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Frontiers In Quantitative Finance

Volatility and Credit Risk Modeling

RAMA CONT

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Preface

When I started my research career in mathematical modeling in finance in the 1990s, I was struck by the huge gap between, on one hand, academic seminars in Paris on mathematical finance, which tended to be rather technical and inaccessible to quants and risk managers and, on the other hand, practitioner seminars that discussed many interesting problems but often failed to link them with the vibrant research community in quantitative finance. A few discussions with Stéphane Denise, a friend who was at the time a fixed-income quant, led to the idea of creating a monthly seminar on quantitative finance, which would be an interface between the academic and practitioner community in quantitative finance: the *Petit Déjeuner de la Finance*. This seminar, which I have been co-organizing since 1998, first with Stéphane Denise and, for the last few years, with Yann Braouezec, has progressively become a platform for exchange of ideas in quantitative finance and risk management between quants, risk managers, and academics, with a long list of speakers, among which are many of the major contributors to the recent developments in quantitative finance. Our recognition goes to those speakers who have made the *Petit Déjeuner de la Finance* a successful seminar and to the sponsoring institutions that have made this seminar possible through their support.

This volume is a selection of recent contributions from the *Petit Déjeuner de la Finance*, dealing with topics of current interest in financial engineering, which arrives in time to celebrate the tenth anniversary of the seminar! The contributing authors, leading quants and academic researchers who have contributed to the recent developments in quantitative finance, discuss emerging issues in quantitative finance, with a focus on portfolio credit risk and volatility modeling.

The volume is divided into two parts. The first part (Chapters 1 through 5) deals with advances in option pricing and volatility modeling in the context of equity and index derivatives. The second part (Chapters 6 through 11) covers recent advances in pricing models for portfolio credit derivatives.

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OPTION PRICING

Chapter 1 deals with the simplest setting for option pricing, a static one-period model. As shown by Alexandre d'Aspremont, this framework, albeit simple, turns out to be quite rich in mathematical structure, with links to harmonic analysis and semidefinite programming. Aspremont derives necessary and sufficient conditions for the absence of static or buy-and-hold arbitrage opportunities in a one period market by mapping the problem to a generalized moment problem: he shows that this no-arbitrage condition is equivalent to the positive semidefiniteness of matrices formed by the market prices of tradeable securities and their products and applies this result to a market with multiple assets and basket call options.

In Chapter 2, Shalom Benaim, Peter Friz, and Roger Lee survey their recent results on the behavior of the Black-Scholes implied volatility at extreme strikes. These results lead to simple and universal formulae that give quantitative links between tail behavior and moment explosions of the underlying on one hand, and growth of the implied volatility smile on the other hand. In addition to surveying former results of the authors, this chapter also includes original, previously unpublished results on this topic.

The emergence of new volatility instruments—variance swaps—has inspired a renewed interest in volatility modeling. Variance swap markets give direct, observable, quotes on future realized variance as opposed to indirect ones via option prices. In Chapter 3, Lorenzo Bergomi proposes a new class of volatility models, which are based on using the forward variances as state variables and calibrating them to the observable variance swap term structure. This approach, known by now as the Bergomi model, has opened new directions in volatility modeling and enables a meaningful analysis of the volatility exposure of cliquet options and other exotic equity derivatives.

Implied volatility asymptotics is also the focus of Pierre Henry-Labordère's contribution, in Chapter 4. Henry-Labordère's results, published here for the first time, enable us to obtain first-order asymptotics for implied volatility for any stochastic volatility model using a geometric approach based on the heat kernel expansion on a Riemannian manifold. This formula is useful for the calibration of such models. Examples that are treated include a *SABR model with a mean-reversion term*, corresponding in this geometric framework to the Poincaré hyperbolic plane.

Chapter 5 discusses jump-diffusion models, which form another large class of models generalizing the Black-Scholes option pricing model to take into account sudden market moves or "jumps" in prices. Peter Tankov and Ekaterina Voltchkova review important properties of jump-diffusion models and show that these models can be used as a practical tool for option pricing

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and hedging, without dwelling on technical details. After introducing several widely used jump-diffusion models, Tankov and Voltchkova discuss Fourier transform–based methods for European option pricing, partial differential equations for barrier and American options, and the existing approaches to calibration and hedging.

CREDIT RISK MODELING

The second part of the book deals with credit risk modeling, a topic which has increasingly occupied the forefront of mathematical modeling in finance in the recent years. Credit risk modeling has witnessed the emergence of a wide variety of approaches that seem to have little in common, both at the single name level—where structural models compete with reduced-form models—and at the level of portfolio credit risk modeling, where "bottom-up" models coexist with "top-down" approaches. This diversity of approaches has created the need for comparative studies of various modeling approaches.

Chris Rogers opens this part with a discussion of the methodology for modeling credit risk in Chapter 6: note that this text, first prepared in 1999, is still relevant in many respects after 10 years and several credit crises! In particular, structural approaches are compared with reduced-form approaches and the choice of state variables is discussed.

The main focus of credit risk modeling in the late decade has been the modeling and pricing of collateralized debt obligations (CDOs), for which static default-time copula models have been frequently used. In Chapter 7, Jean-Paul Laurent and Areski Cousin present a review of factor models for CDO pricing and the link between factor representations and copula representations.

Index CDO markets have led to observable quotes for "default correlation": practitioners speak of implied correlation "smiles" and "skews." In Chapter 8, Erik and Lütz Schlögl explore how this analogy can be taken a step further to extract implied factor distributions from the market quotes for synthetic CDO tranches.

In Chapter 9, Julien Turc and Philippe Very also focus on the implied correlation skew: they introduce a local correlation function that makes default correlation dependent on the state of the economy and argue that this allows one to fit the model to the correlation smile and price exotic CDO products consistently with implied correlations.

The dramatic failure of copula-based models in the 2007 market turmoil has led to a renewed interest in dynamic models of credit risk. The last two chapters present material related to dynamic reduced-form models

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of portfolio credit risk that are, arguably, more amenable to pricing and simulation.

Dynamic reduced-form models of portfolio credit risk can be divided into two categories: in a *bottom-up* model, the portfolio intensity is an aggregate of the constituent intensities, while in a *top-down* model, the portfolio intensity is specified as the intensity of the aggregate loss process. In Chapter 10, Kay Giesecke, who has been among the active contributors to the literature on top-down pricing models, compares these approaches, emphasizing the role of the information (filtration) in the modeling process.

One of the issues in the pricing of portfolio credit derivatives has been the numerical obstacles that arise in the computation of quantities of interest such as CDO tranche spreads and sensitivities, for which the main approach has been quadrature methods and Monte Carlo simulation. In Chapter 11, Cont and Savescu introduce an alternative approach for computing the values of CDO tranche spreads, based on the solution of a system of ordinary differential equations. This approach, which is the analog for portfolio credit derivatives of Dupire's famous equation for call option prices, allows one to efficiently price CDOs and other portfolio credit derivatives without Monte Carlo simulation. This equation can also be used as a first ingredient in efficient calibration of top-down pricing models.

Special thanks go to Stéphane Denise and Yann Braouezec, with whom I have had the pleasure of organizing the *Petit Déjeuner de la Finance* for the past 10 years. I also thank Jennifer McDonald and Caitlin Cornish from Wiley for encouraging this project and for their patience and support.

We hope that this timely volume will be useful to readers eager to know about current developments in quantitative finance. Enjoy!

RAMA CONT
New York, March 2008

About the Editor

Rama Cont is associate professor in the School of Engineering and Applied Science and director of the Center for Financial Engineering at Columbia University (New York), Senior Research Scientist in Mathematics at the Centre National de Recherche Scientifique (CNRS, France), and founding partner of *Finance Concepts*, a risk management advisory based in Paris and New York. He is a coauthor of *Financial Modelling with Jump Processes* (CRC Press, 2003) and *Credit Derivatives and Structured Credit* (John Wiley and Sons, Inc., 2005).

He has been co-organizing since 1998 the *Petit Déjeuner de la Finance* seminar in Paris, from which this volume originated.

About the Contributors

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Lorenzo Bergomi is head of equity derivatives research at Société Générale. After receiving his PhD in theoretical physics in the theory group at CEA, Saclay, Lorenzo spent two years in the physics department at MIT before joining SG in 1997 as a quantitative analyst on the exotics desk. He now heads a research team on equity derivatives at Société Generale, which currently focuses on models and algorithms for exotics, proprietary trading strategies, credit/equity models.

Areski Cousin is a PhD candidate at Institut de Sciences Financieres et Actuarielles (ISFA) actuarial school, University of Lyon. He holds an engineer's degree from Ecole Nationale Superieure d'Informatique et Mathematiques Appliquees de Grenoble (ENSIMAG), France, and a master's degree in actuarial sciences from ISFA, France. His research focuses on dependence modeling of credit portfolios, risk management of CDOs within factor models, and hedging issues for CDOs.

Peter Friz is a reader at the Statistical Laboratory, Cambridge University. Before joining Cambridge, he worked in Jim Gatheral's group at Merrill Lynch, New York, for a year. He holds a PhD from the Courant Institute of Mathematical Sciences (New York University) and continues to work in quantitative finance and stochastic analysis. His recent research in finance includes applications of diusion short-time asymptotics to basket options, pricing of volatility derivatives as inverse problem, Malliavin calculus for jump diusions, and smile asymptotics.

Kay Giesecke is an assistant professor of management science and engineering at Stanford University. Kay's research and teaching address the quantitative modeling of financial risk, in particular credit risk. Kay has served as a

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consultant to financial institutions and the European Commission. Prior to joining Stanford in 2005, he taught financial engineering at Cornell University's School of Operations Research and Information Engineering. He holds an MSc in electrical engineering and economics and a PhD in economics.

Pierre Henry-Labordère works with the equity derivatives quantitative research team at Société Générale as a quantitative analyst. After receiving an engineering degree from Ecole Centrale de Paris and a PhD at Ecole Normale Supérieure (Paris) in superstring theory, he worked in the Theoretical Physics Department at Imperial College (London) and subsequently at Barclays Capital before joining SocGen.

Jean Paul Laurent is professor of finance at Institut des Sciences Financières et Actuarielles (ISFA), Université Claude Bernard (Lyon, France). He is also a consultant with BNP Paribas.

Roger Lee is an assistant professor of mathematics at the University of Chicago, and was previously a member of the mathematics faculty at Stanford University and a visiting member of the Courant Institute at NYU. His research topics include asymptotics of implied volatility; analytic and computational aspects of Fourier methods for option pricing; and robust hedging of path-dependent derivatives, including derivatives on realized volatility.

Chris Rogers holds the chair of statistical science at Cambridge University and leads the quantitative finance group in the Statistical Laboratory. Chris works in the theory of probability and its applications, particularly in quantitative finance. His work in finance includes the potential approach to the term structure of interest rates, complete models of stochastic volatility, portfolio turnpike theorems, improved binomial pricing, robust hedging, liquidity modeling, axiomatics of valuation operators, the equity premium puzzle, duality in optimal investment/consumption, and Monte Carlo valuation of American options. He is coauthor, with Professor David Williams, of the two-volume classic work *Diffusions, Markov Processes, and Martingales*.

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Lutz Schlögl is an executive director at Lehman Brothers in London and responsible for quantitative credit research, ABS modeling, and credit strategy in Europe, as well as global synthetic CDO modeling. Having joined Lehman as an associate in 2000, his research has been focused on the development of models for the pricing and risk management of exotic credit derivatives, in particular correlation products such as default baskets, synthetic CDO tranches, and CDO^2. He holds a PhD in financial economics from the University of Bonn.

Peter Tankov is assistant professor at Paris VII University, where he teaches mathematical finance. He holds a doctoral degree in applied mathematics from Ecole Polytechnique (France) and a bachelor's degree from the University of St. Petersburg. His research focuses on copulas and dependency modeling, applications of Lévy processes in finance, computational finance, and quantitative risk management. He is a coauthor of the book *Financial Modelling with Jump Processes* (CRC Press, 2003).

Julien Turc is head of quantitative strategy at Société Générale Corporate & Investment Banking. His team is active on the credit, fixed income, and foreign exchange markets and provides research to hedge funds and institutional investors worldwide. Over the past 10 years, Julien's research has covered topics ranging from exotic credit derivatives pricing to statistical relative value trading, equity-credit modeling and cross-asset strategies. Julien is a graduate of the Ecole Polytechnique and ENSAE and teaches credit derivatives at Paris VI University.

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Ekaterina Voltchkova is assistant professor in applied mathematics at Université de Toulouse I (France). She holds a doctoral degree in applied mathematics from Ecole Polytechnique (France) and a BS in mathematics from the University of Novoibirsk (Russia). Her research has focused on computational finance, in particular numerical methods for models with jumps and the numerical solution of partial integro-differential equations (PIDEs).