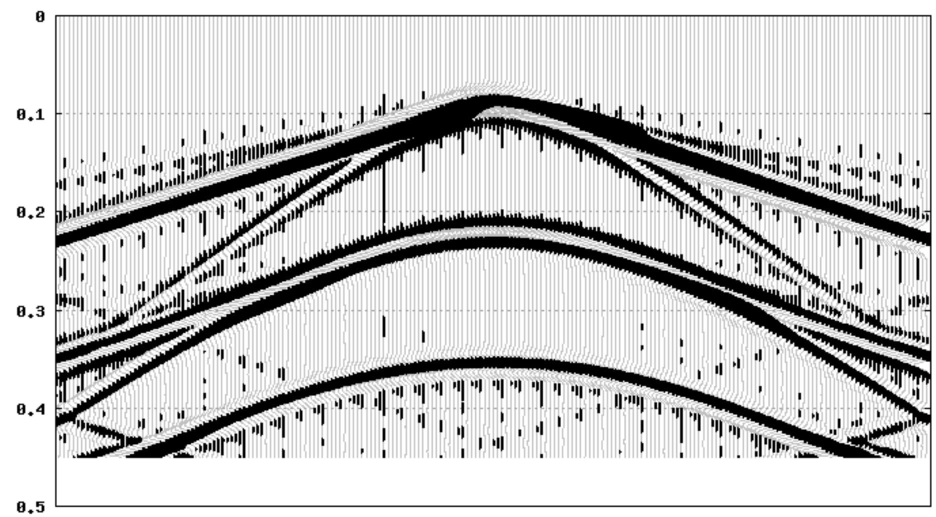
Source in the ice



Source in the ice, without carbon reservoir



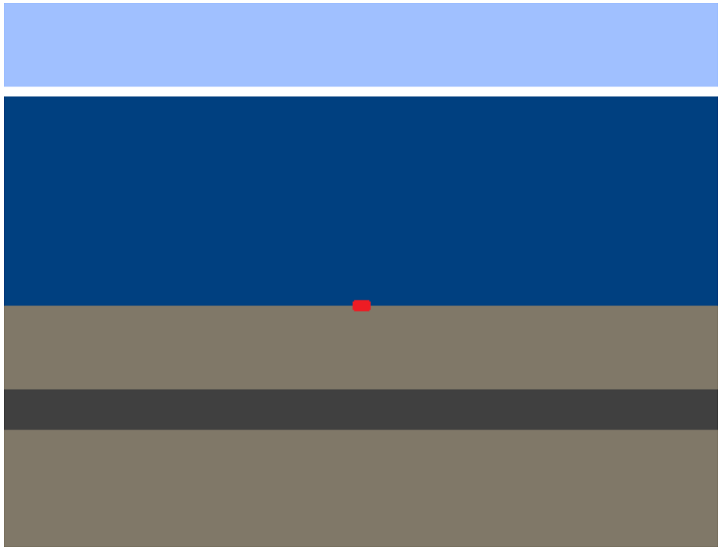
Source in the water



Source in the water, without carbon reservoir

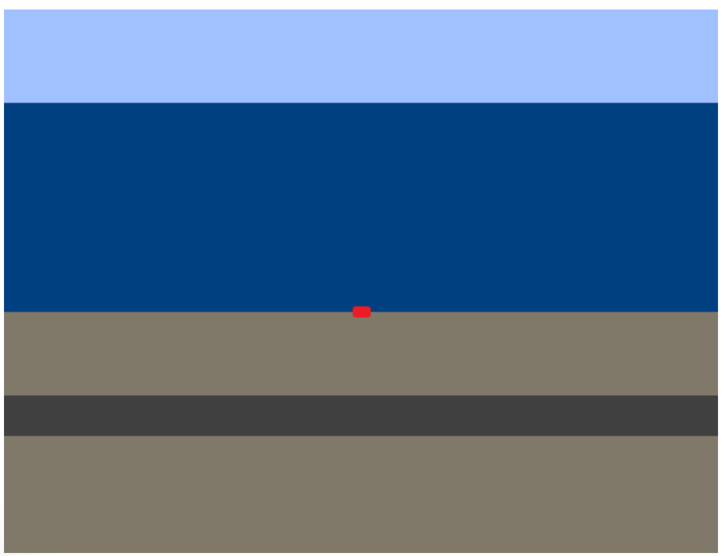
4. Influence of ice. Sources at the seabed.

Problem definitions



With ice

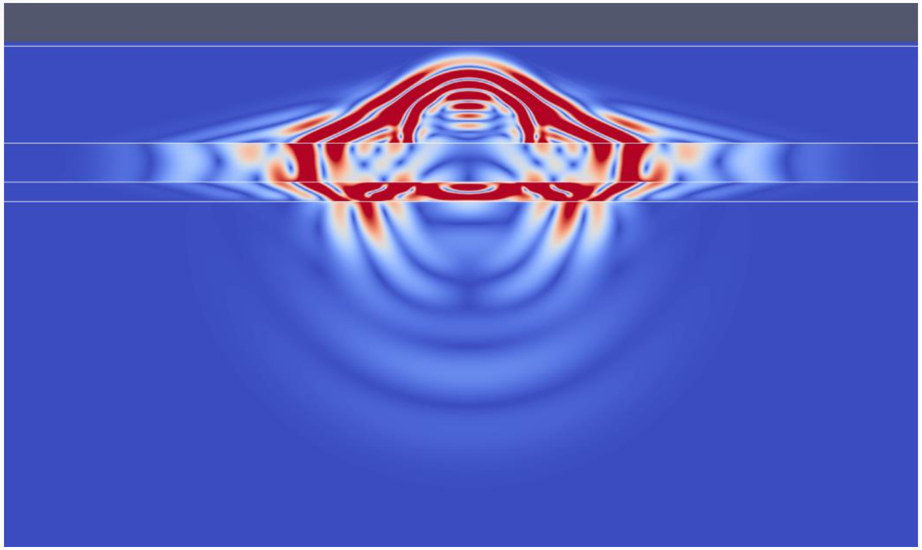
With ice, without carbon reservoir



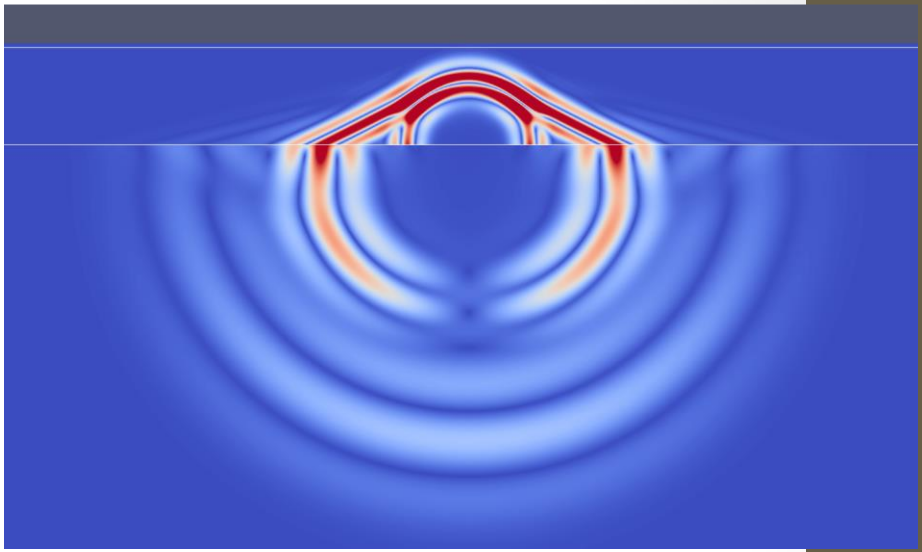
Without ice

Without ice, without carbon reservoir

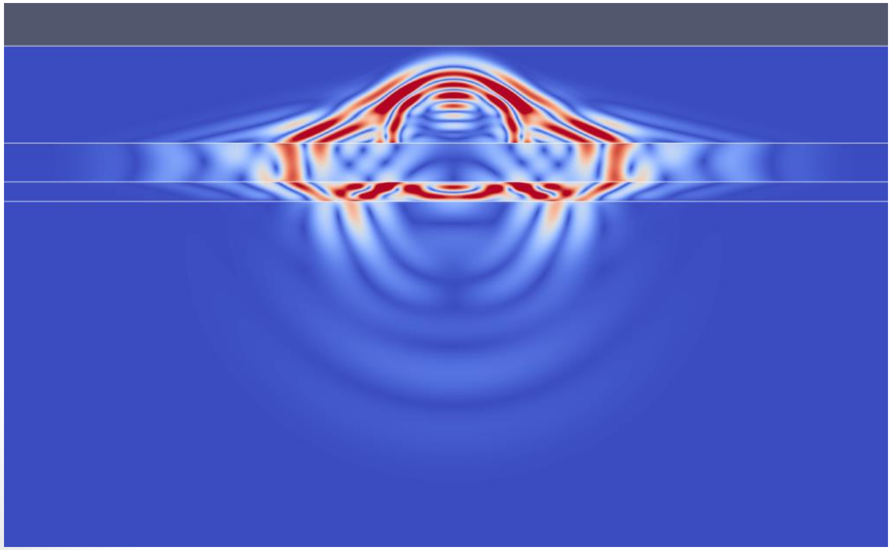
Wave patterns



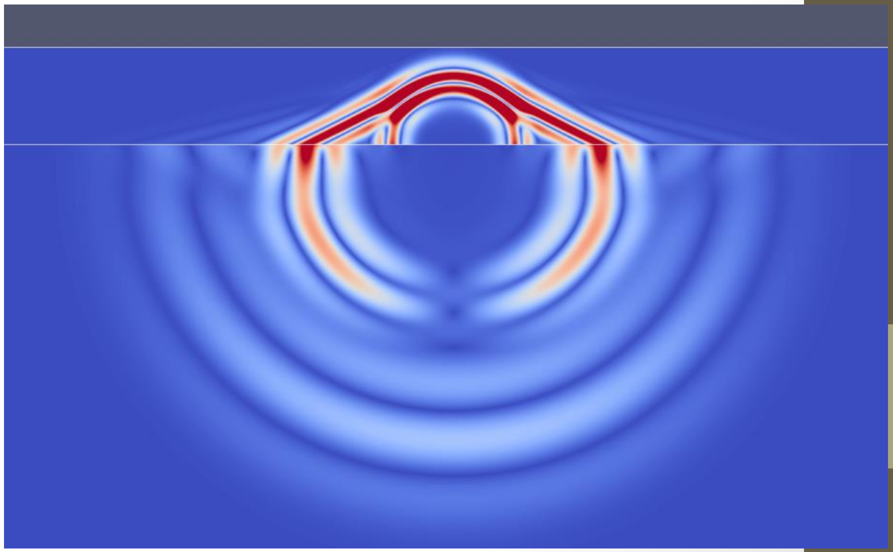
With ice



With ice, without carbon reservoir

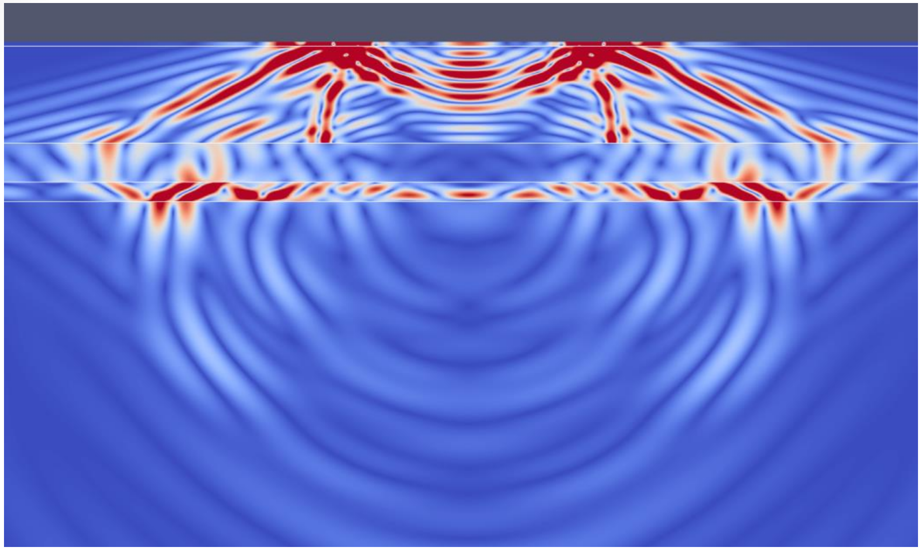


Without ice

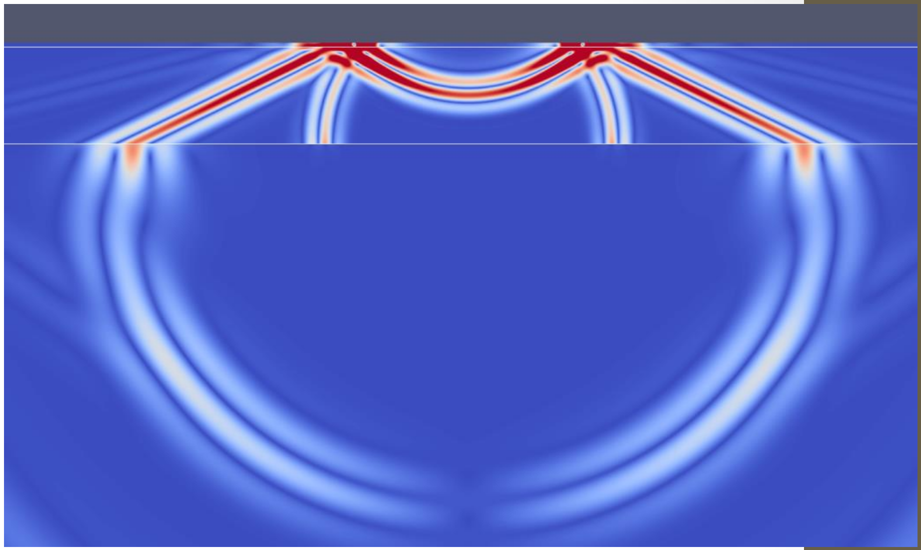


Without ice, without carbon reservoir

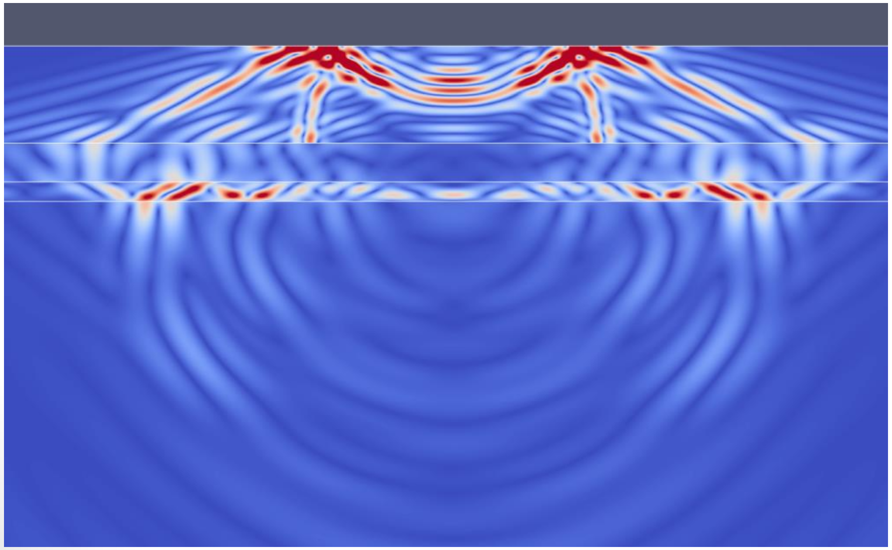
Wave patterns



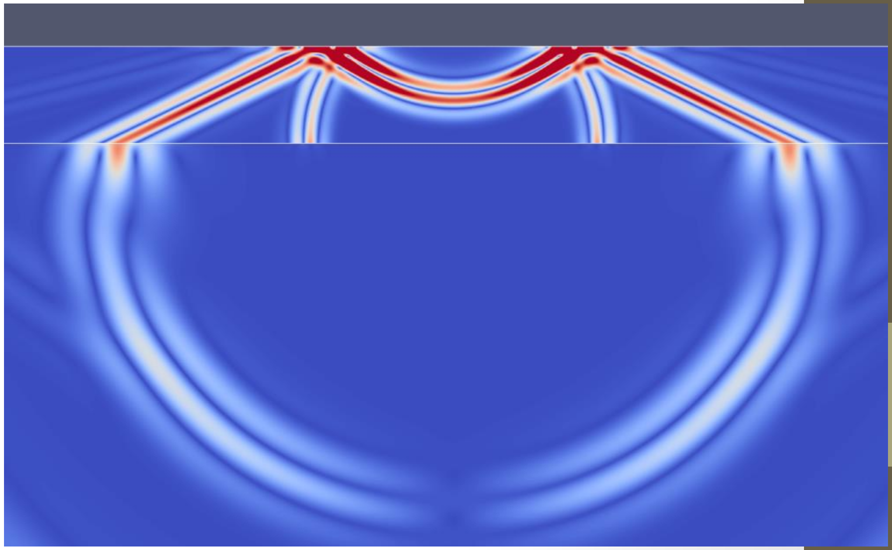
With ice



With ice, without carbon reservoir

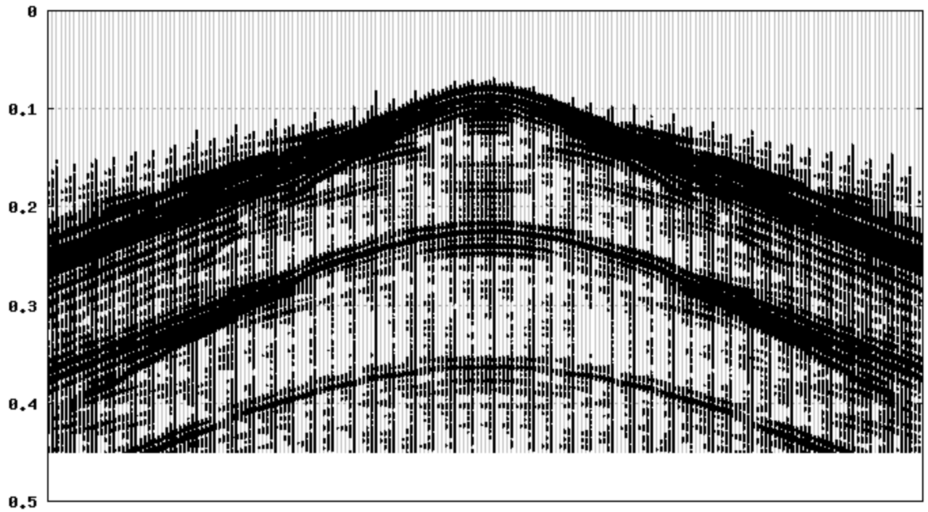


Without ice

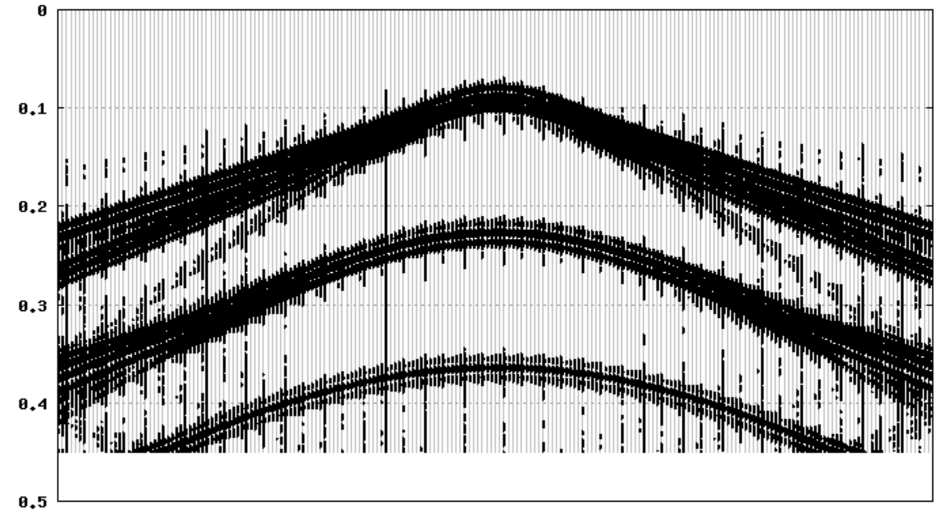


Without ice, without carbon reservoir

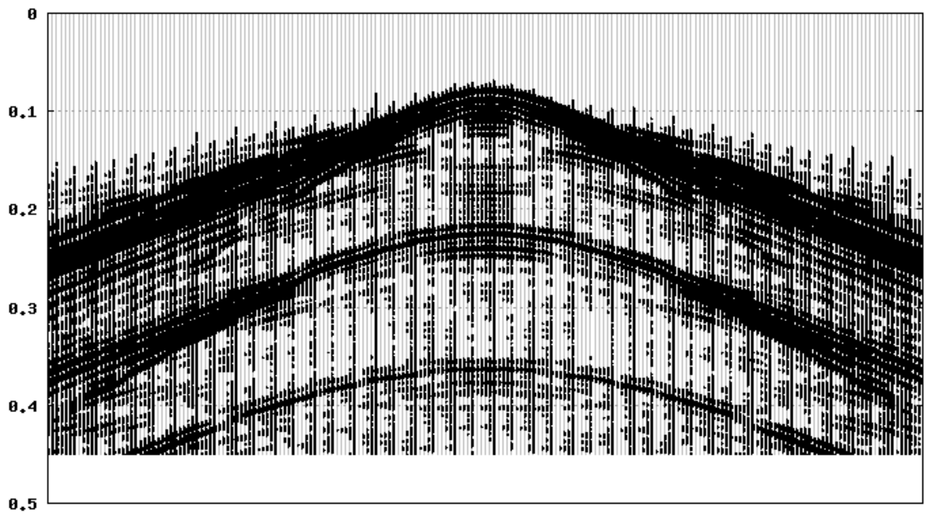
Seismograms, receivers in the water/ice, V



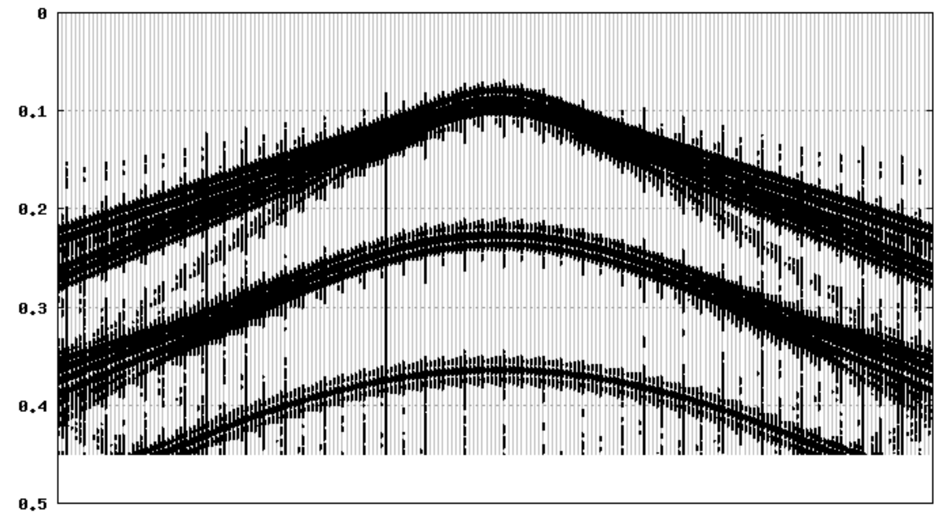
With ice



With ice, without carbon reservoir

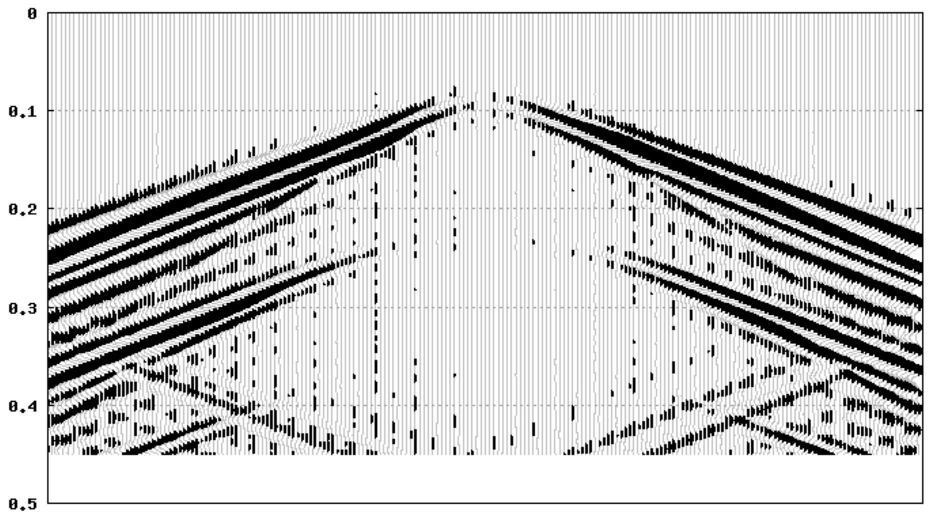


Without ice

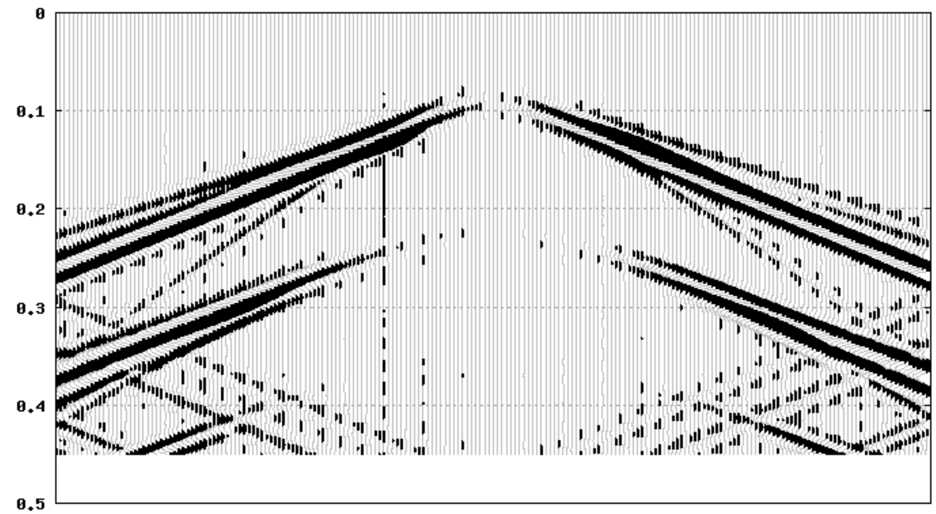


Without ice, without carbon reservoir

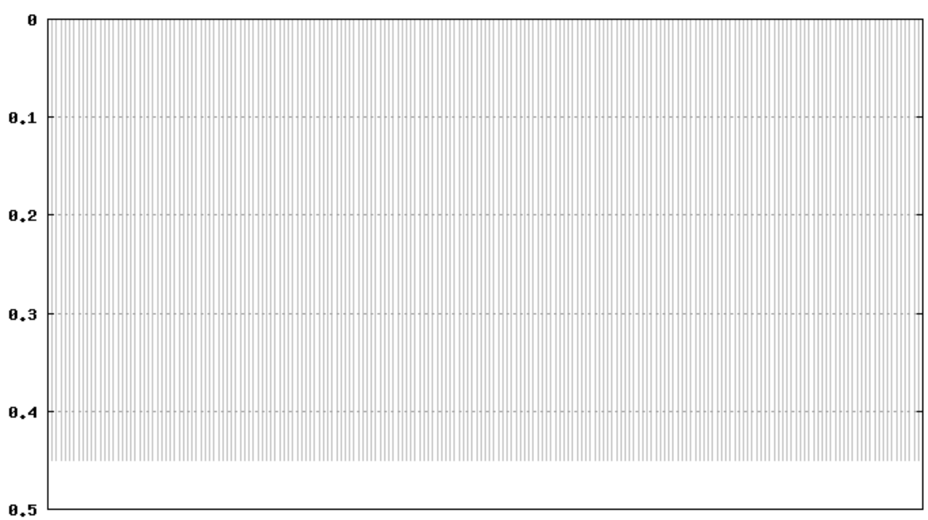
Seismograms, receivers in the water/ice, Vx



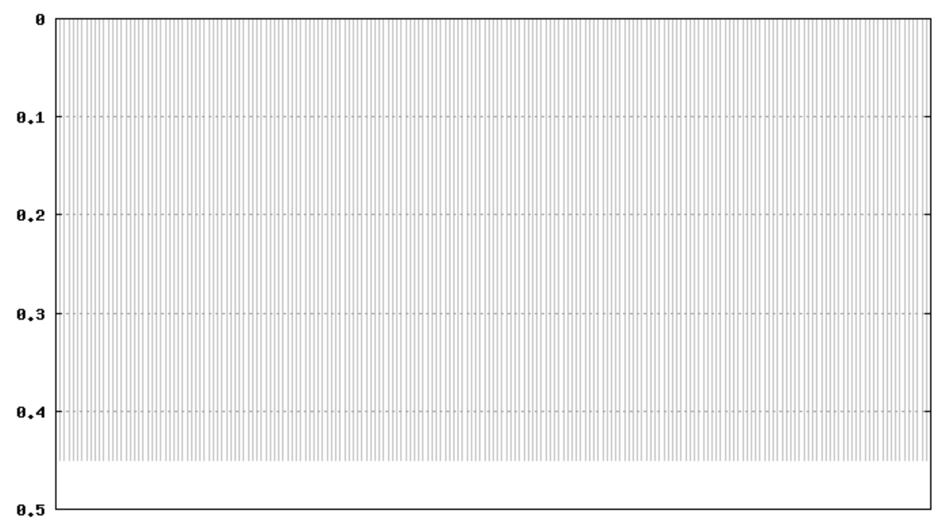
With ice



With ice, without carbon reservoir

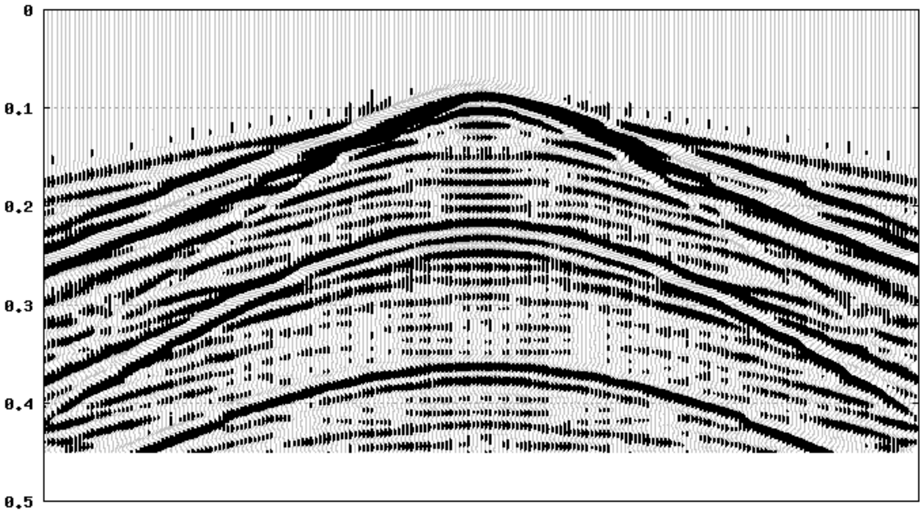


Without ice

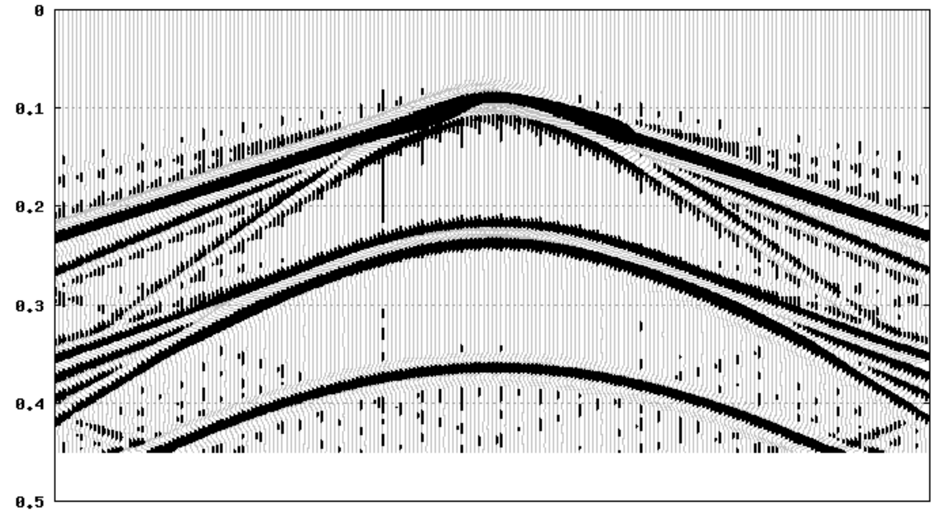


Without ice, without carbon reservoir

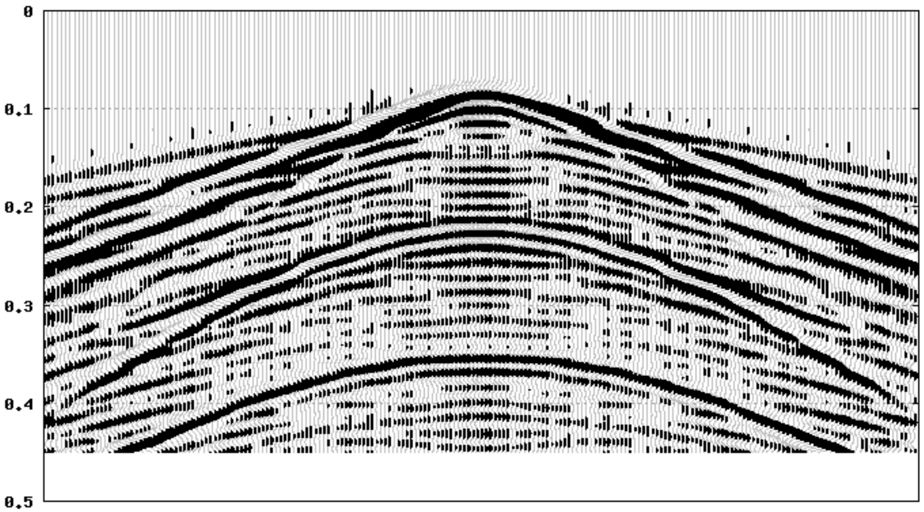
Seismograms, receivers in the water/ice, Vy



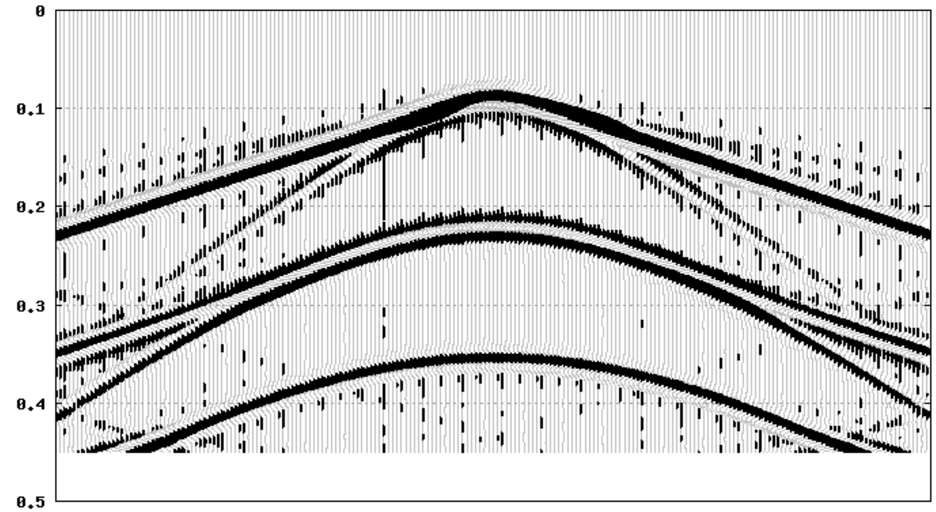
With ice



With ice, without carbon reservoir

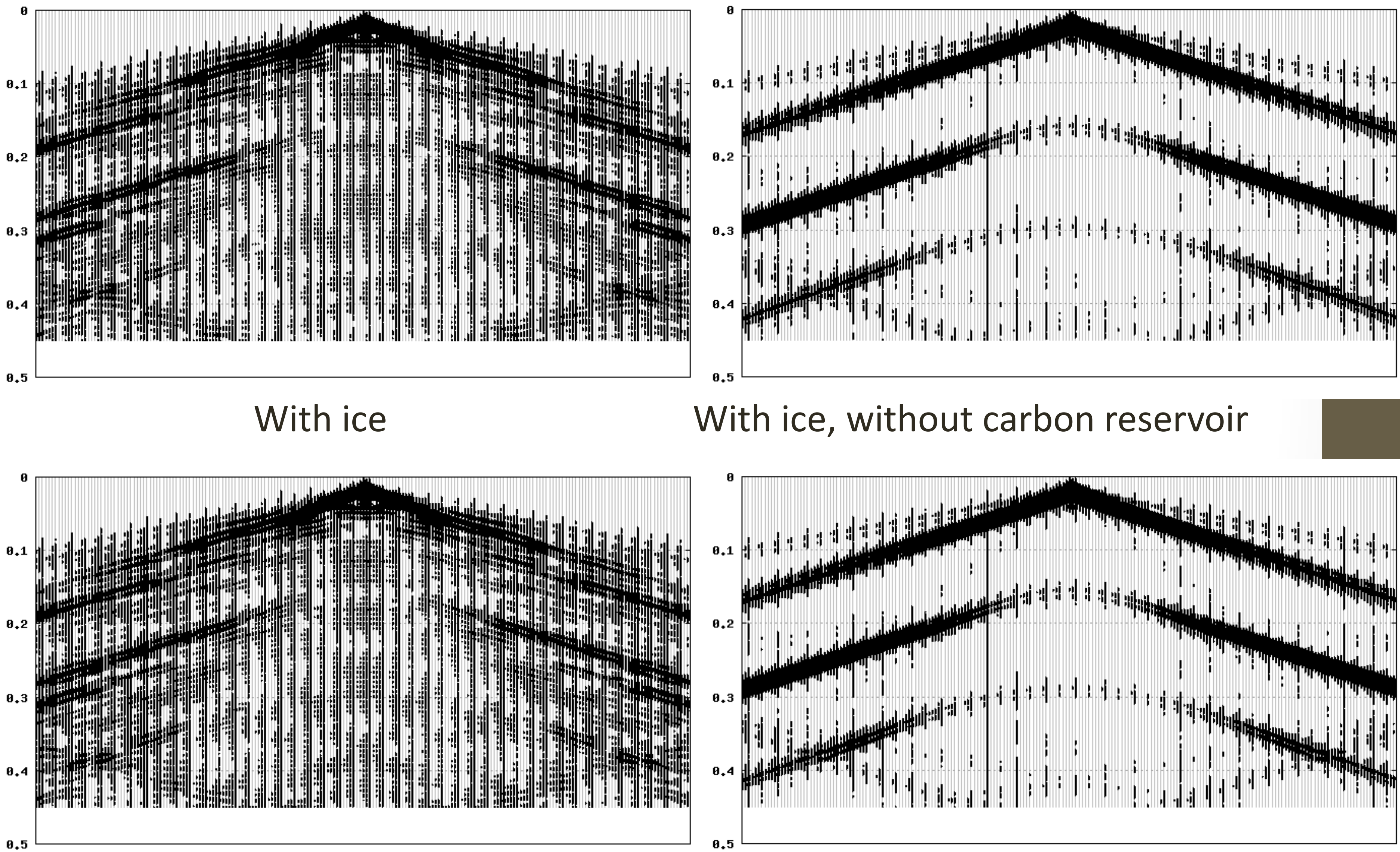


Without ice



Without ice, without carbon reservoir

Seismograms, receivers at the seabed, V



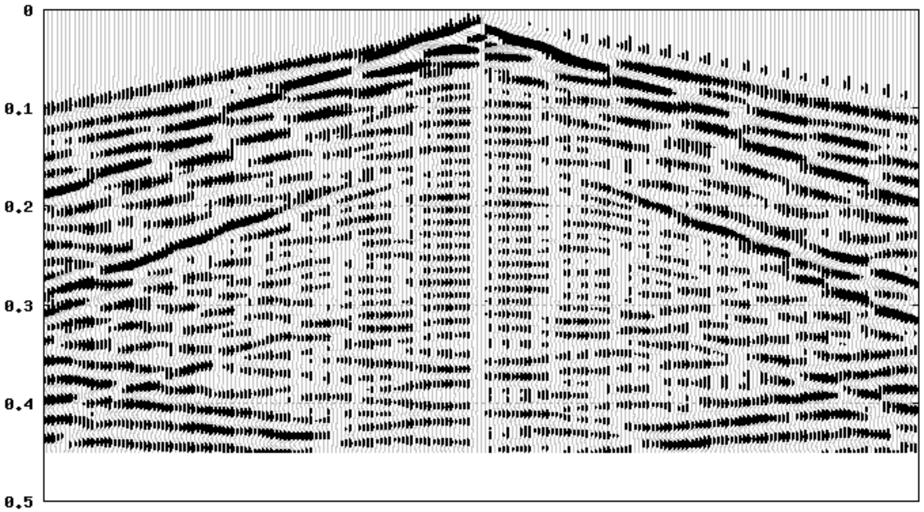
With ice

With ice, without carbon reservoir

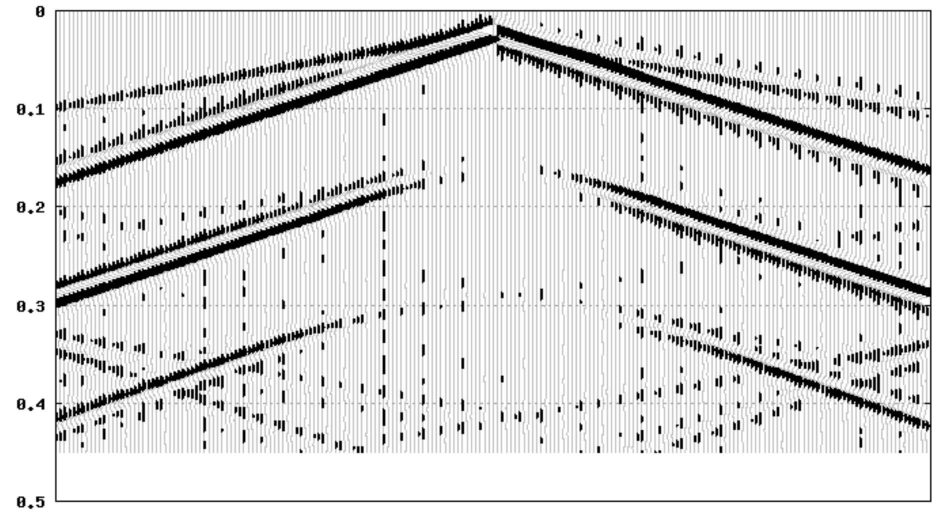
Without ice

Without ice, without carbon reservoir

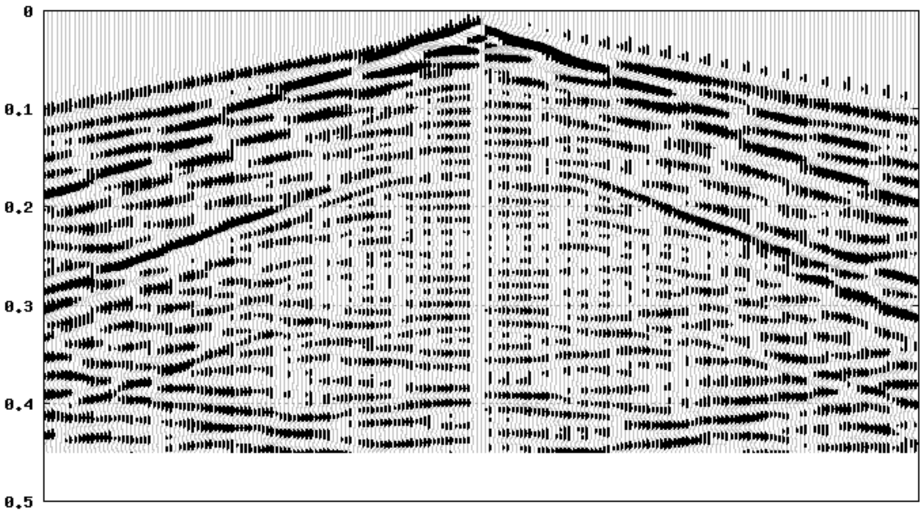
Seismograms, receivers at the seabed, Vx



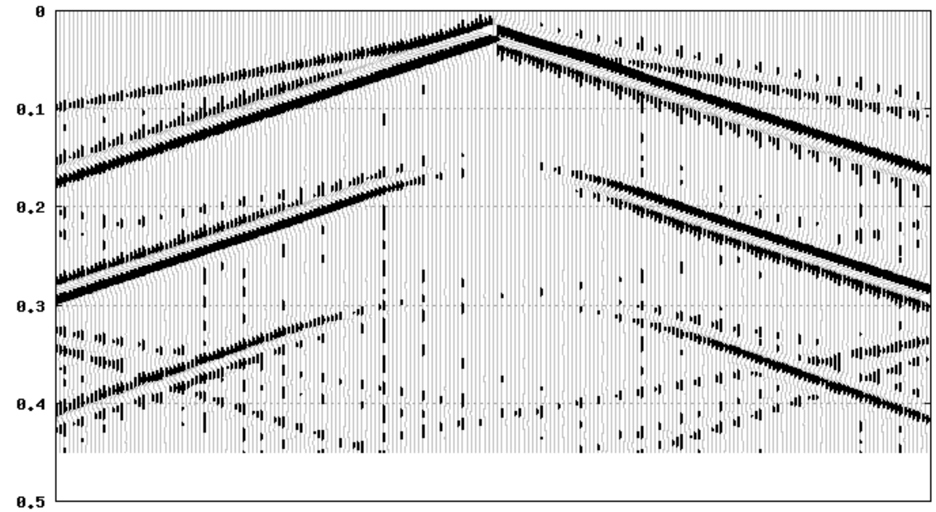
With ice



With ice, without carbon reservoir

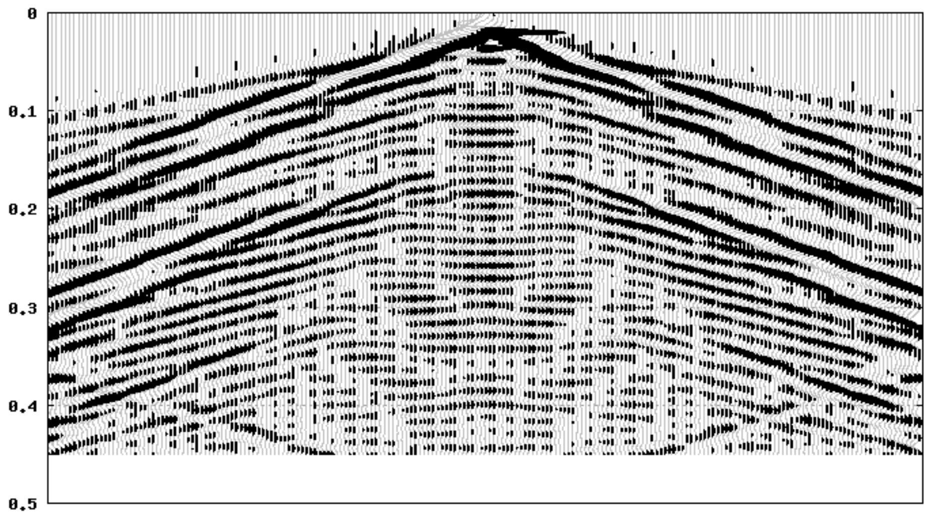


Without ice

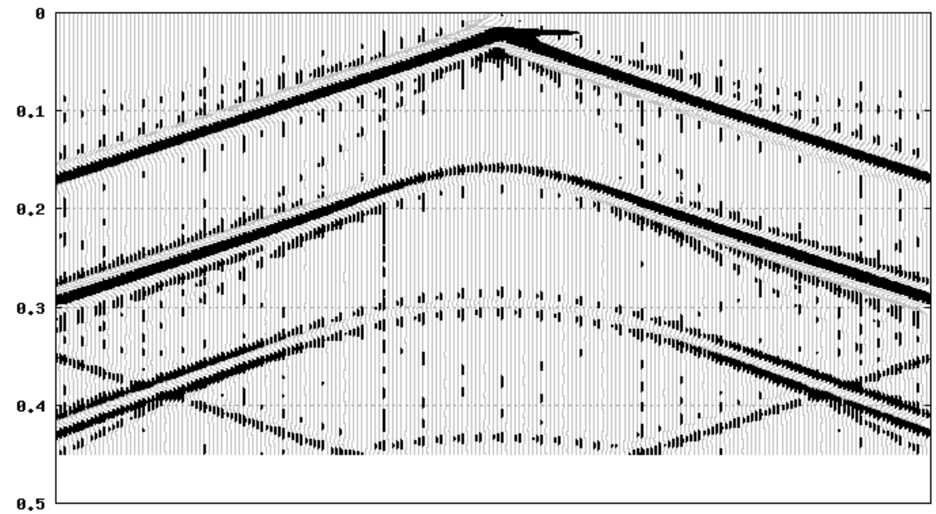


Without ice, without carbon reservoir

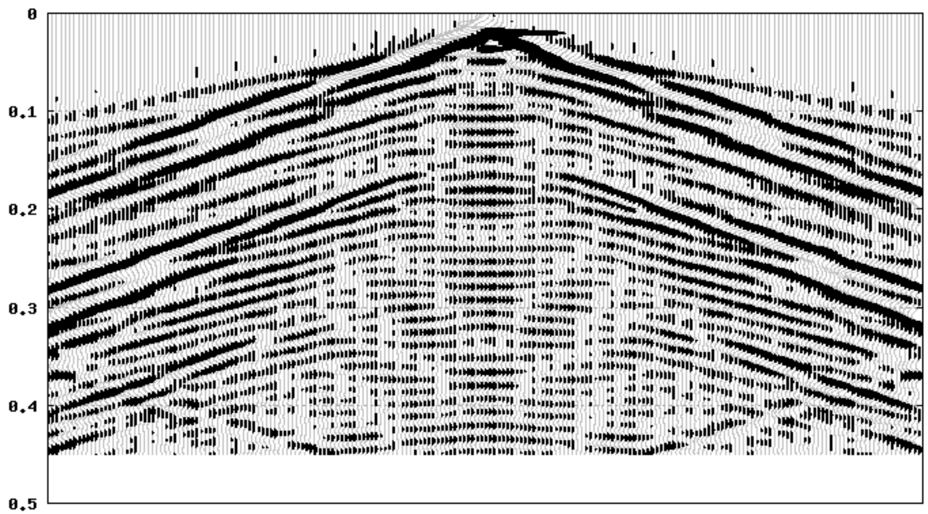
Seismograms, receivers at the seabed, Vy



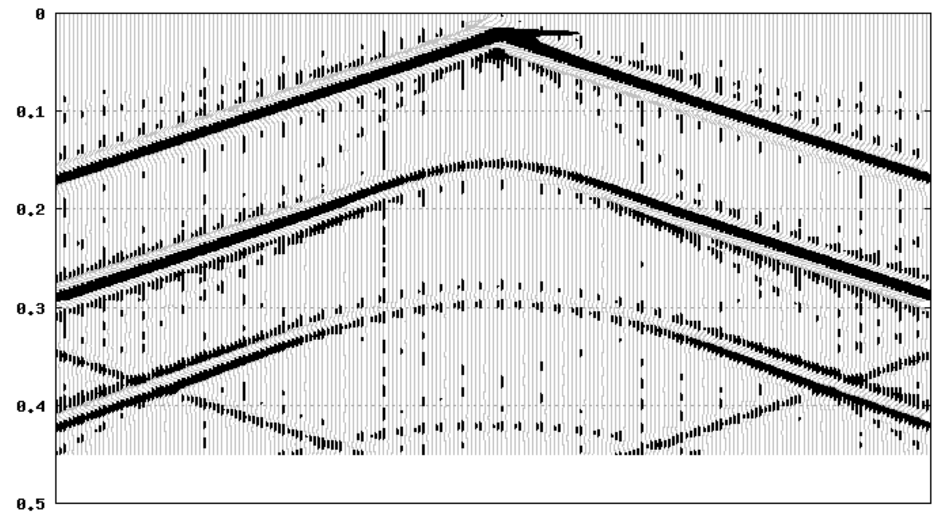
With ice



With ice, without carbon reservoir

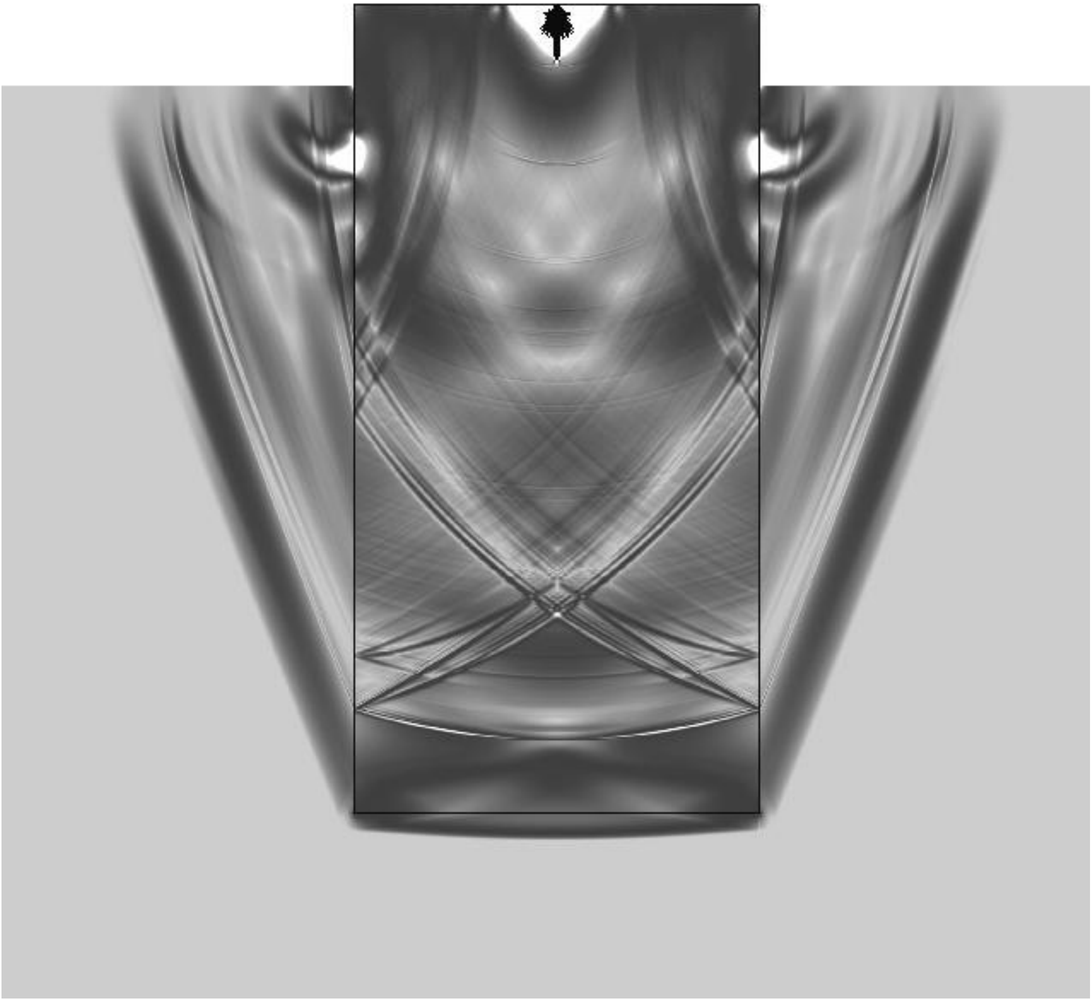


Without ice

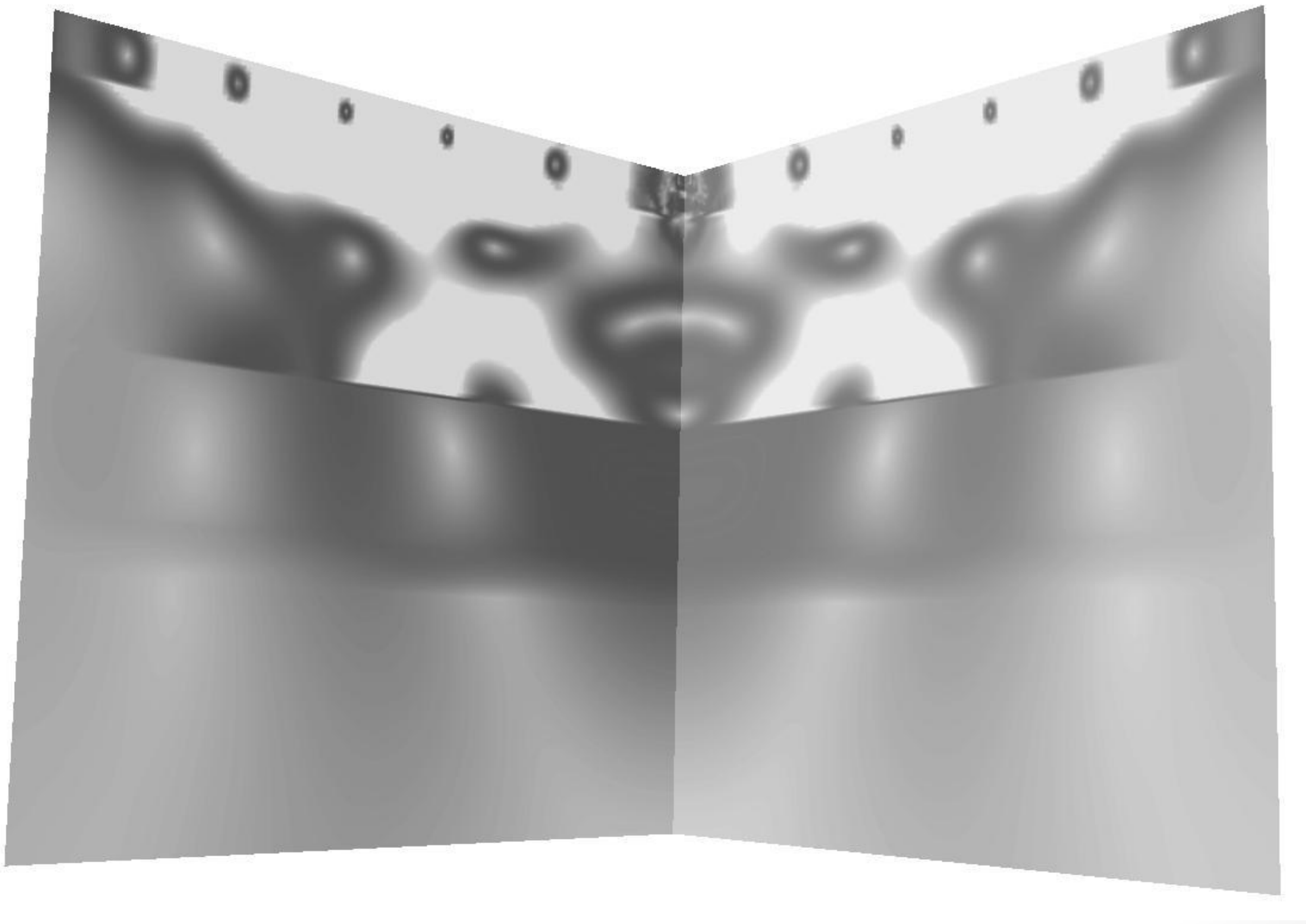


Without ice, without carbon reservoir

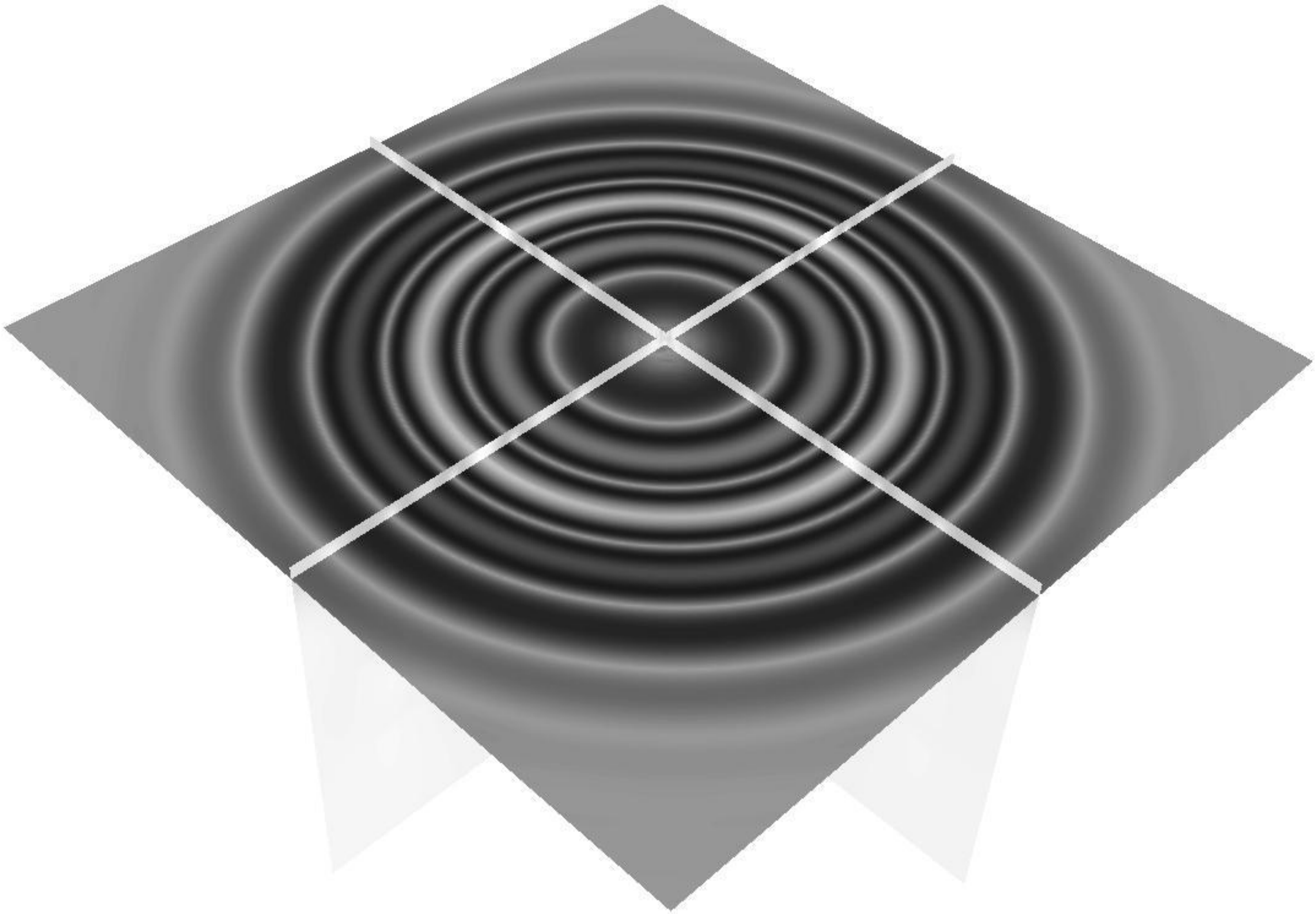
Iceberg under explosion



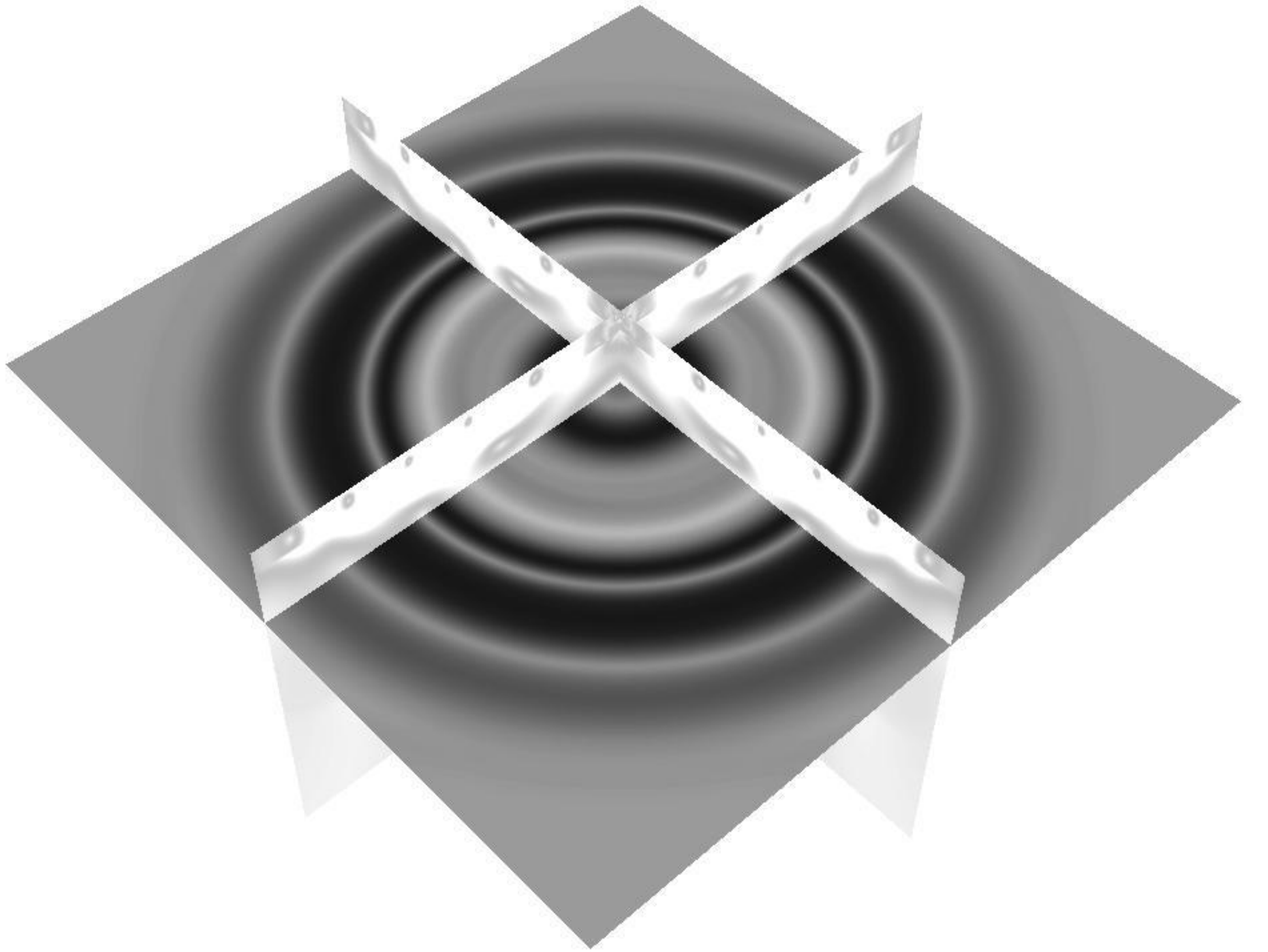
3D Model



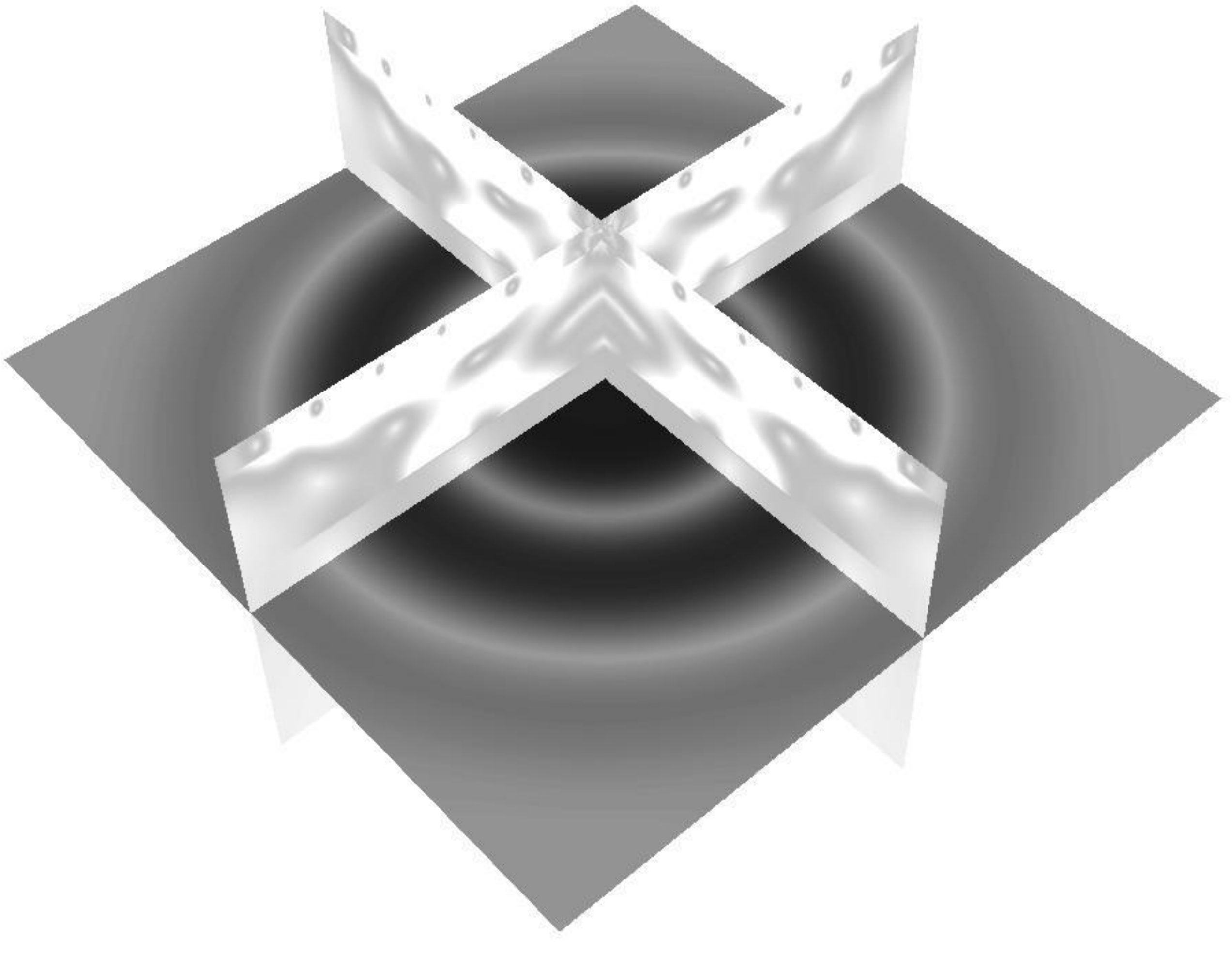
Wave propagation in the ice



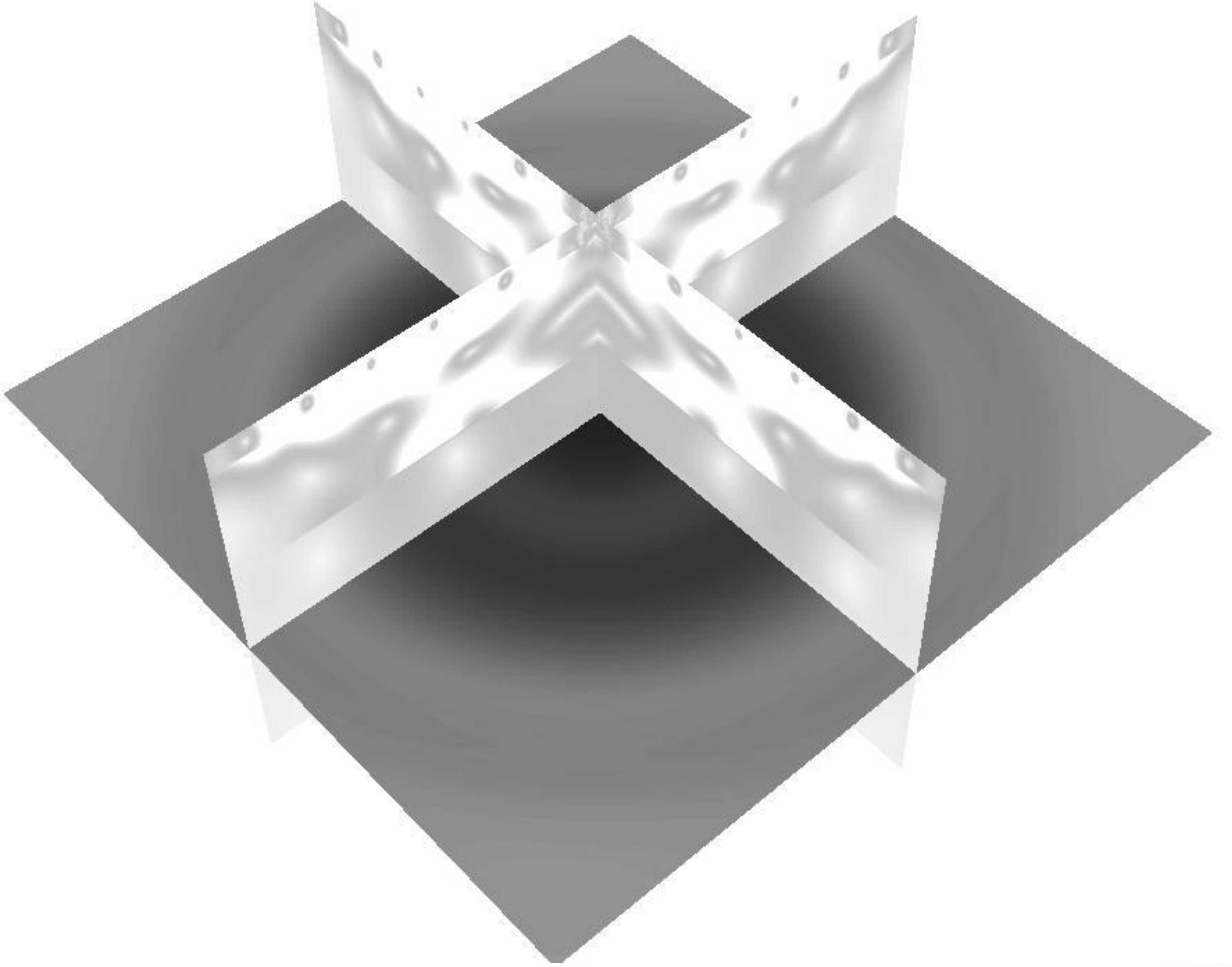
Wave propagation in the water



Wave propagation in the reservoir



Wave propagation in the ground



Conclusions

- Numerical modeling of wave processes in the elastic and acoustic media was done
- We solve problems of seismic exploration in the Arctic shelf.
- We made synthetic seismograms.
- We study wave propagation in the icebergs under explosion.

Further research: immerse interface method

Problem: loss of precision at the boundaries

Possible solution *(Chaoming Zhang, Randall J. LeVeque, Charles S. Peskin, Xin Wen, Shi Jin, and others):*

- “crop” the boundary and the nearest nodes
- apply local scheme in the “cropped” area, taking into consideration its features
- smooth the results obtained in the inner and outer grids.

Thank you for your attention!

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