The Use of Fast Automatic Differentiation Technique for Solving Coefficient Inverse Problems

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In studying and modeling heat propagation in complex porous composite materials both the convective and radiative heat transfer must be taken into account. The thermal conductivity coefficients in this case typically depend on the temperature.

To estimate these coefficients, various models of the medium are used. As a result, one has to deal with a complex nonlinear model that describes the heat propagation in the composite material.

However, another approach is possible: a simplified model is constructed in which the radiative heat transfer is not taken into account, but its effect is modeled by an effective thermal conductivity coefficient that is determined based on experimental data.

We consider one possible statement of the inverse coefficient problem. It is considered based on the Dirichlet problem for the one-dimensional unsteady-state heat equation. The inverse coefficient problem is reduced to a variational problem.

The identification problem of the model parameters

Direct problem:

$$\rho C \frac{\partial T(x,t)}{\partial t} - \frac{\partial}{\partial x}\left(K(T) \frac{\partial T(x,t)}{\partial x} \right) = 0, \qquad (x,t) \in Q,$$

$$T(x,0) = w_0(x), \qquad 0 \le x \le L, \qquad\qquad (1)$$

$$T(0,t) = w_1(t), \qquad T(L,t) = w_2(t), \qquad 0 \le t \le \Theta.$$

A layer of material of width L is considered

$Q = \{ (0 < x < L) \times (0 < t \le \Theta) \}$

x - the Cartesian coordinate of the point in the layer

$T(x,t)$ - the temperature of the material at the point with the coordinate x at the time t

ρ and C are the density and the heat capacity of the material, respectively

$K(T)$ - the coefficient of the convective thermal conductivity

$w_0(x)$, $w_1(t)$, $w_2(t)$ - are given

The cost functional :

$$\Phi\big(K(T)\big) = \int_0^{\Theta}\int_0^{L}\left[T(x,t) - \mathrm{Y}(x,t)\right]^2 \mu(x,t)\,dxdt \;+$$

$$+\,\beta\int_0^{\Theta}\left[K\big(T(0,t)\big)\cdot\frac{\partial T}{\partial x}(0,t) - \mathrm{P}(t)\right]^2 dt$$

$\beta \geq 0$ - a given number; $\mu(x,t) \geq 0$ - a given weighting function

$\mathrm{P}(t)$ - the known heat flux on the left boundary of the domain

The optimal control problem is to find the optimal control

$K(T)$ **and the corresponding optimal solution** $T(x,t)$

of problem (1) that minimizes functional $\Phi\big(K(T)\big)$

THE GRADIENT OF FUNCTIONAL IN THE CONTINUOUS CASE

Let: $$a = \min\left\{ \min_{x\in[0,L]} w_0(x),\ \min_{t\in[0,\Theta]} w_1(t),\ \min_{t\in[0,\Theta]} w_2(t) \right\}$$

$$b = \max\left\{ \max_{x\in[0,L]} w_0(x),\ \max_{t\in[0,\Theta]} w_1(t),\ \max_{t\in[0,\Theta]} w_2(t) \right\}$$

$$T(x,t) \in C^2(Q) \cap C^1(\overline{Q})$$

$$G = \left\{ K(z) : K(z) \in C^1([a,b]),\ K(z) > 0,\ z \in [a,b] \right\}$$ - the class of the feasible control functions

$$E(z) = \int_0^z \rho(\zeta) C(\zeta)\, d\zeta \in C^1([a,b])$$ - the specific internal energy

$\delta_z K(z)$ - the variation of the control function

$\delta_{xt} T(x,t)$ - the variation of the phase variable

The Lagrange functional:

$$I = \Phi\left(K(T)\right) + \int_0^{\Theta}\int_0^{L} \left[p\left(x,t\right) \left(\frac{\partial E(T)}{\partial t} - \frac{\partial}{\partial x}\left(K(T)\frac{\partial T(x,t)}{\partial x} \right) \right) \right] dx\, dt$$

$p\left(x,t\right) \in C^{2}(Q) \cap C^{1}(\bar{Q})$ - an arbitrary function

The first variation of I:

$$\delta I = \delta\Phi\left(K(T)\right) + \int_0^{\Theta}\int_0^{L} \left[p\left(x,t\right) \cdot \delta_{xt} \left(\frac{\partial E(T)}{\partial t} - \frac{\partial}{\partial x}\left(K(T)\frac{\partial T(x,t)}{\partial x} \right) \right) \right] dx\, dt$$

Denote: $\Psi(x,t) = K\left(T(x,t)\right) \cdot \frac{\partial T(x,t)}{\partial x}$

The adjoint problem

$$E\big(T(x,t)\big)\cdot\frac{\partial p(x,t)}{\partial t}+K(T)\cdot\frac{\partial^2 p(x,t)}{\partial x^2}=2\mu(x,t)\cdot\big(T(x,t)-\Upsilon(x,t)\big),$$

$$(x,t)\in Q,$$

$$p(x,\Theta)=0, \qquad (0\le x\le L),$$

$$p(0,t)=-2\beta\cdot\big(\psi(0,t)-\mathrm{P}(t)\big), \qquad p(L,t)=0, \qquad (0\le t\le\Theta)$$

The first variation of the Lagrange functional

$$\delta I=\int\limits_Q\!\!\int\left(\frac{\partial p(x,t)}{\partial x}\cdot\frac{\partial T(x,t)}{\partial x}\right)\cdot\delta_z K(T)\,dx\,dt$$

THE GRADIENT OF FUNCTIONAL

$$\nabla I\left(K(T)\right) = \mathrm{M}(T), \quad T \in [a,b]$$

$$\mathrm{M}(\eta) = \sum_i \int_{\xi_{Ini}^{(i)}(\eta)}^{\xi_{Fin}^{(i)}(\eta)} \left(\frac{\partial p\left(x_i(\xi,\eta), t_i(\xi,\eta)\right)}{\partial x_i} \cdot sign\left(- J_i(\xi,\eta)\right) \frac{\partial t_i(\xi,\eta)}{\partial \xi} \right) d\xi,$$

$$Q = \bigcup_i Q_i, \qquad Q_i \cap Q_j = \varnothing, \quad i \neq j$$

$$J_i(\xi,\eta) = \begin{vmatrix} \dfrac{\partial x_i(\xi,\eta)}{\partial \xi} & \dfrac{\partial t_i(\xi,\eta)}{\partial \xi} \\ \dfrac{\partial x_i(\xi,\eta)}{\partial \eta} \, x_\eta & \dfrac{\partial t_i(\xi,\eta)}{\partial \eta} \end{vmatrix}$$

$$x = x_i(\xi,\eta) \qquad t = t_i(\xi,\eta)$$

$\eta = \mathrm{const}$ - the isolines of the field of temperature

$\xi = \mathrm{const}$ - the lines orthogonal to $\eta = \mathrm{const}$

$\left(\eta_{\mathrm{Ini}}^{(i)}, \eta_{\mathrm{Fin}}^{(i)}\right)$ - the interval o variation of the isothrems in Q_i

$\left(\xi_{Ini}^{(i)}(\eta), \eta\right)$, $\left(\xi_{Fin}^{(i)}(\eta), \eta\right)$ - the coordinates of the endpoints of the isotherm $\eta = const \in \left(\eta_{Ini}^{(i)}, \eta_{Fin}^{(i)}\right)$ contained in Q_i

THE DISCRETE OPTIMAL CONTROL PROBLEM

The interval $[a,b]$ is partitioned by the points $\tilde{T}_0 = a,\ \tilde{T}_1,\ \tilde{T}_2, \ldots, \tilde{T}_N = b$ into N parts.

The function $K(T)$ to be found is approximated by a continuous piecewise linear functions with the nodes at the points $\left\{ (\tilde{T}_n,\ k_n) \right\}_{n=0}^{N}$

so that

$$K(T) = k_n + \frac{k_{n+1} - k_n}{\tilde{T}_{n+1} - \tilde{T}_n}(T - \tilde{T}_n), \qquad \tilde{T}_n \le T \le \tilde{T}_{n+1}, \quad (n = 0, \ldots, N-1)$$

$$k_n = K(\tilde{T}_n)$$

THE DISCRETE OPTIMAL CONTROL PROBLEM

The function $K(T) \quad (T \in [a,b])$ was approximated by a continuous piecewise linear function

Nonuniform grid : $\left\{ \tilde{x}_i \right\}_{i=0}^{I} \qquad \left\{ \tilde{t}^{\,j} \right\}_{j=0}^{J} \qquad T_i^j = T(\tilde{x}_i, \tilde{t}^{\,j})$

Finite difference scheme that approximates the direct problem:

$$\rho_i C_i (h_i + h_{i-1}) \cdot T_i^j - \sigma\tau^j \left(K(T_i^j) + K(T_{i+1}^j) \right) \cdot \frac{T_{i+1}^j - T_i^j}{h_i} +$$

$$+ \sigma\tau^j \left(K(T_i^j) + K(T_{i-1}^j) \right) \cdot \frac{T_i^j - T_{i-1}^j}{h_{i-1}} = D_i^j, \qquad i = \overline{1, I-1}, \qquad j = \overline{1, J},$$

$$D_i^j = \rho_i C_i (h_i + h_{i-1}) T_i^{j-1} + (1-\sigma)\tau^j \left(K(T_i^{j-1}) + K(T_{i+1}^{j-1}) \right) \frac{T_{i+1}^{j-1} - T_i^{j-1}}{h_i} -$$

$$- (1-\sigma)\tau^j \left(K(T_i^{j-1}) + K(T_{i-1}^{j-1}) \right) \frac{T_i^{j-1} - T_{i-1}^{j-1}}{h_{i-1}}$$

$$T_i^0 = (w_0)_i, \qquad (i = \overline{0, I}),$$

$$T_0^j = w_1^j, \qquad T_I^j = w_2^j, \qquad (j = \overline{0, J})$$

$$h_i = \tilde{x}_{i+1} - \tilde{x}_i, \quad i = \overline{0, I-1}, \qquad \tau^j = \tilde{t}^{\,j} - \tilde{t}^{\,j-1}, \qquad j = \overline{1, J},$$

Cost functional is approximated by the function $F = F(k_0, k_1, ..., k_N)$ with the aid of the trapezoids method:

$$\Phi\left(K(T)\right) \approx F = \sum_{j=1}^{J}\sum_{i=1}^{I-1}\left((T_i^j - Y_i^j)^2 \cdot \mu_i^j h_i \tau^j\right) +$$

$$\Phi\left(K(T)\right) \approx F = \sum_{j=1}^{J}\sum_{i=1}^{I-1}\left((T_i^j - Y_i^j)^2 \cdot \mu_i^j h_i \tau^j\right) +$$

$$+ \frac{1-\sigma}{2h_0}\left(K(T_0^{j-1}) + K(T_1^{j-1})\right) \cdot \left(T_1^{j-1} - T_0^{j-1}\right) - \frac{\rho_0 C_0 h_0}{2\tau^j}\left(T_0^j - T_0^{j-1}\right) - P^j \Bigg)^2 \cdot \tau^j \Bigg)$$

Fast Automatic Differentiation technique

$$T_i^j = \Psi\left[(i,j), Z_{(i,j)}, U_{(i,j)}\right]$$

$$\frac{\partial F}{\partial k_n} = F_{k_n} + \sum_{(q,v)\in \overline{K}_{(q,v)}} \Psi_{k_n}\left[(q,v), Z_{(q,v)}, U_{(q,v)}\right]\cdot p_q^v$$

$$p_i^j = \sum_{(q,v)\in \overline{Q}_{(q,v)}} \Psi_{T_i^j}\left[(q,v), Z_{(q,v)}, U_{(q,v)}\right]\cdot p_q^v + F_{T_i^j}$$

$$\overline{Q}_{(i,j)} = \left\{ (q,v):\ T_i^j \in Z_{(q,v)} \right\} \qquad \overline{K}_{(i,j)} = \left\{ (q,v): k_n \in U_{(q,v)} \right\}$$

p_i^j - the values of conjugate variables (impulses)

Canonical form of the discrete direct problem:

$$T_i^j = b_i^j\left(K(T_i^j)+K(T_{i+1}^j)\right)\cdot\left(T_{i+1}^j - T_i^j\right) - a_i^j\left(K(T_i^j)+K(T_{i-1}^j)\right)\cdot\left(T_i^j - T_{i-1}^j\right) + T_i^{j-1} +$$

$$+ c_i^j\left(K(T_i^{j-1})+K(T_{i+1}^{j-1})\right)\cdot\left(T_{i+1}^{j-1} - T_i^{j-1}\right) - d_i^j\left(K(T_i^{j-1})+K(T_{i-1}^{j-1})\right)\cdot\left(T_i^{j-1} - T_{i-1}^{j-1}\right) \equiv \psi_i^j,$$

$$a_i^j = \frac{\sigma\tau^j}{\rho_i C_i h_{i-1}(h_i + h_{i-1})} \qquad b_i^j = \frac{\sigma\tau^j}{\rho_i C_i h_i (h_i + h_{i-1})}$$

$$c_i^j = \frac{(1-\sigma)\tau^j}{\rho_i C_i h_i (h_i + h_{i-1})} \qquad d_i^j = \frac{(1-\sigma)\tau^j}{\rho_i C_i h_{i-1}(h_i + h_{i-1})}$$

The discret adjoint problem

$$p_i^j = \left\{ b_{i-1}^j \cdot \left[K(T_{i-1}^j) + W_i^j - K'(T_i^j)\cdot T_{i-1}^j \right] \right\} \cdot p_{i-1}^j +$$

$$+ \left\{ b_i^j \cdot \left[K'(T_i^j)\cdot T_{i+1}^j - W_i^j - K(T_{i+1}^j) \right] - a_i^j \cdot \left[K(T_{i-1}^j) + W_i^j - K'(T_i^j)\cdot T_{i-1}^j \right] \right\} \cdot p_i^j +$$

$$+ \left\{ - a_{i+1}^j \cdot \left[K'(T_i^j)\cdot T_{i+1}^j - W_i^j - K(T_{i+1}^j) \right] \right\} \cdot p_{i+1}^j +$$

$$+ \left\{ c_{i-1}^{j+1} \cdot \left[K(T_{i-1}^j) + W_i^j - K'(T_i^j)\cdot T_{i-1}^j \right] \right\} \cdot p_{i-1}^{j+1} +$$

$$+ \left\{ - a_{i+1}^j \cdot \left[K'(T_i^j)\cdot T_{i+1}^j - W_i^j - K(T_{i+1}^j) \right] \right\} \cdot p_{i+1}^j +$$

$$+ \left\{ c_{i-1}^{j+1} \cdot \left[K(T_{i-1}^j) + W_i^j - K'(T_i^j)\cdot T_{i-1}^j \right] \right\} \cdot p_{i-1}^{j+1} +$$

$$+ \left\{ 1 + c_i^{j+1} \cdot \left[K'(T_i^j)\cdot T_{i+1}^j - W_i^j - K(T_{i+1}^j) \right] \right\} \cdot p_i^{j+1} -$$

$$- \left\{ d_i^{j+1} \cdot \left[K(T_{i-1}^j) + W_i^j - K'(T_i^j)\cdot T_{i-1}^j \right] \right\} \cdot p_i^{j+1} +$$

$$+ \left\{ - d_{i+1}^{j+1} \cdot \left[K'(T_i^j)\cdot T_{i+1}^j - W_i^j - K(T_{i+1}^j) \right] \right\} \cdot p_{i+1}^{j+1} + \frac{\partial F}{\partial T_i^j}$$

$$K'(T_i^j) = \frac{\partial K(T)}{\partial T}(T_i^j) = \begin{cases} \dfrac{k_{n+1} - k_n}{\tilde{T}_{n+1} - \tilde{T}_n}, & \text{если } \tilde{T}_n \le T_i^j \le \tilde{T}_{n+1}, \\ 0, & \text{в противном случае,} \end{cases} \qquad W_i^j = \frac{\partial \left(K(T)\cdot T \right)}{\partial T}(T_i^j) = K'(T_i^j)\cdot T_i^j + K(T_i^j)$$

$$p_i^{J+1} = 0, \quad i = \overline{1, I}, \quad p_0^j = p_I^j = 0, \quad j = \overline{1, J},$$

Gradient of the cost function of the discreet optimal control problem

$$\frac{\partial F}{\partial k_n} = \beta \cdot \sum_{j=0}^{J} \left(\frac{\partial F}{\partial K(T_0^j)} \frac{\partial K(T_0^j)}{\partial k_n} + \frac{\partial F}{\partial K(T_1^j)} \frac{\partial K(T_1^j)}{\partial k_n} \right) + \sum_{m=0}^{J} \sum_{l=0}^{I} \left(\sum_{j=1}^{J} \sum_{i=1}^{I-1} \frac{\partial \psi_i^j}{\partial K(T_l^m)} \cdot p_i^j \right) \cdot \frac{\partial K(T_l^m)}{\partial k_n},$$

$$\frac{\partial F}{\partial K(T_0^j)} = \frac{\partial F}{\partial K(T_1^j)} = A^j \cdot \frac{\sigma}{2h_0} \left(T_1^j - T_0^j \right) + A^{j+1} \cdot \frac{1-\sigma}{2h_0} \left(T_1^j - T_0^j \right)$$

$$A^0 = A^{J+1} = 0$$

$$A^j = 2\tau^j \left(\frac{\sigma}{2h_0} \left(K(T_0^j) + K(T_1^j) \right) \cdot \left(T_1^j - T_0^j \right) + \frac{1-\sigma}{2h_0} \left(K(T_0^{j-1}) + K(T_1^{j-1}) \right) \cdot \left(T_1^{j-1} - T_0^{j-1} \right) \right) -$$

$$- \rho_0 C_0 h_0 \left(T_0^j - T_0^{j-1} \right) - 2\tau^j \mathrm{P}^j \qquad j = 1, \ldots, J$$

$$\frac{\partial K(T_i^j)}{\partial k_n} = \begin{cases} 1 - \dfrac{T_i^j - \tilde{T}_n}{\tilde{T}_{n+1} - \tilde{T}_n}, & \text{если } \tilde{T}_n \le T_i^j \le \tilde{T}_{n+1}, \\ 0, & \text{в противном случае,} \end{cases}$$

$$\frac{\partial K(T_i^j)}{\partial k_{n+1}} = \begin{cases} \dfrac{T_i^j - \tilde{T}_n}{\tilde{T}_{n+1} - \tilde{T}_n}, & \text{если } \tilde{T}_n \le T_i^j \le \tilde{T}_{n+1}, \\ 0, & \text{в противном случае.} \end{cases}$$

The value of the gradient of the objective function, calculated according to these formulas is precise for the selected approximation of the optimal control problem.

The machine time needed for calculation the gradient components using the approach presented here (based on the FAD-methodology) is not more than machine time needed for solving one direct problem.

NUMERICAL RESULTS

The first series of computations

1. $L = 1, \quad \Theta = 1, \quad Q = (0,1) \times (0,1)$

$$\rho(T, x) \equiv C(T, x) \equiv 1, \quad 0 \le x \le 1$$

$$w_0(x) = \sin x, \quad 0 \le x \le 1,$$

$$w_1(t) = 0, \quad 0 \le t \le 1$$

$$w_2(t) = \sin 1 \exp(-4t), \quad 0 \le t \le 1$$

$$\Upsilon(x, t) = \sin x \exp(-4t), \quad (x, t) \in Q$$

$$a = 0, \quad b = \sin 1$$

$\mu(x) \equiv 1 \qquad \beta = 0$ - the field functional

$$K(T) \equiv 4$$

When approximating the "experimental" field of temperatures by its analytical value $\Upsilon_i^j = \Upsilon(\tilde{x}_i, \tilde{t}^{\,j}) = \sin \tilde{x}_i \cdot \exp(-4\tilde{t}^{\,j})$:

$F_{ini} = 1.403072 \cdot 10^{-4}$ $F_{opt} = 3.476330 \cdot 10^{-26}$

$\max|GR_{ini}| = 4.201177 \cdot 10^{-5}$ $\max|GR_{opt}| = 6.061304 \cdot 10^{-17}$

The maximum deviation of the resulting coefficient of thermal conductivity from its analytical value $K(T) \equiv 4$ did not exceed $1.0 \cdot 10^{-5}$

If $\Upsilon_i^j = T_i^j$ - the solution of the direct problem (1) :

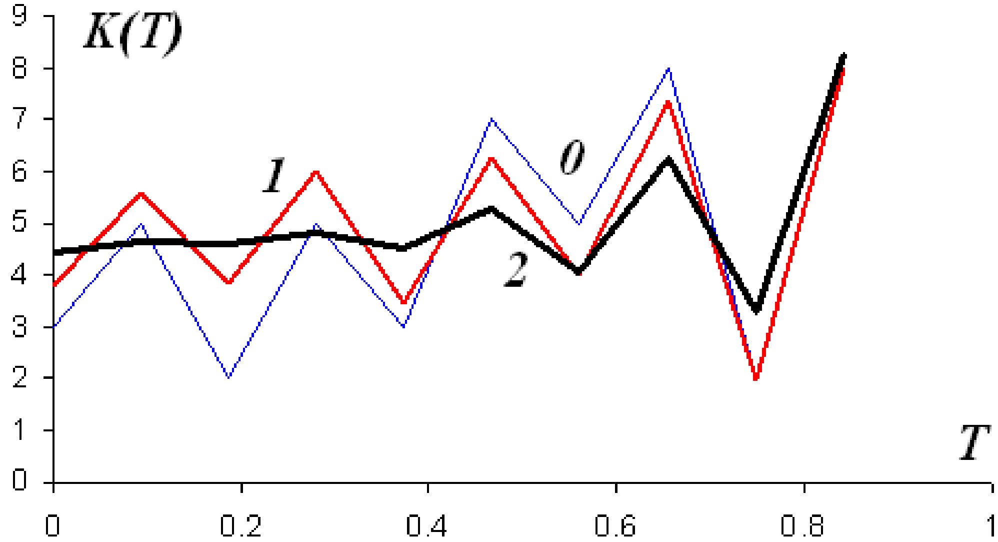
$F_{ini} = 3.0196 \cdot 10^{-2}$ $F_{opt} = 2.9909 \cdot 10^{-26}$

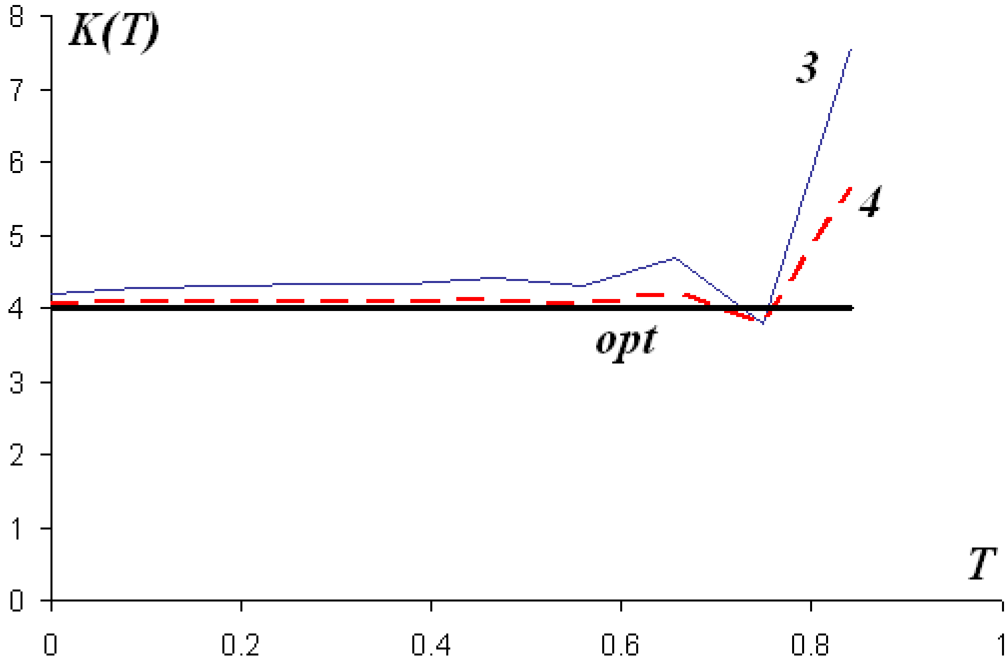
$\max|GR_{ini}| = 5.4852 \cdot 10^{-2}$ $\max|GR_{opt}| = 5.2778 \cdot 10^{-17}$

The coefficient of thermal conductivity coincides with $K(T) \equiv 4$ accurate to the machine precision

The optimal control

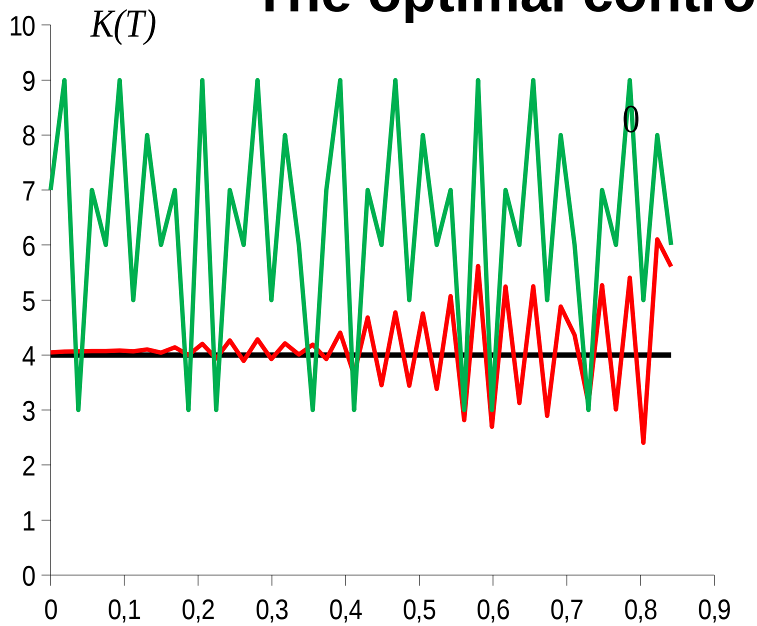
N=9





The optimal control

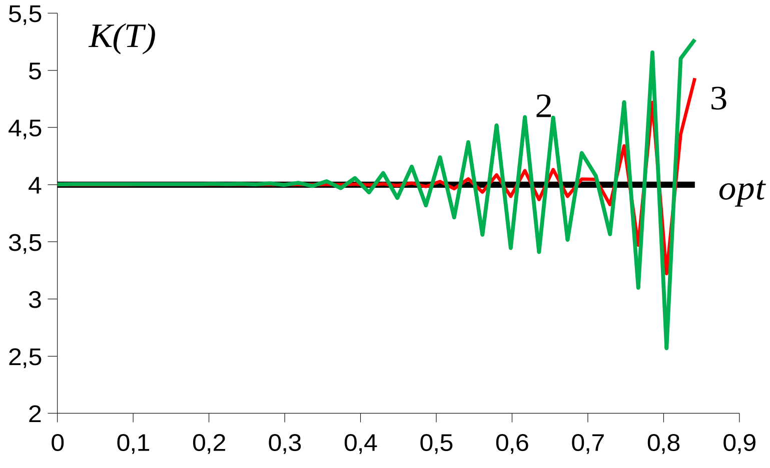
N=45



opt

1

T



T

2.

$$\mu(x) \equiv 0 \qquad \beta = 1$$

The functional is the thermal flux on the boundary

$$K_{ini}(T) = T$$

$$F_{ini} = 1.7408 \qquad F_{opt} = 1.8560 \cdot 10^{-20}$$

$$\max|GR_{ini}| = 6.8059 \cdot 10^{-1} \qquad \max|GR_{opt}| = 8.5559 \cdot 10^{-14}$$

The maximum deviation of the resulting coefficient of thermal conductivity

from its analytical value $K(T) \equiv 4$ did not exceed $3.5 \cdot 10^{-7}$

3.

$\mu(x) \equiv 1$ $\beta = 1$ The "mixed" functional

$$K_{ini}(T) = T$$

$F_{ini} = 1.7710$ $F_{opt} = 6.4531 \cdot 10^{-20}$

$\max|GR_{ini}| = 7.3544 \cdot 10^{-1}$ $\max|GR_{opt}| = 1.9782 \cdot 10^{-13}$

The maximum deviation of the resulting coefficient of thermal conductivity

from its analytical value $K(T) \equiv 4$ did not exceed $6.0 \cdot 10^{-7}$

The second series of computations

The input data of the problem coincide with first exemple except for the following ones:

$$w_0(x) = \sqrt[m]{m(1.5 - x)}, \quad 0 \le x \le 1$$

$$w_1(t) = \sqrt[m]{m(t + 1.5)}, \quad 0 \le t \le 1$$

$$w_2(t) = \sqrt[m]{m(t + 0.5)}, \quad 0 \le t \le 1$$

$$\Upsilon(x, t) = \sqrt[m]{m(t + 1.5 - x)}, \quad x, t \in Q$$

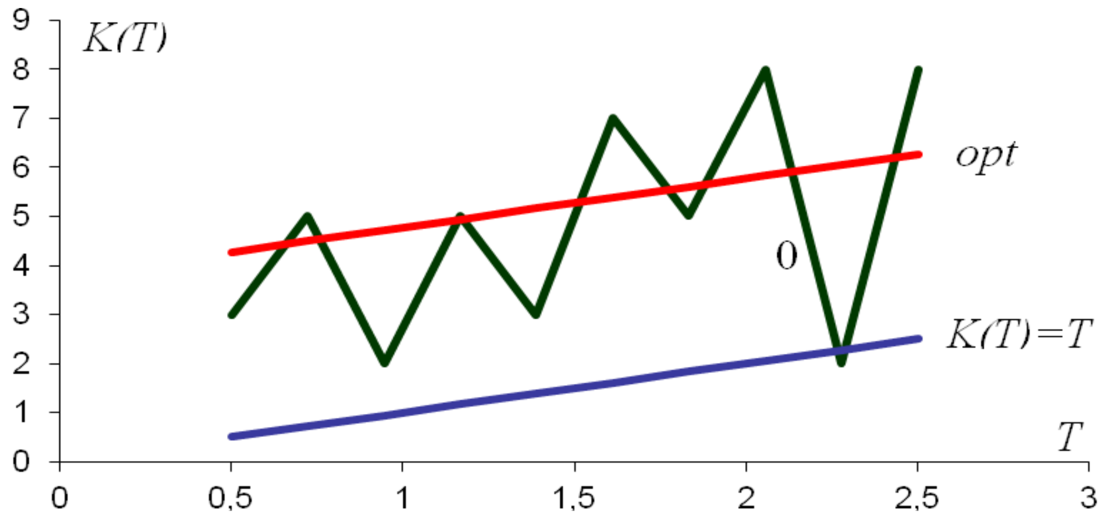
$$a = \sqrt[m]{0.5m}, \quad b = \sqrt[m]{2.5m}$$

$m > 0$ - an arbitrary real number

$$K(T) = T^m$$

1. $m = 1$

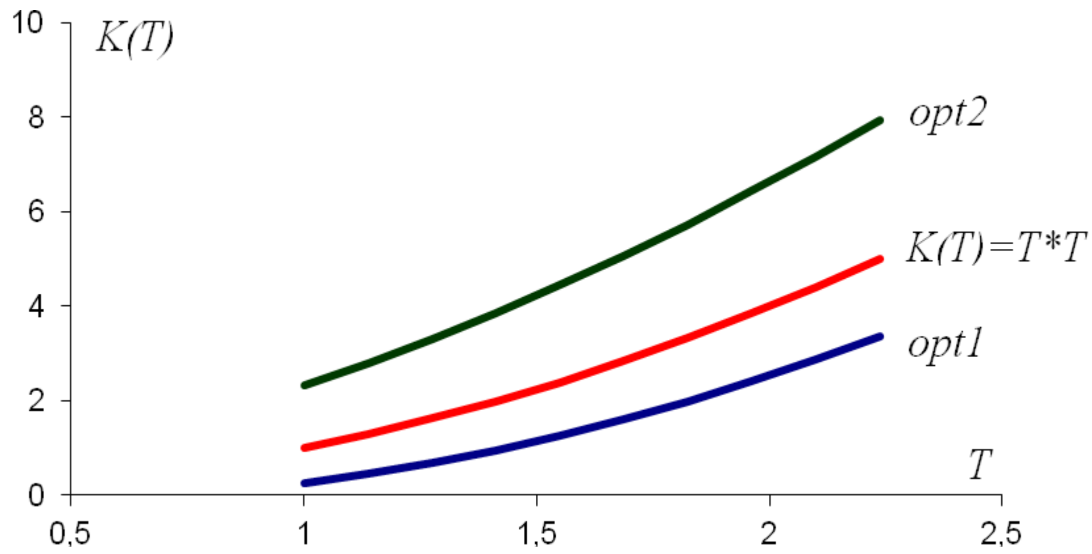
$\mu(x) \equiv 1 \qquad \beta = 0 \qquad$ - the field functional



The solution of the identification problem for the coefficient of convective thermal conductivity is not unique

2. $m = 2$

$\mu(x) \equiv 1$ $\beta = 0$ - the field functional



The solution of the identification problem for the coefficient of convective thermal conductivity is not unique

Necessary conditions for non-uniqueness of solution of the inverse problem

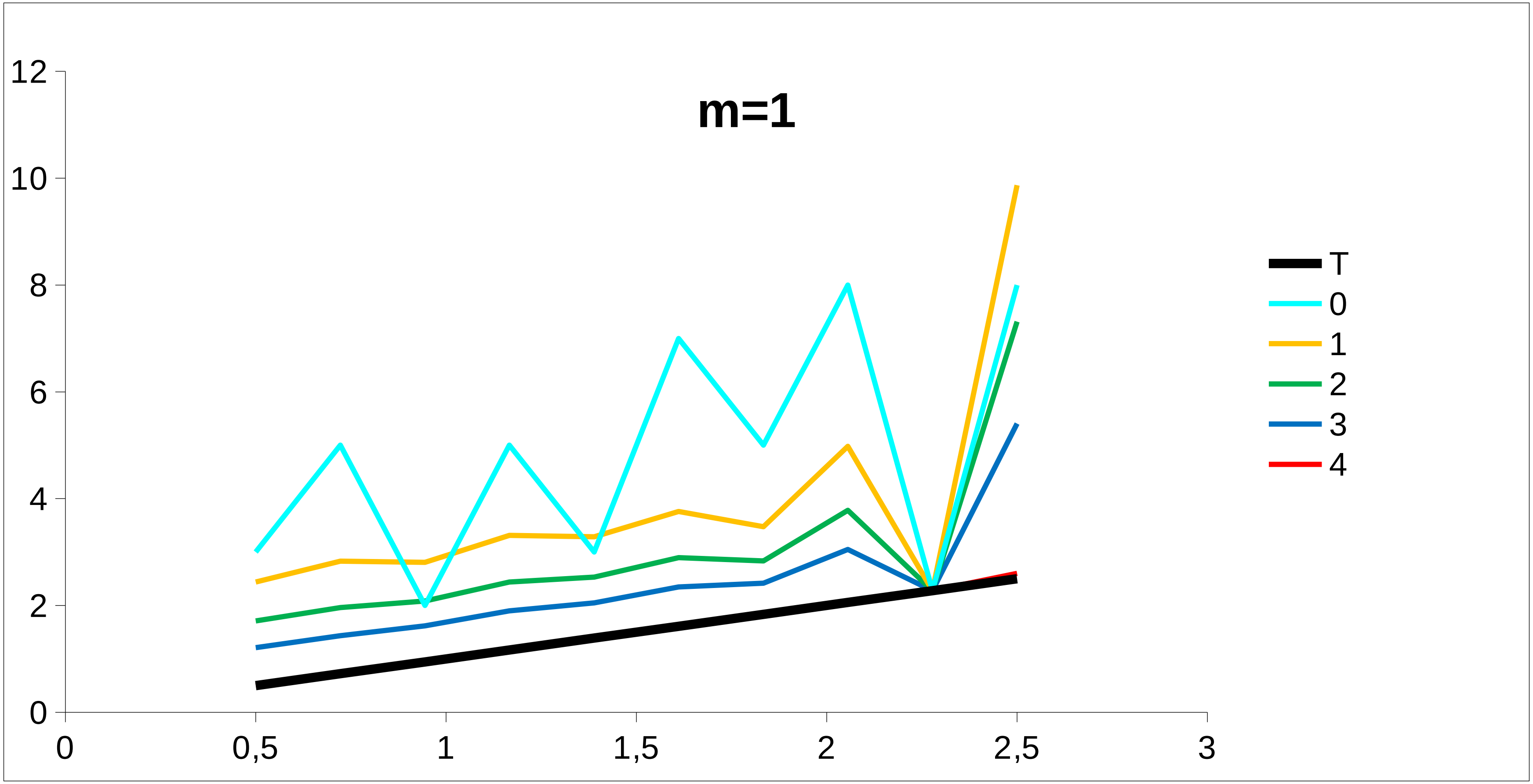
The derivative $x'(t)$ of the isoline $x = x(t)$

is proportional to the derivative $\dfrac{\partial T(x,t)}{\partial x}$, $\quad x'(t) \sim \dfrac{\partial T(x,t)}{\partial x}$

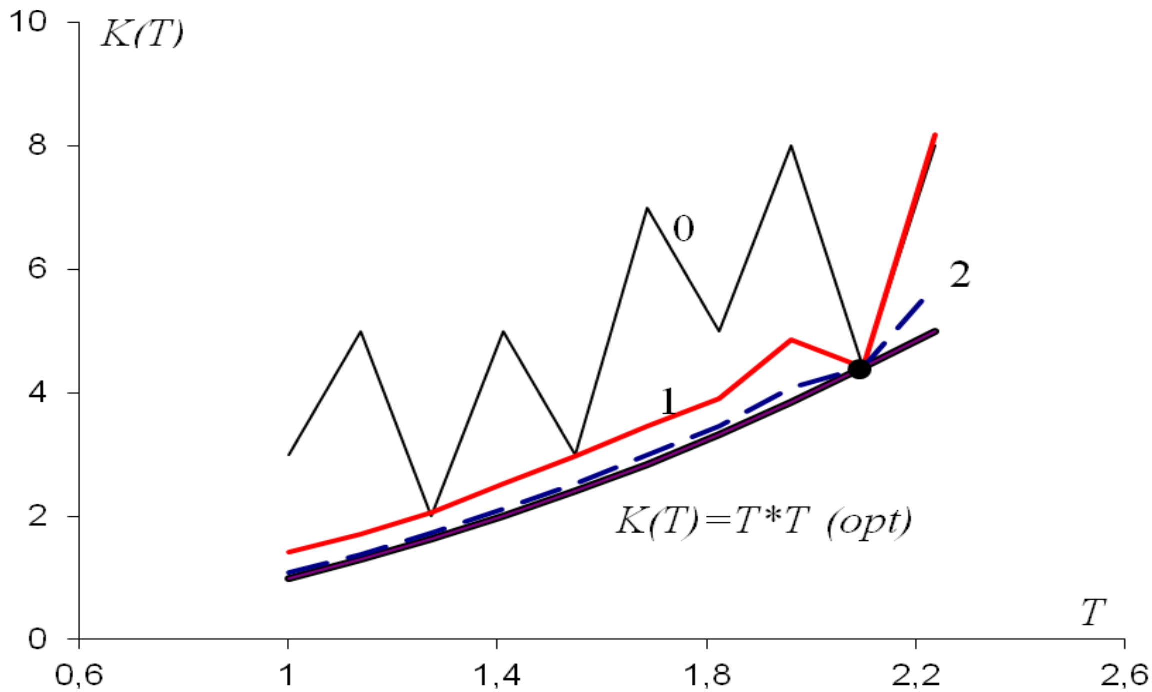
The proportionality coefficient does not change along the isoline and depends only on the temperature $T(x,t)$ on it .

To single out a unique solution of the optimal control problem, we suggest specifying a point T_* at which the thermal conductivity coefficient is known: $K_* = K(T_*)$

If the approximate function $K(T)$ passes through the given point (T_*, K_*) at each step of the minimization process, then the solution to the inverse problem is unique



m=2



the flux functional and the "mixed" functional :

the approximate values of $K(T)$ converged to the limiting function $K_{\text{opt}}(T) = T$

independently of the initial approximation; the solution was unique

The third series of computations

the experimental field does not belong to the reachability domain determined by the controls (thermal conductivity coefficients) from the feasible set

$$\rho(\Upsilon)C(\Upsilon)\frac{\partial\Upsilon(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K_c(\Upsilon)\frac{\partial\Upsilon(x,t)}{\partial x} - q_0(x)\Upsilon^4(x,t)\right) = 0, \quad (x,t)\in Q,$$

$$Q = \{(0 < x < 1)\times(0 < t < 1)\},$$

$$\Upsilon(x,0) = 1, \quad 0 \le x \le 1$$

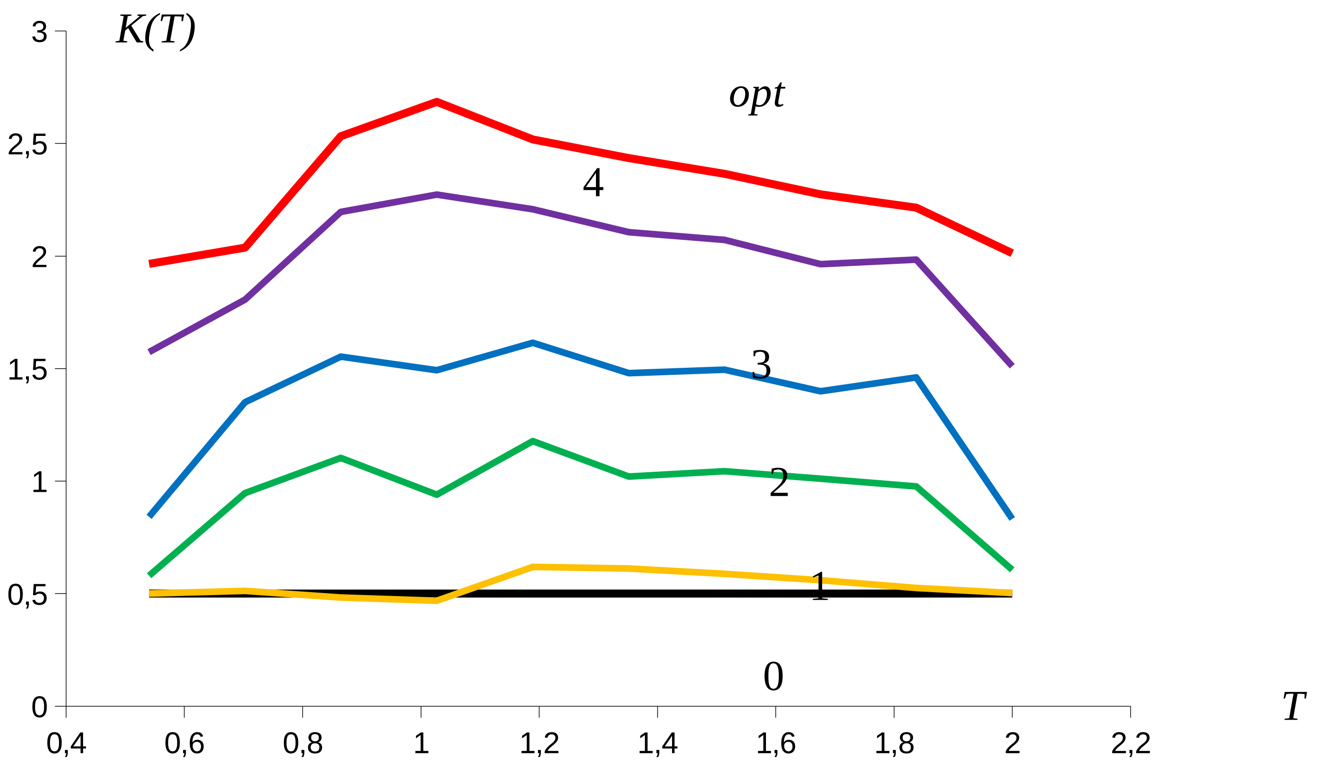
$$\Upsilon(0,t) = \cos t, \quad \Upsilon(L,t) = t+1, \quad 0 \le t \le 1$$

$$K_c(\Upsilon) = 2 + 3\Upsilon, \quad \rho(\Upsilon) = C(\Upsilon) = 1, \quad a \le \Upsilon \le b$$

$$q_0(x) = 0.5, \quad 0 \le x \le 1$$

$K_c(\Upsilon)$ - the coefficient of the convective thermal conductivity

$q_0 = q_0(x)$ - a given function



$\mu(x) \equiv 1 \qquad \beta = 0$ - the field functional

$F_{ini} = 1.0609 \cdot 10^{-3}$ $\qquad$ $F_{opt} = 1.1631 \cdot 10^{-5}$

$\max \left| GR_{ini} \right| = 2.8158 \cdot 10^{-3}$ $\qquad$ $\max \left| GR_{opt} \right| = 1.4103 \cdot 10^{-10}$

CONCLUSIONS

It is recommended to solve the identification problem several (3–4) times choosing each time a different function as the initial approximation.

If this gives the same solution of the optimal control problem, then it may be considered as the solution of the identification problem.

If the solution of the optimal control problem depends on the initial approximation, then an additional condition (e.g., a point at which the thermal conductivity coefficient is known) should be specified in order to solve the identification problem.

Thank you for your attention.