Migration imaging in elastic media using Born approximation

Voynov O.Ya., Golubev V.I., Zhdanov M.S.
Moscow Institute of Physics and Technology

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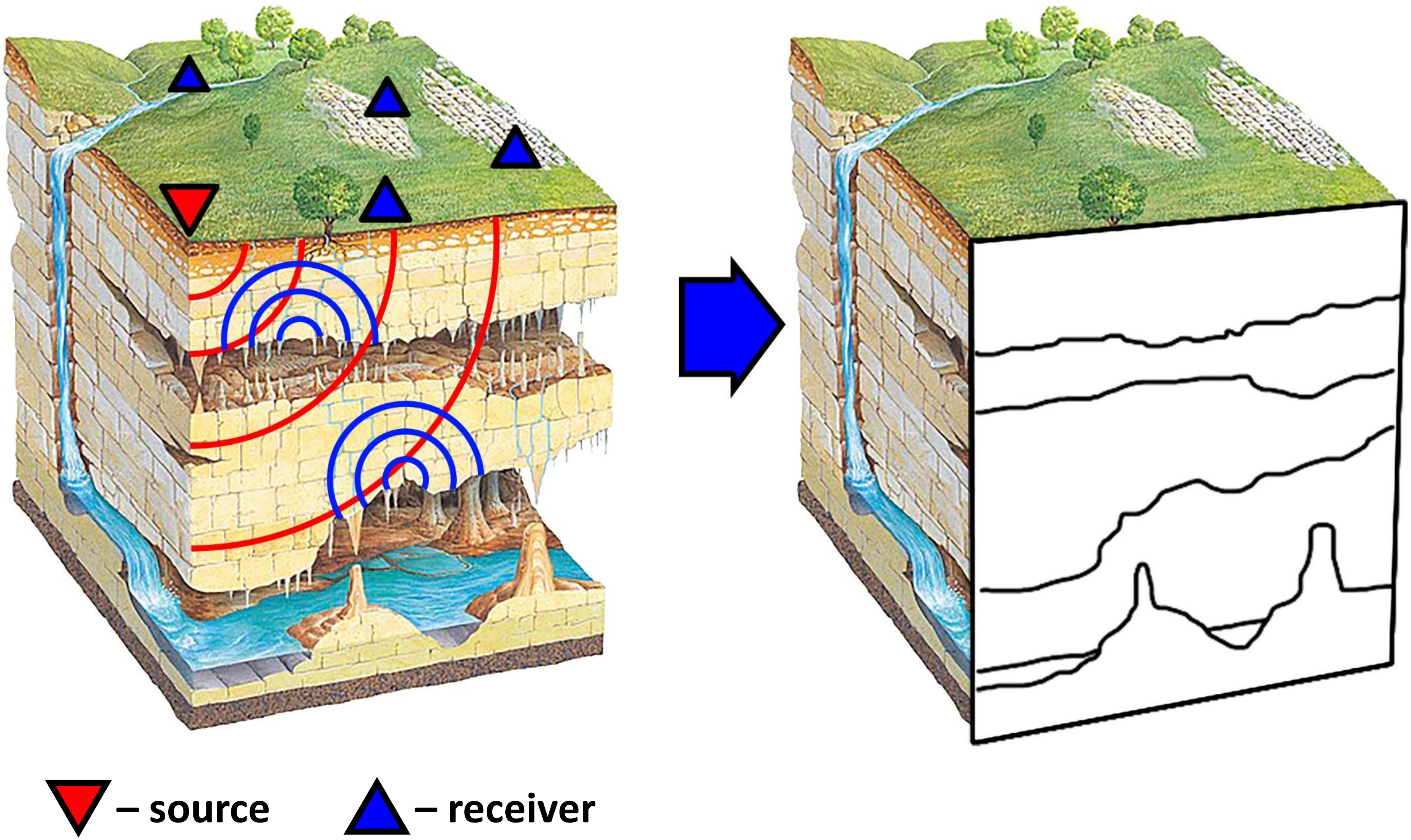
Plan

1. Problem statement
2. Main formulae
3. Comparison of acoustic and elastic Born migration

Seismic migration imaging



Formulae

Lame equation:

$$\hat{\Lambda}\vec{u} - \frac{\partial^2 \vec{u}}{\partial t^2} = -\vec{f}^e, \qquad \hat{\Lambda} = c_p^2 \nabla\nabla \cdot - c_s^2 \nabla \times \nabla \times$$

Background and anomalous parts:

$$c_\alpha^2 = c_{\alpha,b}^2 + \Delta c_\alpha^2, \qquad \alpha \in \{p, s\},$$

$$\hat{\Lambda} = \hat{\Lambda}_b + \Delta\hat{\Lambda}, \qquad \vec{u} = \vec{u}^i + \vec{u}^s$$

Equations for incident and scattered fields:

Born approximation

$$\hat{\Lambda}_b \vec{u}^i - \frac{\partial^2 \vec{u}^i}{\partial t^2} = -\vec{f}^e, \qquad \hat{\Lambda}_b \vec{u}^s - \frac{\partial^2 \vec{u}^s}{\partial t^2} = -\Delta\hat{\Lambda}\left(\vec{u}^i + \cancel{\vec{u}^s}\right)$$

Acoustic wave equation:

$$\Delta P - s^2 \partial_t^2 P = -F^e$$

Background and anomalous parts:

$$s^2 = s_b^2 + \Delta s^2,$$

$$P = P^i + P^s$$

Equations for incident and scattered fields:

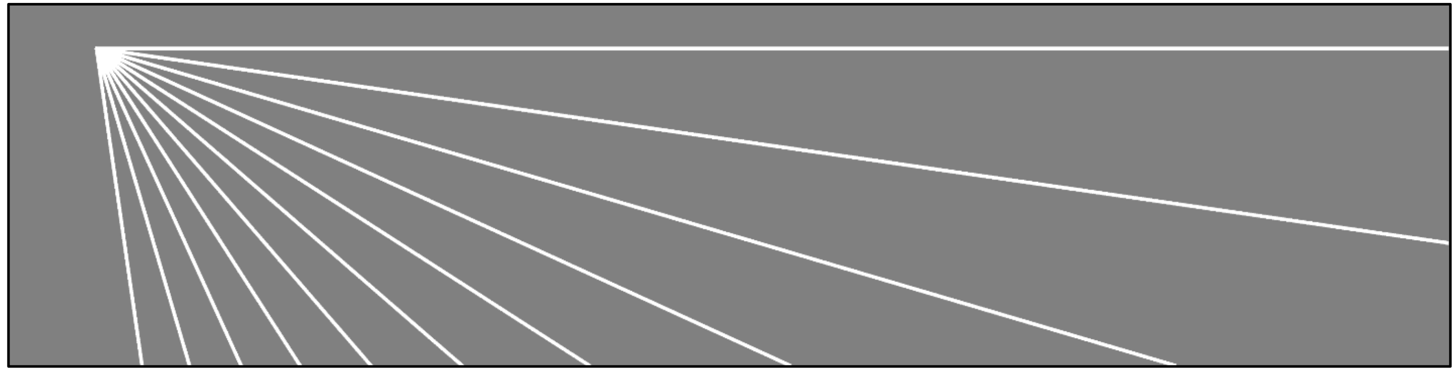
Born approximation

$$\Delta P^i - s_b^2 \partial_t^2 P^i = -F^e, \qquad \Delta P^s - s_b^2 \partial_t^2 P^s = \Delta s^2 \partial_t^2 \left(P^i + \cancel{P^s}\right)$$

Acoustic vs Elastic Born migration

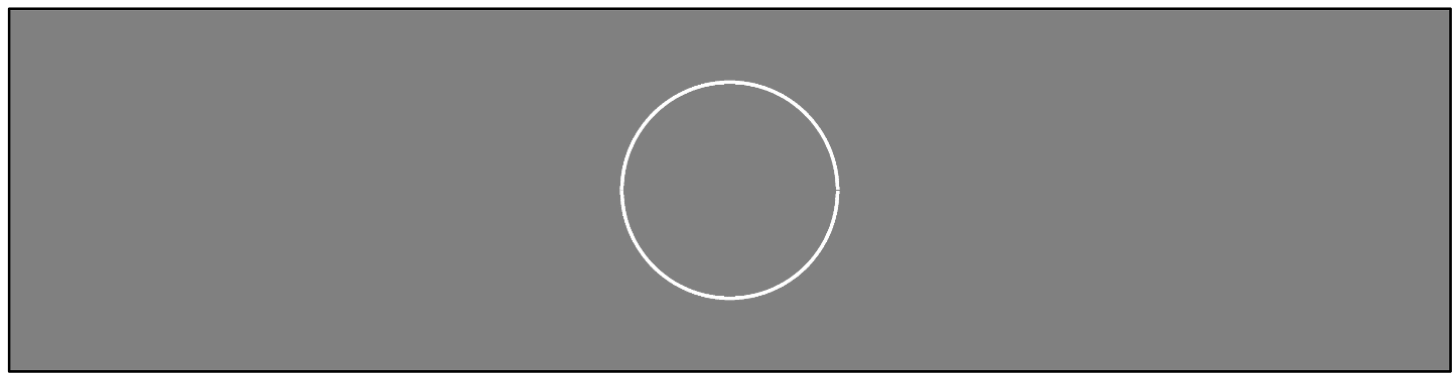
Media

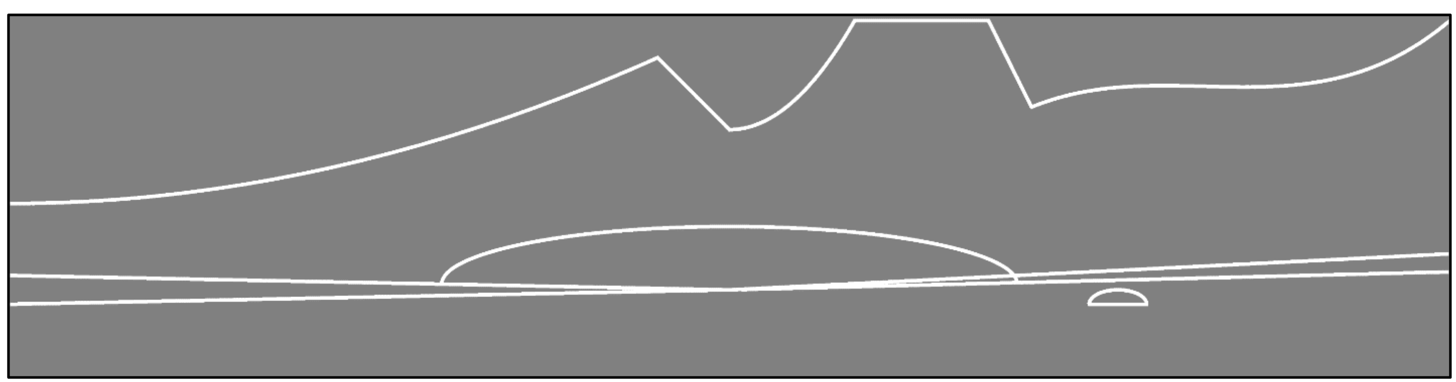
$x \times z = 10 \times 2.5$ km,
$c_{p,b} = 2.5$ km/s,
$c_{s,b} = 1.25$ km/s,
$\Delta c_\alpha^2 / c_{\alpha,b}^2 = 0.01$,
$\rho = 2.5$ t/m^3

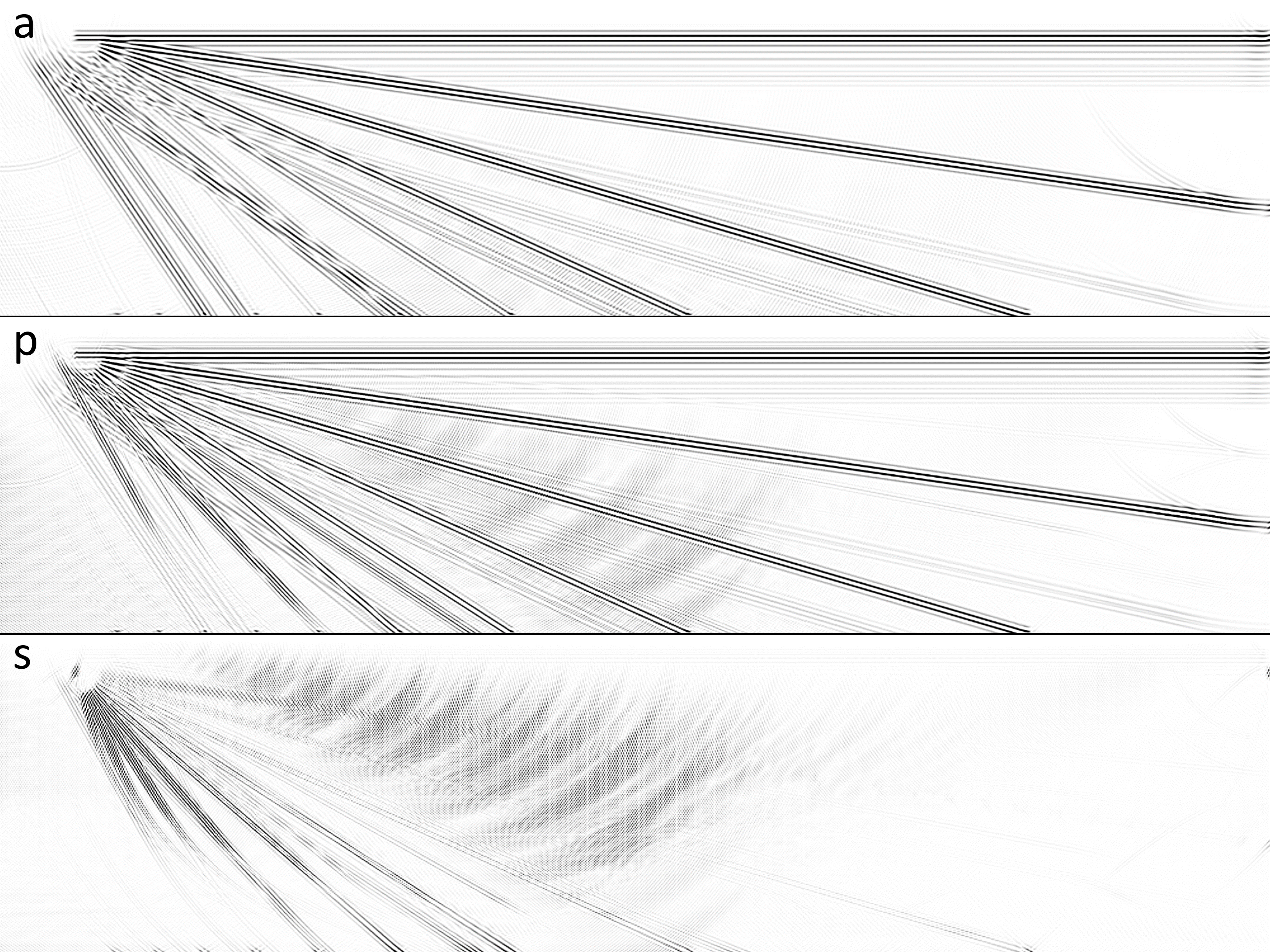


Data

only z-component of scattered field



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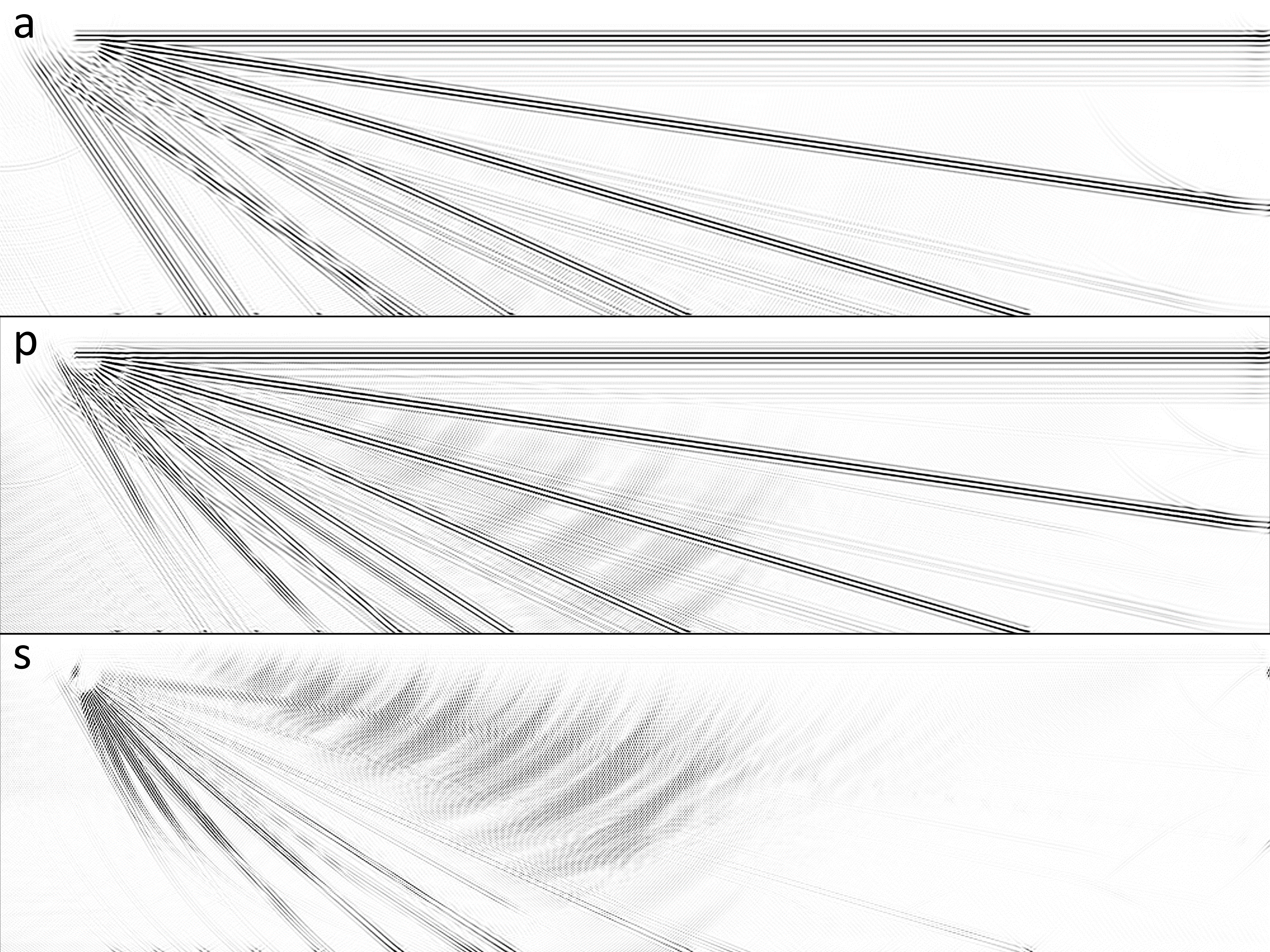
$$P = P^i + P^s$$

Equations for incident and scattered fields:

$$\Delta P^i - s_b^2 \partial_t^2 P^i = -F^e, \qquad \Delta P^s - s_b^2 \partial_t^2 P^s = \Delta s^2 \partial_t^2 \left(P^i + P^s\right)$$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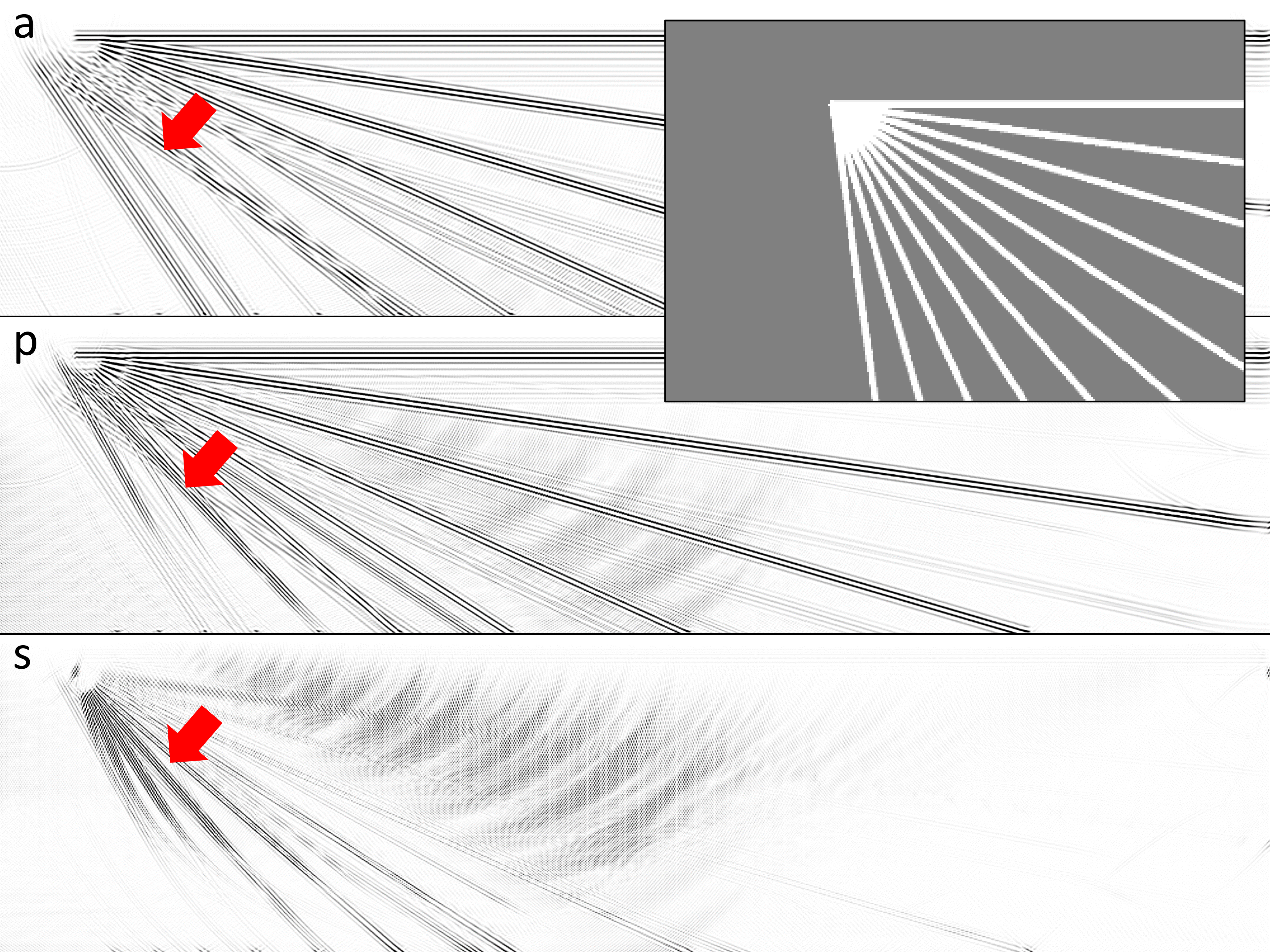
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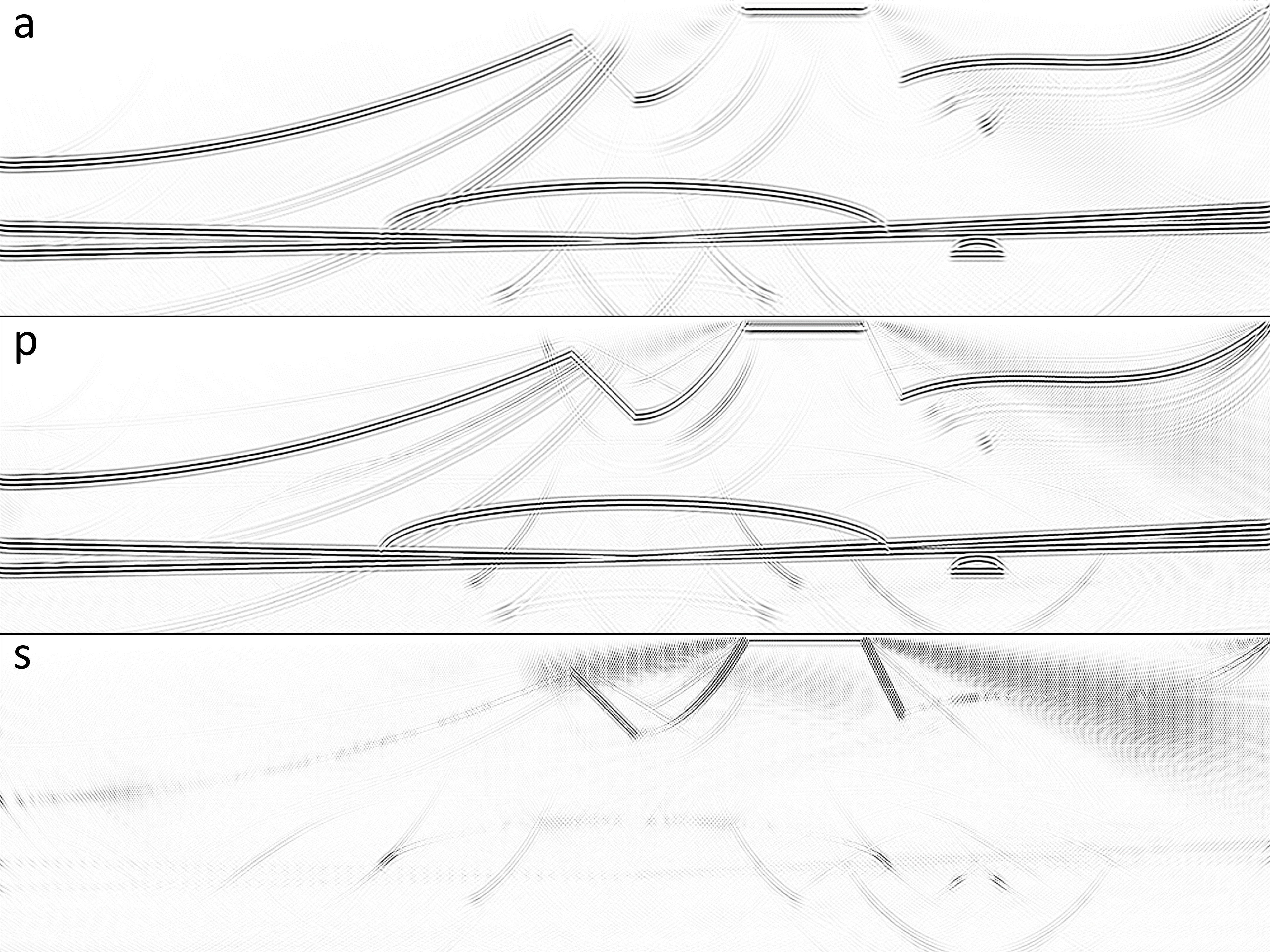
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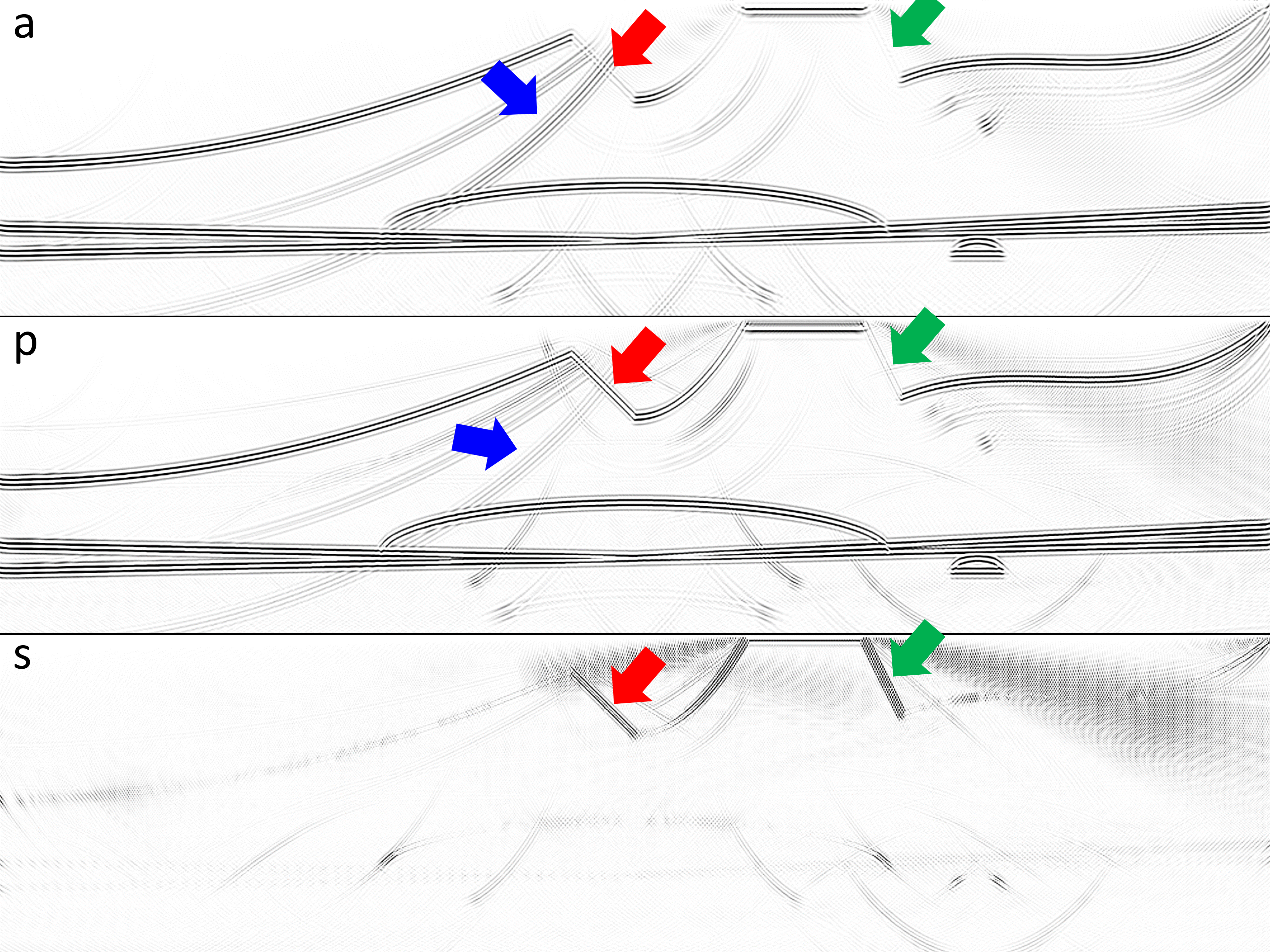

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Results

- Algorithm of elastic migration based on Born approximation was proposed and developed
- It has been shown to locate steep interfaces better than acoustic algorithm and to have less strongly pronounced false boundaries

Media

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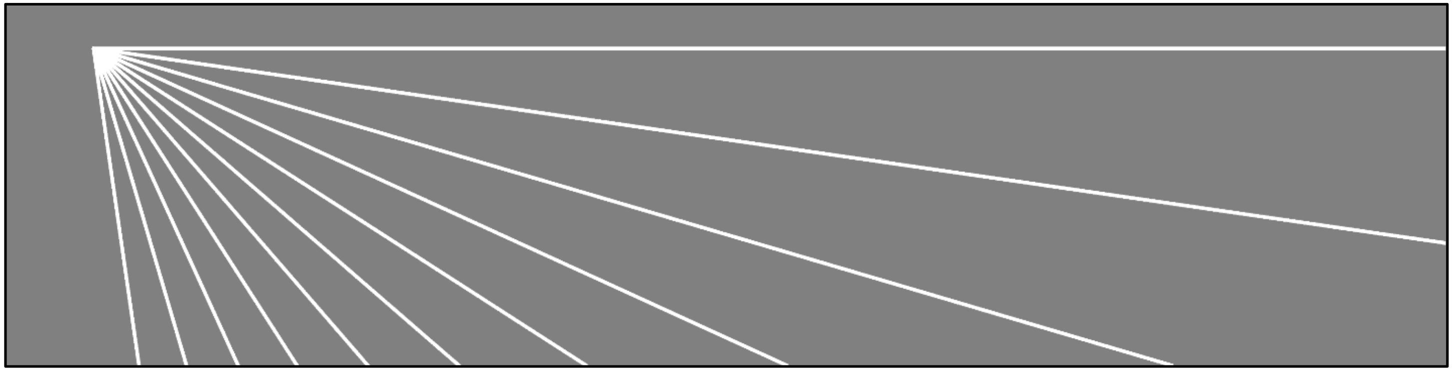

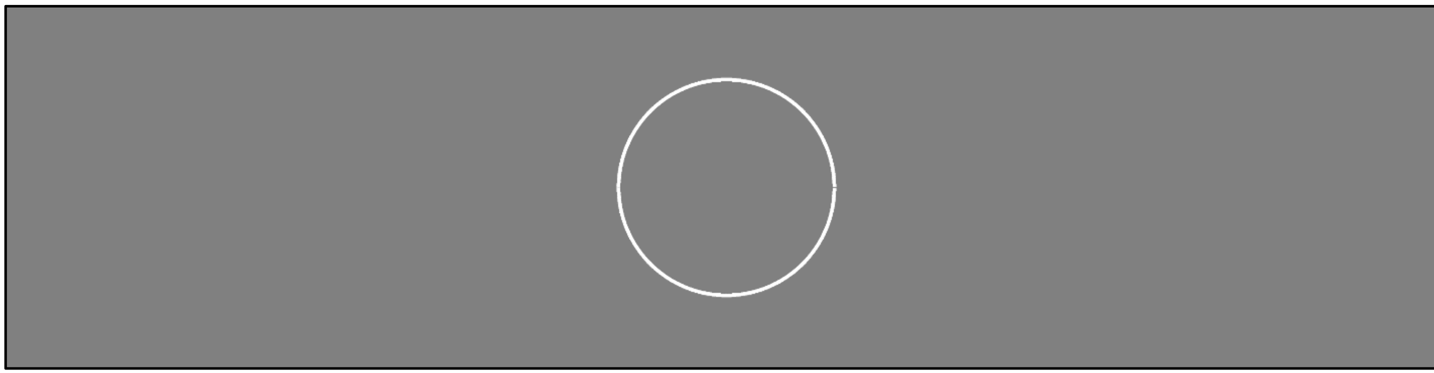

Data

(only z-component of scattered field)
$z = 15$ m,
$\Delta x = 10$ m,
$\Delta t = 2$ ms,
$t \in [0, 4]$ s,
$F(t) =$
$(1 - 2\pi^2 f_M^2 t^2) \cdot$
$\cdot\, e^{-\pi^2 f_M^2 t^2}$,
$f_M = 25$ Hz,
$\vec{f} = (0\ 0\ 1)^T$