Module 1: Explicit methods

- Description of the prototype code for:

\[
(L) \; v(t, x) = (\partial_t + \frac{1}{2}\partial_{xx})v(t, x) = 0, \quad v(T, x) = g(x) = 1_{\{x \geq 0\}}.
\]

- Exercise 1:
  1. enlarge the domain according to the time to maturity;
  2. extend the code to include the payoff of a call options, i.e. \( g(x) = (e^x - K)_+ \);
  3. check the results for different values of the \( cfl \) parameter;
  4. compare with null boundary condition.

- Exercise 2:
  1. extend to Black and Scholes model with no interest rates (addition of a first-order term):

\[
\mathcal{L} = \partial_t + \frac{\sigma^2}{2}(\partial_{xx} - \partial_x)
\]

- implement the centered explicit scheme and check the results simply monotonicity condition is satisfied;
  2. add the interest rate \( r \) (addition of a zero-order term)

\[
\mathcal{L} = \partial_t + \frac{\sigma^2}{2}(\partial_{xx} - \partial_x) + r(\partial_x - 1);
\]

- implement the upwind scheme and check the results when the simply monotonicity condition is satisfied;
  3. extend the previous codes (both the centered and the upwind explicit schemes) to the CEV local volatility model (variable coefficients), i.e.

\[
\sigma(x) = \sigma e^{(\beta-1)x}, \quad \sigma > 0, \; \beta \in [0, 1].
\]

This model is degenerate, and has unbounded coefficients. Set \( h_0 \) and \( h_1 \) such that the simply monotonicity condition is satisfied for the restriction of the coefficients on the domain \([R_1, R_2]\), and verify the stability of the approximate solutions.