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When I started my research career in mathematical modeling in finance in the 1990s, I was struck by the huge gap between, on one hand, academic seminars in Paris on mathematical finance, which tended to be rather technical and inaccessible to quants and risk managers and, on the other hand, practitioner seminars that discussed many interesting problems but often failed to link them with the vibrant research community in quantitative finance. A few discussions with Stéphane Denise, a friend who was at the time a fixed-income quant, led to the idea of creating a monthly seminar on quantitative finance, which would be an interface between the academic and practitioner community in quantitative finance: the Petit Déjeuner de la Finance. This seminar, which I have been co-organizing since 1998, first with Stéphane Denise and, for the last few years, with Yann Braouezec, has progressively become a platform for exchange of ideas in quantitative finance and risk management between quants, risk managers, and academics, with a long list of speakers, among which are many of the major contributors to the recent developments in quantitative finance. Our recognition goes to those speakers who have made the Petit Déjeuner de la Finance a successful seminar and to the sponsoring institutions that have made this seminar possible through their support.

This volume is a selection of recent contributions from the Petit Déjeuner de la Finance, dealing with topics of current interest in financial engineering, which arrives in time to celebrate the tenth anniversary of the seminar! The contributing authors, leading quants and academic researchers who have contributed to the recent developments in quantitative finance, discuss emerging issues in quantitative finance, with a focus on portfolio credit risk and volatility modeling.

The volume is divided into two parts. The first part (Chapters 1 through 5) deals with advances in option pricing and volatility modeling in the context of equity and index derivatives. The second part (Chapters 6 through 11) covers recent advances in pricing models for portfolio credit derivatives.
OPTION PRICING

Chapter 1 deals with the simplest setting for option pricing, a static one-period model. As shown by Alexandre d’Aspremont, this framework, albeit simple, turns out to be quite rich in mathematical structure, with links to harmonic analysis and semidefinite programming. Aspremont derives necessary and sufficient conditions for the absence of static or buy-and-hold arbitrage opportunities in a one period market by mapping the problem to a generalized moment problem: he shows that this no-arbitrage condition is equivalent to the positive semidefiniteness of matrices formed by the market prices of tradeable securities and their products and applies this result to a market with multiple assets and basket call options.

In Chapter 2, Shalom Benaim, Peter Friz, and Roger Lee survey their recent results on the behavior of the Black-Scholes implied volatility at extreme strikes. These results lead to simple and universal formulae that give quantitative links between tail behavior and moment explosions of the underlying on one hand, and growth of the implied volatility smile on the other hand. In addition to surveying former results of the authors, this chapter also includes original, previously unpublished results on this topic.

The emergence of new volatility instruments—variance swaps—has inspired a renewed interest in volatility modeling. Variance swap markets give direct, observable, quotes on future realized variance as opposed to indirect ones via option prices. In Chapter 3, Lorenzo Bergomi proposes a new class of volatility models, which are based on using the forward variances as state variables and calibrating them to the observable variance swap term structure. This approach, known by now as the Bergomi model, has opened new directions in volatility modeling and enables a meaningful analysis of the volatility exposure of cliquet options and other exotic equity derivatives.

Implied volatility asymptotics is also the focus of Pierre Henry-Labordère’s contribution, in Chapter 4. Henry-Labordère’s results, published here for the first time, enable us to obtain first-order asymptotics for implied volatility for any stochastic volatility model using a geometric approach based on the heat kernel expansion on a Riemannian manifold. This formula is useful for the calibration of such models. Examples that are treated include a SABR model with a mean-reversion term, corresponding in this geometric framework to the Poincaré hyperbolic plane.

Chapter 5 discusses jump-diffusion models, which form another large class of models generalizing the Black-Scholes option pricing model to take into account sudden market moves or “jumps” in prices. Peter Tankov and Ekaterina Voltchkova review important properties of jump-diffusion models and show that these models can be used as a practical tool for option pricing.
and hedging, without dwelling on technical details. After introducing several widely used jump-diffusion models, Tankov and Voltchkova discuss Fourier transform–based methods for European option pricing, partial differential equations for barrier and American options, and the existing approaches to calibration and hedging.

**CREDIT RISK MODELING**

The second part of the book deals with credit risk modeling, a topic which has increasingly occupied the forefront of mathematical modeling in finance in the recent years. Credit risk modeling has witnessed the emergence of a wide variety of approaches that seem to have little in common, both at the single name level—where structural models compete with reduced-form models—and at the level of portfolio credit risk modeling, where “bottom-up” models coexist with “top-down” approaches. This diversity of approaches has created the need for comparative studies of various modeling approaches.

Chris Rogers opens this part with a discussion of the methodology for modeling credit risk in Chapter 6: note that this text, first prepared in 1999, is still relevant in many respects after 10 years and several credit crises! In particular, structural approaches are compared with reduced-form approaches and the choice of state variables is discussed.

The main focus of credit risk modeling in the late decade has been the modeling and pricing of collateralized debt obligations (CDOs), for which static default-time copula models have been frequently used. In Chapter 7, Jean-Paul Laurent and Areski Cousin present a review of factor models for CDO pricing and the link between factor representations and copula representations.

Index CDO markets have led to observable quotes for “default correlation”: practitioners speak of implied correlation “smiles” and “skews.” In Chapter 8, Erik and Lutz Schlögl explore how this analogy can be taken a step further to extract implied factor distributions from the market quotes for synthetic CDO tranches.

In Chapter 9, Julien Turc and Philippe Very also focus on the implied correlation skew: they introduce a local correlation function that makes default correlation dependent on the state of the economy and argue that this allows one to fit the model to the correlation smile and price exotic CDO products consistently with implied correlations.

The dramatic failure of copula-based models in the 2007 market turmoil has led to a renewed interest in dynamic models of credit risk. The last two chapters present material related to dynamic reduced-form models
of portfolio credit risk that are, arguably, more amenable to pricing and simulation.

Dynamic reduced-form models of portfolio credit risk can be divided into two categories: in a bottom-up model, the portfolio intensity is an aggregate of the constituent intensities, while in a top-down model, the portfolio intensity is specified as the intensity of the aggregate loss process. In Chapter 10, Kay Giesecke, who has been among the active contributors to the literature on top-down pricing models, compares these approaches, emphasizing the role of the information (filtration) in the modeling process.

One of the issues in the pricing of portfolio credit derivatives has been the numerical obstacles that arise in the computation of quantities of interest such as CDO tranche spreads and sensitivities, for which the main approach has been quadrature methods and Monte Carlo simulation. In Chapter 11, Cont and Savescu introduce an alternative approach for computing the values of CDO tranche spreads, based on the solution of a system of ordinary differential equations. This approach, which is the analog for portfolio credit derivatives of Dupire’s famous equation for call option prices, allows one to efficiently price CDOs and other portfolio credit derivatives without Monte Carlo simulation. This equation can also be used as a first ingredient in efficient calibration of top-down pricing models.

Special thanks go to Stéphane Denise and Yann Braouezec, with whom I have had the pleasure of organizing the Petit Déjeuner de la Finance for the past 10 years. I also thank Jennifer McDonald and Caitlin Cornish from Wiley for encouraging this project and for their patience and support.

We hope that this timely volume will be useful to readers eager to know about current developments in quantitative finance. Enjoy!

RAMA CONT
New York, March 2008
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