Algebraic Domain Decomposition Methods

an M2 Internship proposal
supervised by **Nicole Spillane** (CNRS Researcher)
at **Centre de Mathématiques Appliquées of école Polytechnique** in Palaiseau.

Context: Applied Mathematics within High Performance Computing

The internship focuses on the efficient solution of large scale linear systems:

$$Ax = b,$$

where $A \in \mathbb{R}^{n \times n}$ is the problem matrix, $b \in \mathbb{R}^n$ is the right hand side, and n is potentially very large ($n \gg 10^9$). Scientists and engineers routinely have access to supercomputers with thousands, or many more, processing units to solve these problems. But hardware only tells part of the story. Indeed, in order to be useful, these supercomputers require dedicated software and algorithms that maintain the same efficiency when the workload grows (scalability), and when the problems become more difficult (robustness). This is where applied mathematics are necessary. **The emphasis of the internship is on <u>algebraic</u> <u>domain decomposition</u> solvers.**

- Algebraic solvers are the ones that can be applied simply by typing $x = A \backslash b$. This is in contrast to many state of the art linear solvers that require some additional knowledge on the problem being solved: underlying continuous equations, discretization method... This is a limitation for the applicability of the solvers in every day computing by scientists and engineers that are not specialists in numerical analysis.

- Domain decomposition solvers are based on a partition of the solution space $\mathbb{R}^n$ into N subspaces called the subdomains and spanned by the lines in a matrix denoted R^n. The inverse of $\mathbf{A}$ is then approximated by a sum of inverses of smaller subproblems posed in each subdomain:

$$A^{-1} \approx \sum_{s=1}^{N} {R_s}^{\top} (\tilde{A}_s)^{-1} R_s.$$

prolongation
local solve
restriction

Objectives of the Internship

Domain decomposition methods [?] are state of the art linear solvers for elliptic PDEs with varying coefficients. Spectral coarse space methods are particularly efficient [?]. Their algebraic counterparts are not as advanced and remain an open challenge. The objectives of the internship are to:

(i) study AWG, an algebraic domain decomposition preconditioner introduced in [?] by Loïc Gouarin and Nicole Spillane,

(ii) together with Nicole Spillane, propose an improvement on the theoretical and/or on the computational side. An open source code is already available for AWG (`https://github.com/gouarin/GenEO`)

Keywords

Numerical linear algebra, numerical analysis, scientific computing, domain decomposition, linear solver, preconditioner, parallel computing, high performance computing

Candidate Skills

I am looking for a final year master student with a strong background in scientific computing and/or numerical analysis for PDEs. The ideal candidate should have some experience with programming in Python, possibly with MPI.

Duration

5 to 6 months starting during the first semester of 2025 (flexible).

Grant

Depending on candidate profile ($\geq$ 640 euros per month). The internship will be partly funded by ANR project DARK.

Application

Send your resume to nicole.spillane@polytechnique.edu and feel free to contact me with any questions.

References

[1] L. Gouarin and N. Spillane. Fully algebraic domain decomposition preconditioners with adaptive spectral bounds. *Electronic Transactions on Numerical Analysis*, 2024. In Press.

[2] N. Spillane, V. Dolean, P. Hauret, F. Nataf, C. Pechstein, and R. Scheichl. Abstract robust coarse spaces for systems of PDEs via generalized eigenproblems in the overlaps. *Numer. Math.*, 126(4):741–770, 2014.

[3] A. Toselli and O. Widlund. *Domain decomposition methods – algorithms and theory.*, volume 34 of *Springer Ser. Comput. Math.* Berlin: Springer, 2005.