Efficient Preconditioning in 3D Marine Electromagnetic Geophysical Modeling

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One of several Marine Electromagnetic (EM) Acquisition Setups

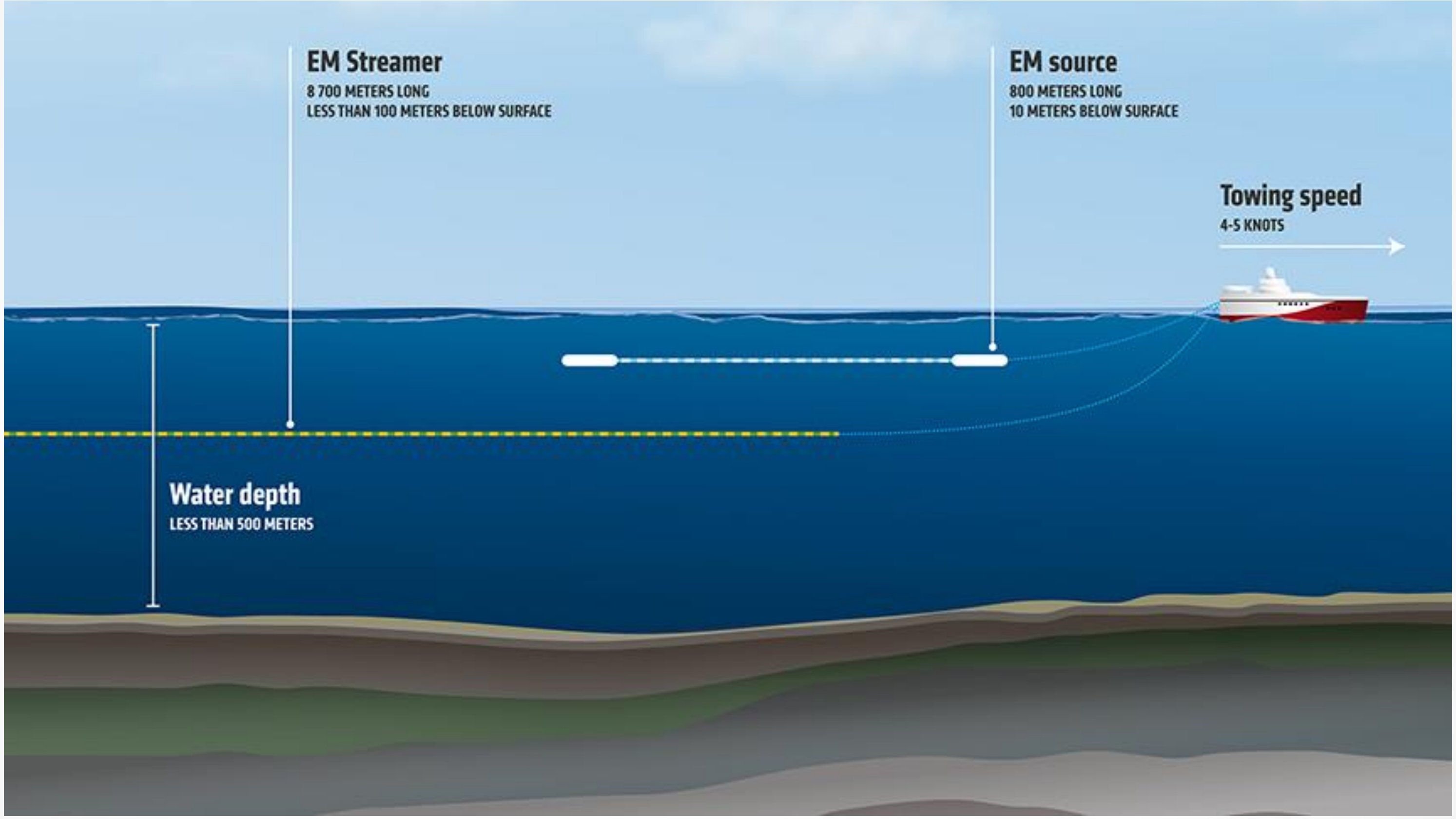
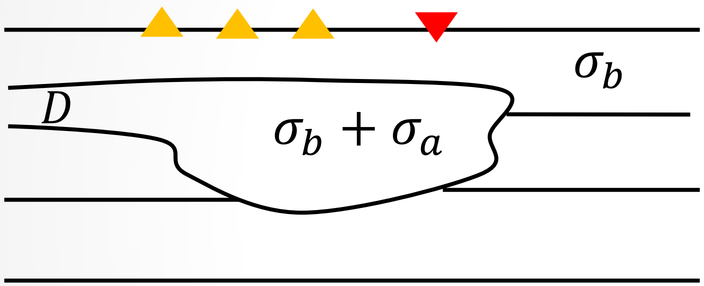


Figure courtesy of pgs.com

To plan a survey and interpret collected data, we have to solve low-frequency Maxwell's equation repeatedly.

Electrical Conductivity Model



$$\sigma = \begin{cases} \sigma_a + \sigma_b & \text{in} \quad D, \\ \sigma_b & \text{in} \ \mathbb{R}^3 \backslash \bar{D}. \end{cases}$$

Let's consider a 3D heterogeneous conductivity Earth model composed of layered background with conductivity $\sigma_b(z)$ and anomalous inclusions (bodies) D with conductivity

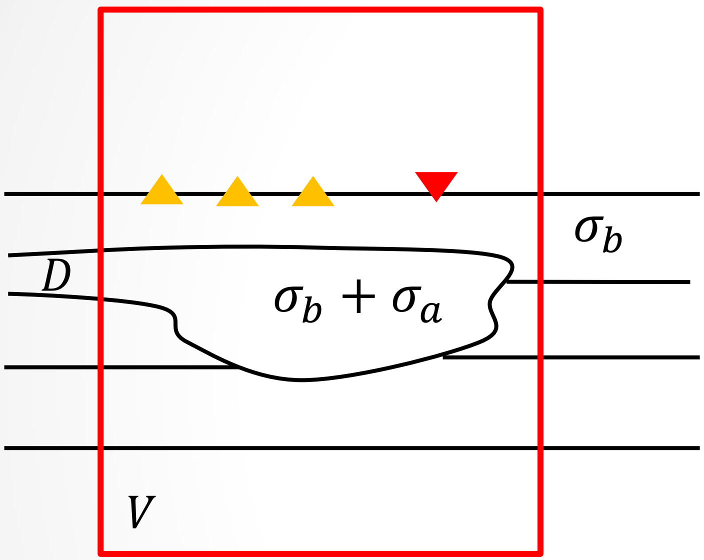
$\sigma_a + \sigma_b$.

To have some simple measure of control on lateral contrast of the model, we assume there exist α and β such that,

$$\alpha \, \sigma_b(z) \leq \ \sigma(x, y, z) \ \leq \beta \, \sigma_b(z)$$
$$0 < \alpha \leq 1 \leq \beta < \infty,$$

This inequality insures that the anomalous inclusions are neither perfect conductors nor insulators.

Low-frequency Maxwell's equations



Within some finite volume V, we formulate the secondary field low-frequency Maxwell's equations:

$$rot\ rot\ E_a - i\omega\mu_0\sigma_b E_a = i\omega\mu_0\sigma_a(E_a + E_b).$$

$$E_a \times \nu = 0.$$

Here E_a is unknown, while layered Earth resposne E_b can be easily computed quasi-analytically.

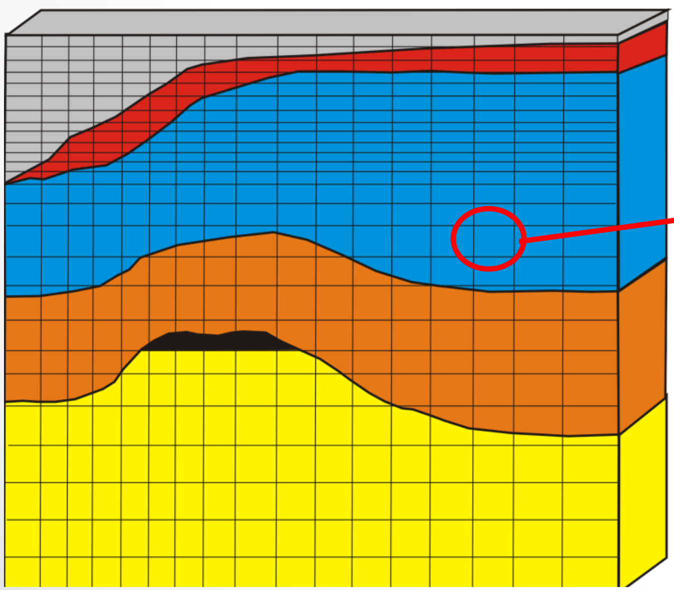
$$E = E_a + E_b$$

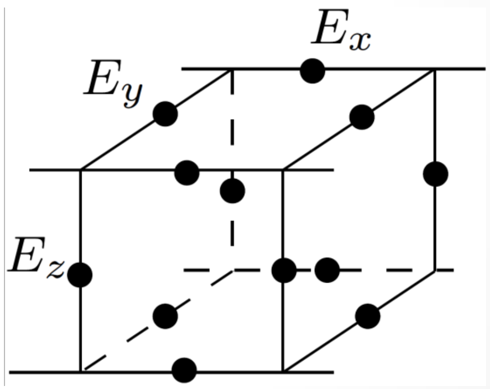
We will discuss efficient solution of the finite-difference (FD) discretization of the later equations.

FD System - 1

We use edge-based electric fields and edge-sampled conductivities on a non-uniform grid, $N_x \times N_y \times N_z$.

The total number of
unknowns is $n \approx 3N_xN_yN_z$.





FD System - 2

We will use the following notations for FD operators and unknowns,

$$E_b \approx e_b,\ E_a \approx e_a$$

$\sigma(x,y,z) \approx \Sigma$, (diagonal matrices)

$$\sigma_b(z) \approx \Sigma_b,\ \sigma_a(x,y,z) \approx \Sigma_a,$$

$$\sigma_{i+\frac{1}{2}\,j\,k} = \frac{h^x_{i+\frac{1}{2}}}{h^y_j\,h^z_k}\left(\int\limits_{x_i}^{x_{i+1}}\left(\int\limits_{z_{k-\frac{1}{2}}}^{z_{k+\frac{1}{2}}}\int\limits_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}\sigma(x,y,z)dydz\right)^{-1}dx\right)^{-1}$$

$$rot\,rot\ - i\omega\mu_0\sigma I \approx A,$$

$$rot\,rot\ - i\omega\mu_0\sigma_b I \approx A_b$$

FD secondary field formulation:

$$Ae_a = i\omega\mu_0\,\Sigma_a e_b \quad \text{or} \quad A_b e_a = i\omega\mu_0\Sigma_a(e_a + e_b).$$

This problems have typically 1 to 10 million unknowns. Their efficient solution is of major importance.

Major Preconditioning Approaches

Sever major approaches are applicable to this problem.

Some of them are

- Geometric and algebraic multigrid,
- ILU, ILUt, etc,
- Domain decomposition methods,
- Discrete separation of variables.

We will base our presentation on discrete separation of variables, since it provides decent spectral properties of the preconditioned problem (will be proved later) and very memory economical.

FD GF Preconditioner – 1

Matrix A_b can be efficiently factorized,

so that complexity to compute $A_b^{-1}u$ is at most $O(N_x N_y N_z (N_x + N_y))$ operations and auxiliary memory $O(n)$.

Consequently, we may used as a preconditioner,

$$A_b^{-1} A\, e_a = i\omega\mu_0\; A_b^{-1}\, \Sigma_a e_b \quad \text{or} \quad e_a = i\omega\mu_0\; A_b^{-1}\, \Sigma_a (e_a + e_b)$$

This is pretty much an equivalent of the IE formulation

since A_b^{-1} is the FD Green's function (GF) of the layered media.

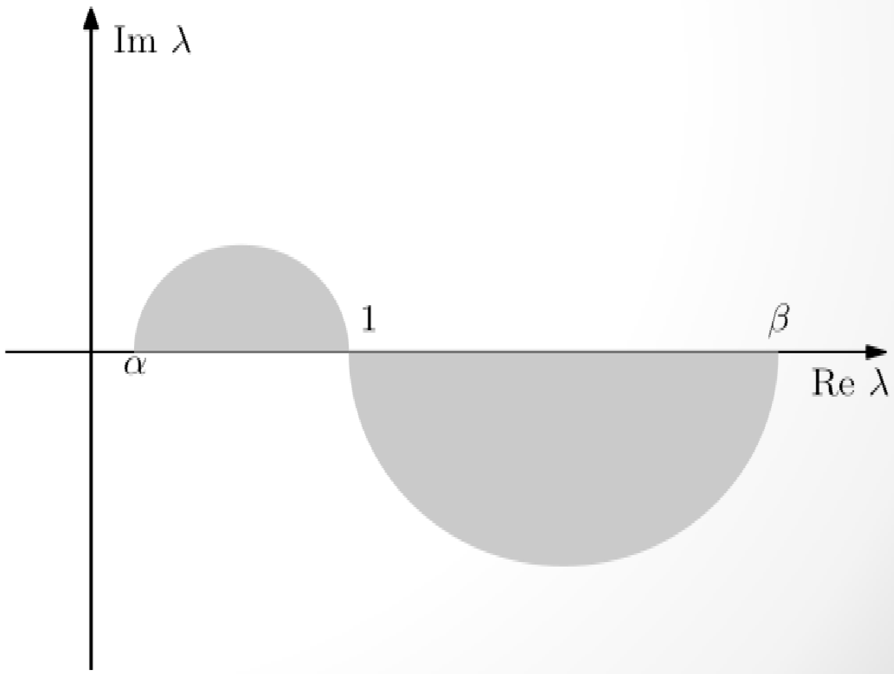
How good will be this preconditioner?

FD GF Preconditioner – 2

We studied the respective eigenvalue problem,

$$A_b^{-1}\, A\, v = \lambda\, v,$$

to understand properties of this preconditioner.



$$cond\left(A_b^{-1}A\right) \approx \frac{|\lambda_{\max}|}{|\lambda_{\min}|} \le \frac{\beta}{\alpha}$$

Contraction Operator – 1

Let us try to improve the later result. Our formulation of the Maxwell's equations imply the energy equality,

$$\int_V \sigma_b |E_a|^2 dV + Re \int_V E_a^* \cdot J_a dV = 0.$$

It also holds at the discrete level,

$$\| \Sigma_b^{\frac{1}{2}} e_a \|^2 + Re(e_a^*, j_a) = 0.$$

The equality can be used to transform the FD system. Introduce,

$$K_1 = \frac{1}{2}(\Sigma + \Sigma_b)\Sigma_b^{-1/2}, \qquad K_2 = \frac{1}{2}(\Sigma - \Sigma_b)\Sigma_b^{-1/2},$$
$$\tilde{e}_a = K_1 e_a.$$

Contraction Operator – 2

Then e_a will satisfy,

$$(I - C)\tilde{e}_a = f,$$

where,

$$C = \left(2i\omega\mu_0\,\Sigma_b^{\frac{1}{2}}\,A_b^{-1}\,\Sigma_b^{\frac{1}{2}} + I\right)K_2K_1^{-1},$$

$$f = i\omega\mu_0\Sigma_b^{\frac{1}{2}}A_b^{-1}\Sigma_a e_b.$$

Interestingly,

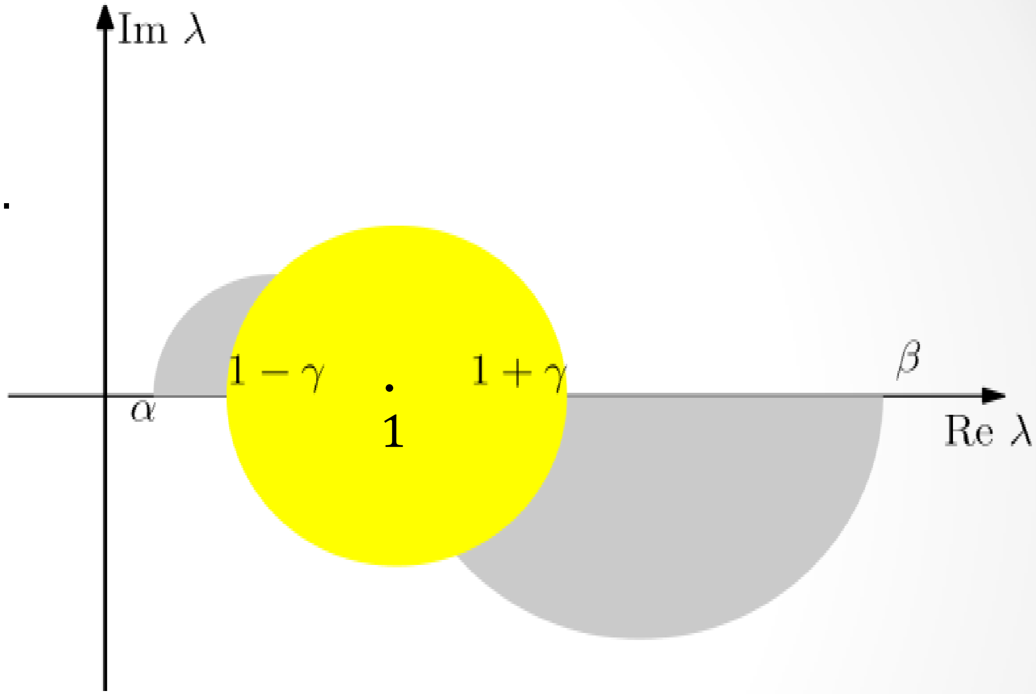
$$\|C\| < 1.$$

Thus we will refer this transformation as the contraction operator (CO) preconditioner.

Contraction Operator – 3

$$(I - C)u = \lambda\, u.$$

$$\|C\| \leq 1 - 2\,\min(\alpha, 1/\beta) =: \gamma.$$

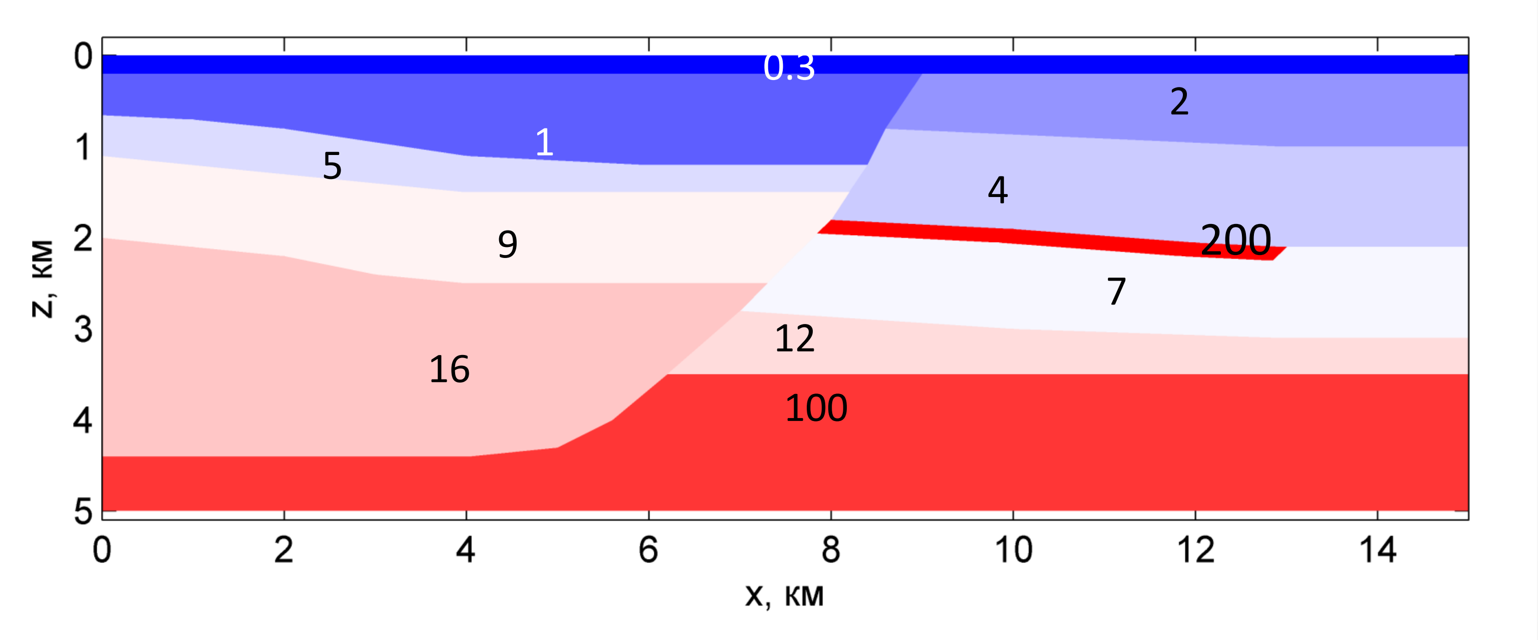


It can be proved,

$$\color{red}{cond\ (I - C) \leq \max(1/\alpha\,, \beta).}$$

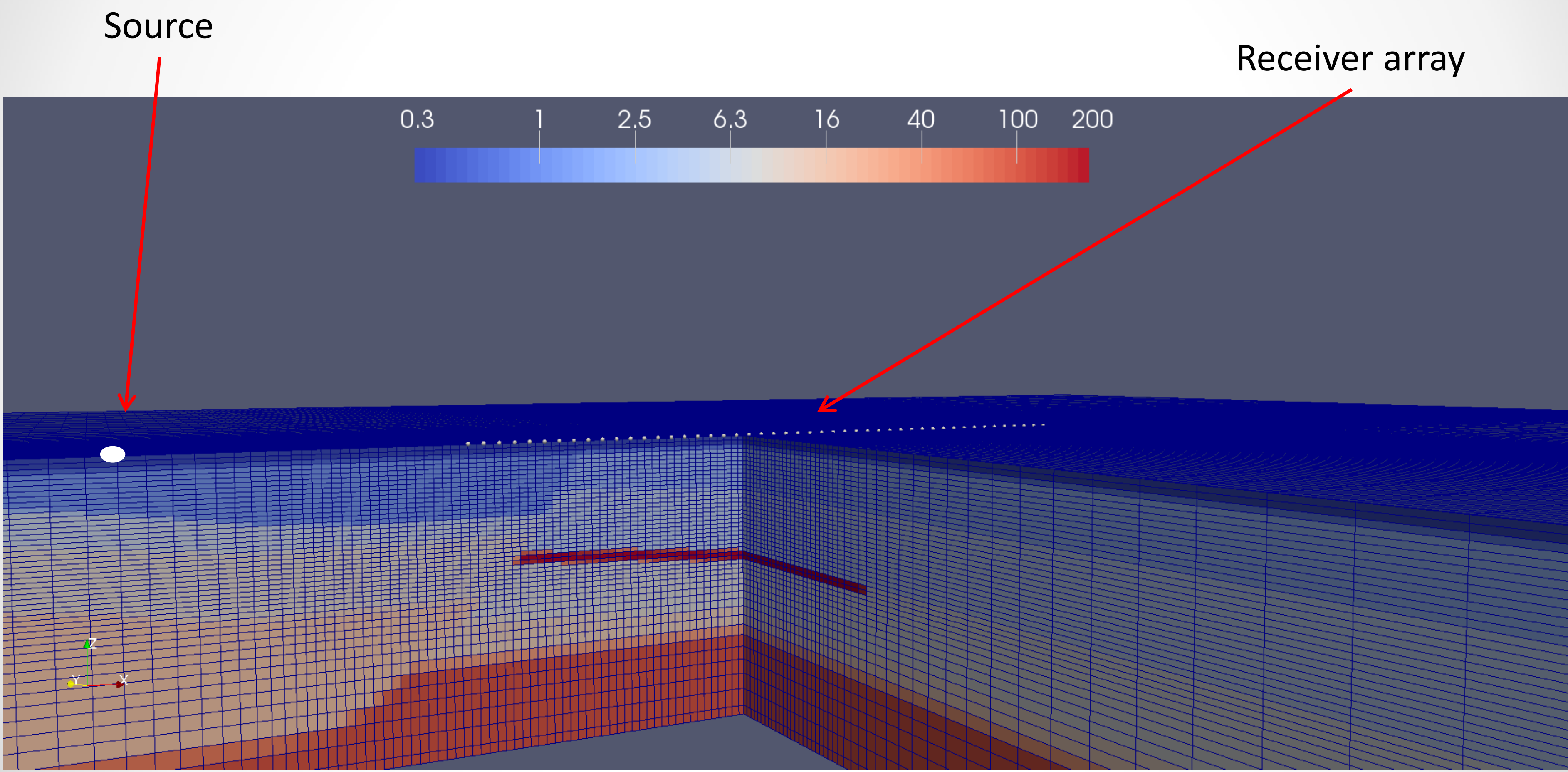
Comparison of the two condition numbers leads us to a conclusion. When the bodies are only resistive <u>or</u> conductive, the covergence of iterative solvers will be similar is similar. In case of resistive <u>and</u> conductive bodies, CO will provide faster convergence.

Marine resistivity model of a hydrocarbon deposit



Ohm·m indicated

Sampled model/ towed source and receiver array



Colors indicate Ohm·m

Performance comparison

172 x 96 x 83 computational grid,

4’034’327 unknowns

BiCGStab, ε=1e-8

Performance of the solver at one of the source positions:

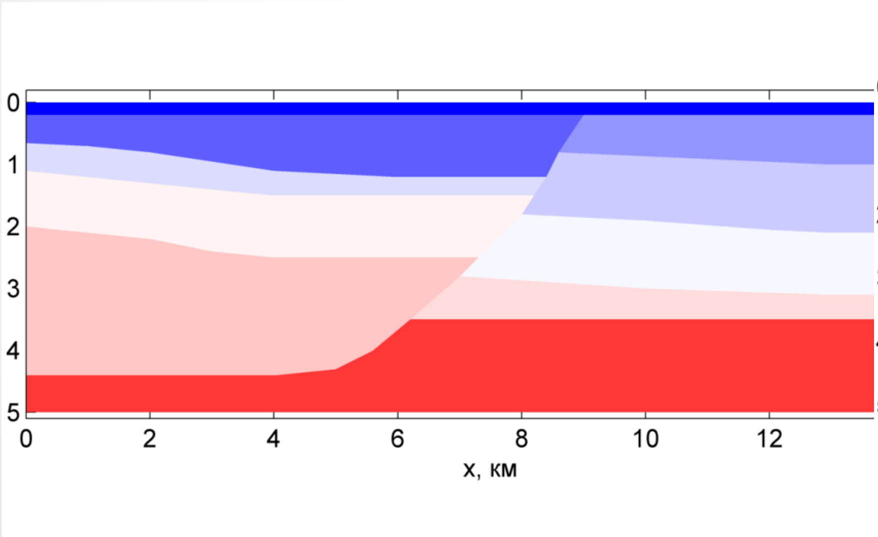
FD GF Preconditioner Iterations/ time, s	Contraction operator Iterations/ time, s
78 / 445	31 / 180

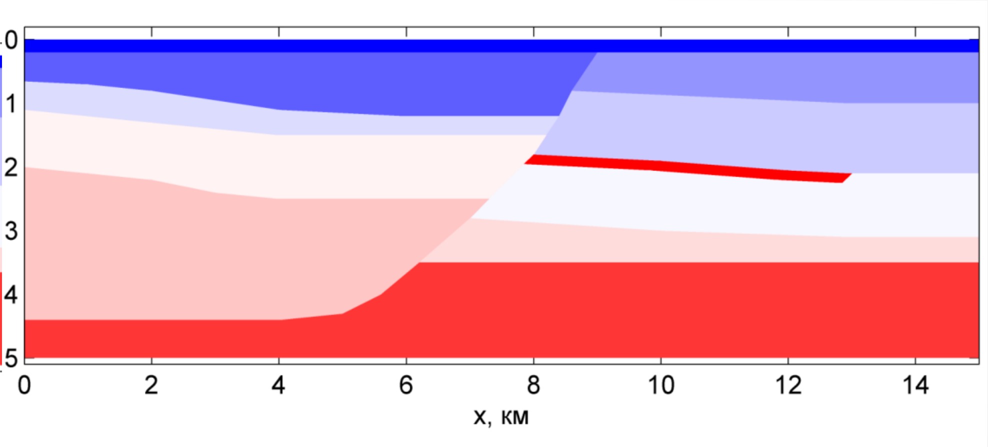
We observed a speed up of 2.5 times!

Towed Streamer Data Sensitivity – 1

We modeled responses at 32 setup locations for models with and without deposit.

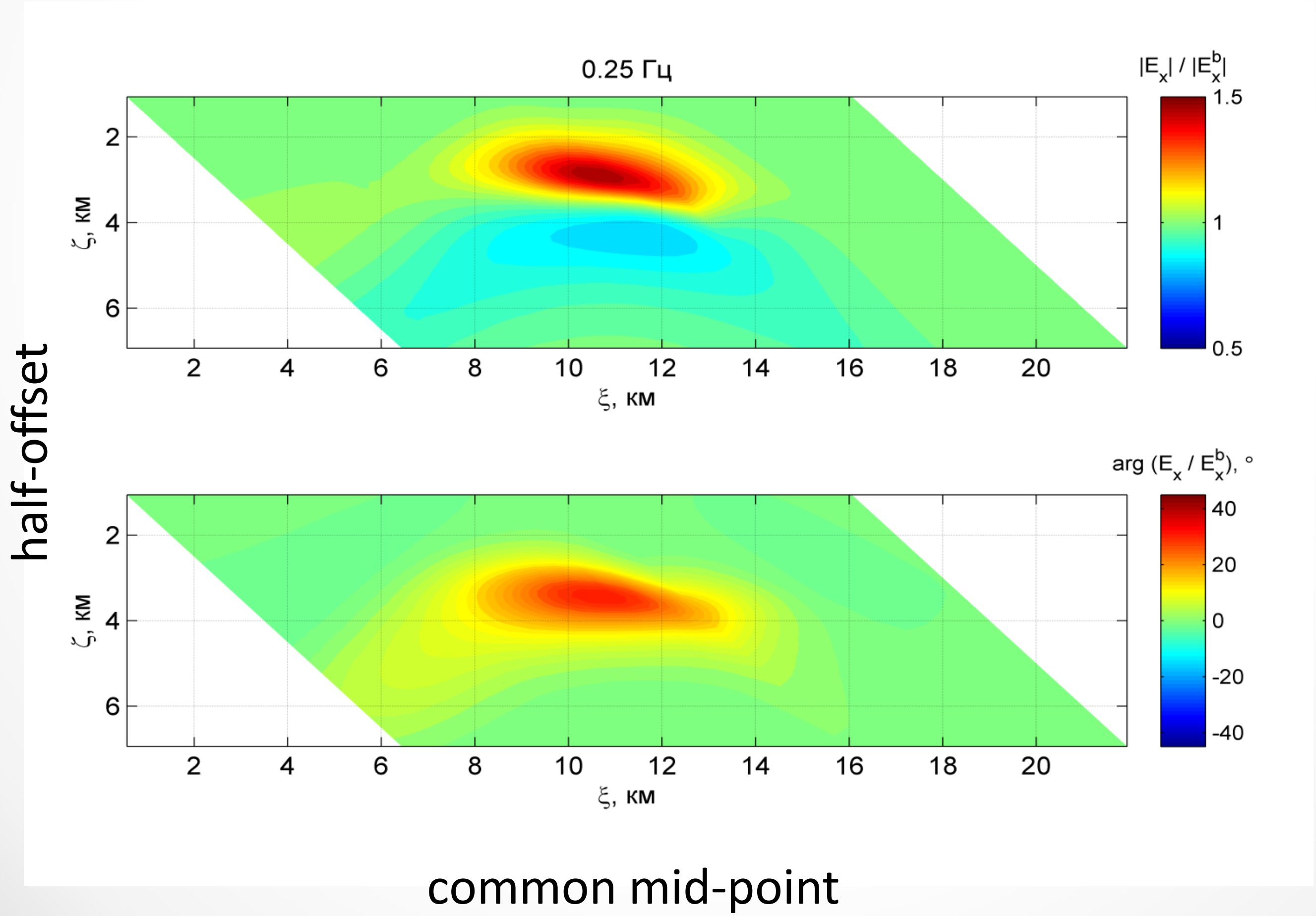
vs.





Towed Streamer Data Sensitivity – 2

Below is the ratio the responses. We good data sensitivity: 48% amplitude anomaly, 32° phase anomaly.



Summary

- We designed, analyzed, and tested two preconditioners for 3D electromagnetic low-frequency modeling.

- Our analysis and tests showed that convergence of iterative solvers applied to CO preconditioned system is faster or same than that applied to GF preconditioned system.

- We also demonstrated applicability of the approaches to marine geophysical EM modeling.