Vertical Integration and Risk Management in Competitive Markets of Non-Storable Goods *

René Aïd[†] Arnaud Porchet[‡] Nizar Touzi[§]

29th September 2006

Abstract

This paper provides an analysis of vertical integration and its benefits for risk diversification in competitive markets of non-storable goods. We study the interactions between spot, forward and retail markets, and the impact of vertical integration on these interactions. In this setting, we present an equilibrium model for the three markets, where a set of actors are specialized in upstream or downstream segments or both if they are integrated. They must decide at time $t = 0$ their retail market share and forward positions under uncertainty before time $t = 1$ where production and supply occur. The objective of each actor is to maximize a mean-variance utility function, and the equilibrium can be characterized explicitly.

We show that vertical integration and forward hedging are two levers for diversifying demand and spot prices risks. We prove that they exhibit similar properties relatively to their impact on retail prices and actors' utility. We also show that, in the presence of highly risk averse downstream actors, vertical integration is more efficient to diversify risk.

Keywords: Electricity Market, Spot, Forward, Retail, Perfect Competition, Equilibrium, Mean Variance Utility, Market Share, Hedging, Production Management, Vertical Integration, Risk.

***Acknowledgement**. The authors are grateful to Profs. Jean Tirole, Robert B. Wilson and G. Chemla for their helpful comments. The paper also benefited from numerous discussions with Jérôme Wirth and Cyrille Strugarek at EDF R&D.

[†]EDF R&D - Département OSIRIS. 1, avenue du Général de Gaulle, F-92141 Clamart Cedex FRANCE. Tel: 33 (0)1 47 65 46 86, email: rene.aid@edf.fr

[‡]Corresponding author. Université Paris Dauphine CEREMADE, ENSAE CREST and EDF R&D. EDF R&D - Département OSIRIS, 1, avenue du Général de Gaulle, F-92141 Clamart Cedex FRANCE. Tel: 33 (0)1 47 65 36 71, email: arnaud.porchet@ensae.fr

[§]Centre de Mathématiques Appliquées, Ecole Polytechnique Paris, France, and Imperial College London. touzi@cmap.polytechnique.fr

1 Introduction

Understanding the determinants of vertical integration has been the focus of much attention and a unified theory has still not emerged. This question has been clarified from several different perspectives, all of which virtually rely on some form of imperfections (see for instance the surveys by Perry [17] and Joskow [13]). In particular, the presence of uncertainties has been proved to be an argument in favor of vertical integration. In Williamson [18] or Bolton and Whinston [5], contractual incompleteness makes it more efficient to integrate vertically. Indeed, long-term contracts can be costly and difficult, if not impossible, to specify in every possible state of the world. Opportunistic behaviours, appearing when the contractual relations are misspecified, induce inefficiencies. Vertical integration then allows for cooperative adaptation, better decision-making and risk reduction. Arrow [3] develops a model in which vertical integration is sought for acquiring valuable private information about the production process. In Green [11] firms integrate vertically to avoid rationing. In Hendrikse and Peters [12] and Carlton [6], uncertainty in demand, rationing, lack of market flexibility and risk aversion are the determinants of vertical integration. When markets are competitive and in the absence of frictions, risk diversification can still be a reason for integration, as showed in Perry [17]. In Emons [9], downstream firms integrate upwards to ensure supply at a lower price.
As mentioned above, long-term contracts are a common alternative to vertical integration. They are theoretically more flexible means of ensuring supply and price stability but are usually difficult to write thus leading to complex and sometimes misspecified contracts. Relations between vertical integration and long-term contracts have been studied by Klein [15] and Joskow [14] from the point of view of incomplete contracts. In the special case of electricity markets, Chao, Oren and Wilson argue in [7]-[8] that a certain level of vertical integration is efficient when the market fails to provide a full set of hedging instruments.

The main objective of this paper is to clarify and quantify the impact of vertical integration from the point of view of risk diversification and discuss the similarities between vertical integration and long-term contracts. We aim at understanding the fundamental mechanisms of risk diversification operating in retail, forward and spot markets, together with the relationship linking each market's equilibrium price. We only focus on risk and do not consider either strategic behavior nor market power. Therefore we concentrate on perfectly competitive markets and are guaranteed that risk diversification considerations are completely responsible for the properties of our equilibrium analysis. We also ignore any kind of profit sharing rules other than vertical integration.
To this end, we develop an equilibrium model of perfectly competitive retail, forward and spot markets for a non-storable good. At time $t = 0$, downstream firms (or downstream subsidiaries of integrated firms) choose their retail market shares and forward positions for time $t = 1$. At that time, upstream firms (or downstream subsidiaries) produce the good,

sell it to downstream entities on the spot market, which deliver it to end-customers. We suppose that the final demand is random and inelastic and that uncertainty is revealed at time $t = 1$ before production occurs. Decisions at time $t = 0$ must then be taken under uncertain demand and spot price. We suppose that the good is non-storable so that no production can occur at time $t = 0$ and be stored until time $t = 1$. This corresponds to the models studied by Allaz [1] or Bessembinder and Lemmon [4], where we consider in addition the retail activity. Assuming that agents' preferences are defined by a mean-variance utility function and that they disregard any influence they could have on the equilibrium price or on the other actors' decisions, we derive the equilibrium prices and exchanged quantities on the three markets in closed forms.

In this setting, we show that vertical integration and forward hedging are two levers for achieving risk diversification, that exhibit similar properties. First, they both have a downward impact on retail price. Second, they are both means for actors with low generation capacity to corner larger market shares. Third, they both tend to decrease downstream firms' utility when upstream firms are only partially integrated. Fourth, the impact of one of these levers one retail price and utilities is drastically reduced in the presence of the other. Nevertheless, we also observe some discrepancies between these two levers due to a strong asymmetry between upstream and downstream firms in terms of risk. Indeed, downstream firms have to take decisions under uncertainty, while upstream firms respond to it. In addition, in the absence of forward hedging, vertical integration and demand elasticity, profit of upstream firms are not impacted by retail price, whereas downstream revenues are impacted by spot price. Therefore, downstream firms are more exposed to risk. As a consequence, we observe that, first, vertical integration restores this symmetry while forward hedging does not. Second, vertical integration is more robust towards high risk aversions in the sense that it can achieve risk diversification when forward hedging fails. Third, vertical integration can also increase downstream firms' utility provided that they have sufficiently high risk aversion. Fourth, a non-integrated economy can be a stable equilibrium whereas a situation where no actors trade forward contracts is almost never stable a stable equilibrium. Finally, we prove that the inelasticity assumption of demand to retail price is not restrictive and that our conclusions prevail in this setting.
The paper is organized as follows. We formulate the equilibrium problem in Section 2, and compare two different situations. First, in Section 3, we consider the equilibrium when there is no forward market. Then, in Section 4, we introduce the forward market and solve the associated equilibrium problem. After having developed the model, we illustrate in Section 5 some case studies in the electricity sector with data from the French market. Finally, in Section 6.1, we discuss the extension of the model to the case of an elastic demand.

2 The model

In this section we describe perfectly competitive retail, forward and spot markets for a non-storable good, and define an equilibrium on these markets.

2.1 The markets

We consider a set $\mathcal{P}$ of producers of a non-storable good (upstream firms) selling their production on spot and forward wholesale markets. A set $\mathcal{R}$ of retailers (downstream firms) sources on the markets and delivers the good to end-customers, whose demand is denoted by D. We also suppose that all actors have access to wholesale markets and we allow for the presence of purely speculative actors (traders) who have no production nor retail subsidiaries. We denote by $\mathcal{K}$ the set of all actors: producers, retailers and traders. We emphasize the fact that an actor is not necessarily specialized in a single activity, but can own subsidiaries of different kinds. Hence the subsets $\mathcal{P}$ and $\mathcal{R}$ of $\mathcal{K}$ are possibly intersecting, leading to four different kinds of actors: pure retailers who buy on the markets and deliver to end-customers, pure producers who produce and sell their production on the wholesale markets, pure traders who only speculate between the spot and forward markets, and integrated producers who produce, trade on the markets and also deliver to end-customers.
There are two dates in the model: $t = 0$ and $t = 1$. We suppose that demand D is random, inelastic and corresponds to the demand at the future date $t = 1$ (we discuss the case of elastic demand in Section 6.1). Since the good is non-storable, production can only occur at that time $t = 1$ when the demand uncertainty is observed. Demand cannot be served by producing at time $t = 0$ and storing the good until time $t = 1$. On the other hand, decisions regarding forward and retail contracts must be made at time $t = 0$, before demand uncertainty is revealed. We suppose here that a single product is offered to all retail customers, that these customers are indifferent as to the choice of a retailer and are only concerned with the level of retail price. We finally assume a perfectly competitive environment: all actors compete disregarding any influence they could have on the equilibrium price, or the other actors' behaviour.

In Subsection 6.2 we consider the case where decisions on the retail and forward market are not taken simultaneously. We prove that, under the assumption of competitive markets, the equilibrium remains unchanged if forward market decisions are taken prior to retail market decisions or vice-versa.

At time $t = 0$, downstream firms choose their market shares $\alpha_k \in [0, 1]$, $k \in \mathcal{R}$. We denote by p the retail price. Demand satisfaction at time $t = 1$ imposes the market-clearing constraint:

$$1 = \sum_{k \in \mathcal{R}} \alpha_k \ . \tag{2.1}$$

The actors also take positions on the forward market at that date. We denote by q the forward price and by f_k, $k \in \mathcal{K}$, the forward positions (where $f_k > 0$ represents a purchase). We also assume a perfectly competitive forward market where the actors must meet the market-clearing constraint:

$$0 = \sum_{k \in \mathcal{K}} f_k \ . \tag{2.2}$$

Finally, at time $t = 1$, when demand uncertainty is resolved, the actors take positions on the spot market. We denote by P the spot price and by G_k, $k \in \mathcal{K}$, the spot positions (where $G_k > 0$ represents a purchase). As in the case of retail and forward markets, it is assumed that the actors follow the rules of perfect competition on the spot market. The market-clearing constraint reads:

$$0 = \sum_{k \in \mathcal{K}} G_k \ . \tag{2.3}$$

Together with the spot positions, producers choose their generation levels $S_k, \ k \in \mathcal{P}$, that must meet demand D:

$$D = \sum_{k \in \mathcal{P}} S_k \ . \tag{2.4}$$

Each actor aims at maximizing its profit, i.e. sum of its gains on the retail, forward and spot markets minus its production costs:

$$p \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - q f_k - P G_k - c_k \left(S_k \right) \mathbf{1}_{\{k \in \mathcal{P}\}} \ .$$

Non-storability imposes that the net volume of good bought, sold or produced by actor k is zero (no inventory is possible):

$$0 = \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - f_k - G_k - S_k \mathbf{1}_{\{k \in \mathcal{P}\}} \ ,$$

allowing us to discard variable G_k and write the profit function as:

$$(p - P) \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} + (P - q) f_k + \left(P S_k - c_k \left(S_k \right) \right) \mathbf{1}_{\{k \in \mathcal{P}\}} \ .$$

Three terms appear in this expression. The first one is the profit made by a retailer who satisfies a demand $\alpha_k D$ at retail price p by sourcing on the spot market at price P. The second one is the profit made by a trader buying a volume f_k on the forward market at price q and selling it on the spot market at price P. The third term is the profit made by a producer who generates a volume S_k at cost $c_k(S_k)$ and sells it on the spot market at price P.

We finally suppose that the preferences of actor k are described by a mean-variance utility function with risk aversion coefficient λ_k. We will use the following notation for the mean variance utility function:

$$\mathrm{MV}_{\lambda_k}[\xi] := \mathbb{E}[\xi] - \lambda_k \mathrm{Var}[\xi] \ .$$

Notice that MV_{λ_k} has the inconvenience of not being monotonic, which implies possible negative prices at equilibrium. Nevertheless, as shown in [16], it can always be seen as a second order expansion of a monotonic Von Neumann-Morgenstern utility function.

2.2 Equilibrium on the spot market

Demand D at time $t = 1$ is supposed to be exogenous to all actors and totally inelastic to both retail and spot prices. See Section 6.1 for the extension to elastic demand. Demand D is described by a random variable on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. In this setting, the equilibrium on the spot market is straightforward and independent of any decision taken at time $t = 0$. For this reason, we start the analysis with this market.

Each producer $k \in \mathcal{P}$ is characterized by a cost function $x \mapsto c_k(x)$ defined on $\mathbb{R}_+$ and satisfying the Inada conditions:

$$c_k'(0+) = 0, \ c_k'(+\infty) = +\infty \ . \tag{2.5}$$

We also suppose that the functions c_k are continuously differentiable and strictly convex. The cost $c_k(x)$ represents the cost for actor k to produce a quantity x. Producer k's generation profit on the spot market reads:

$$PS_k - c_k(S_k) \ . \tag{2.6}$$

At time $t = 1$, when entering the spot market, the actors know the realization of demand uncertainty D, and decisions on the retail and forward markets have already been taken. The spot market competitive equilibrium is thus classically given by:

$$P^* \ = \ C'(D) \quad , \quad S_k^* \ = \ (c_k')^{-1}(P^*) \ , \tag{2.7}$$

where the aggregate cost function C is defined by:

$$C(x) \ := \ \sum_{k \in \mathcal{P}} c_k \circ (c_k')^{-1} \circ \left(\sum_{k \in \mathcal{P}} (c_k')^{-1} \right)^{-1} (x) \ ,$$

and verifies:

$$C'(x) \ = \ \left(\sum_{k \in \mathcal{P}} (c_k')^{-1} \right)^{-1} (x) \ ,$$

so that the random variable

$$C(D) \quad = \quad \sum_{k\in\mathcal{P}} c_k\left(S_k^*(P^*)\right)$$

is the sum of the production costs over all producers. At equilibrium each producer produces at marginal cost.

The equilibrium on the spot market only depends on the exogenous demand D and is therefore independent of any other equilibrium prior to time $t=1$. This results from the non-storability condition and the inelasticity assumption on D. Note that this situation is different from [1], where the demand elasticity to spot price implies a dependency of the spot price to forward positions and a reduction of the market power of the producers. In the following, the equilibrium spot price P^* and the generation profit

$$\Pi_k^g \quad := \quad \left(P^* S_k^* - c_k\left(S_k^*\right)\right)\mathbf{1}_{\{k\in\mathcal{P}\}} \tag{2.8}$$

will act as exogenous random variables and we suppose that their distribution is known by all actors. We then replace variables P and S_k by P^* and S_k^* found previously, and we define the profit function of actor k

$$\Pi_k(p,q,\alpha_k,f_k) \quad := \quad \Pi_k^r(p,\alpha_k) + \Pi_k^t(q,f_k) + \Pi_k^g\ , \tag{2.9}$$

where Π_k^g is defined by (2.8) and

$$\Pi_k^r(p,\alpha_k) \quad := \quad (p-P^*)\alpha_k D\mathbf{1}_{\{k\in\mathcal{R}\}} \tag{2.10}$$

$$\Pi_k^t(q,f_k) \quad := \quad (P^*-q)f_k\ . \tag{2.11}$$

Here, Π_k^r is the net retail profit derived from supplying a retail demand by sourcing on the spot market, and Π_k^t is the net trading profit earned by buying on the forward market and selling on the spot market. Finally, Π_k^g is the net generation profit gained by producing and selling production on the spot market.

Remark 2.1. The model presented in this article does not actually require the assumption of inelastic demand and competitive equilibrium on the spot market. It can be handled similarly without these assumptions, as soon as the equilibrium on the spot market only depends on the retail price. Nonetheless, for clarity purpose, we choose to stick to this framework and address possible generalizations of the model in Section 6.1. $\diamond$

2.3 Competitive Equilibrium

In order to define an equilibrium, we introduce the following two sets:

$$\mathbf{A} \quad := \quad \left\{(\alpha_k)_{k\in\mathcal{K}} \in [0,1]^{|\mathcal{K}|}\ :\ \forall k\notin\mathcal{R},\ \alpha_k = 0 \text{ and } \sum_{k\in\mathcal{K}}\alpha_k = 1\right\} \tag{2.12}$$

$$\mathbf{F} \quad := \quad \left\{(f_k)_{k\in\mathcal{K}} \in \mathbb{R}^{|\mathcal{K}|}\ :\ \sum_{k\in\mathcal{K}} f_k = 0\right\}\ . \tag{2.13}$$

Definition 2.1. *An equilibrium of the retail-forward equilibrium problem is a quadruple* $(p^*,\ q^*,\ \alpha^*,\ f^*) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbf{A} \times \mathbf{F}$ *such that:*

$$(\alpha_k^*, f_k^*) \quad = \quad \underset{\alpha_k, f_k}{\operatorname{argmax}} \, \mathrm{MV}_{\lambda_k} \left[\Pi_k \left(p^*, q^*, \alpha_k, f_k\right)\right], \ \forall k \in \mathcal{K} \ . \tag{2.14}$$

This defines a simultaneous competitive equilibrium on both markets. Each actor submits a supply function specifying his levels of forward purchase and market share for each price level. Each actor chooses his supply function taking the prices as given and without taking into account the presence of competitors. Then, the auctioneer collects all supply functions and sets the prices that ensure market clearing and demand satisfaction. See Subsection 6.2 for other definitions of the equilibrium that introduce sequentiality between forward and retail market.

3 Equilibrium without a forward market

In this section, we focus on equilibria in the absence of a forward market. We derive the explicit formulation of the equilibrium and analyse the results. In the absence of a forward market, we define the profit function without a forward position:

$$\Pi_k^0(p, \alpha_k) := \Pi_k(p, 0, \alpha_k, 0) = \Pi_k^r(p, \alpha_k) + \Pi_k^g \ .$$

In this context, definition 2.1 reduces to:

Definition 3.1. *An equilibrium of the retail equilibrium problem is a pair* $(p^*,\ \alpha^*) \in \mathbb{R}_+ \times \mathbf{A}$ *such that:*

$$\alpha_k^* \quad = \quad \underset{\alpha_k}{\operatorname{argmax}} \, \mathrm{MV}_{\lambda_k} \left[\Pi_k^0 \left(p^*, \alpha_k\right)\right], \ \forall k \in \mathcal{K} \ . \tag{3.1}$$

3.1 Characterization of the equilibrium

Let

$$\Pi_I^g \quad := \quad \sum_{k \in \mathcal{R} \cap \mathcal{P}} \Pi_k^g \tag{3.2}$$

be the aggregate generation profit realized by all integrated producers, i.e. firms running both generation and supply units, and let

$$\Pi^r \quad := \quad \sum_{k \in \mathcal{R}} \Pi_k^r(p^*, \alpha_k^*) \ = \ (p^* - P^*)D \tag{3.3}$$

be the aggregate retail profit realized by all retailers at equilibrium. We also define

$$\Lambda := \left(\sum_{k \in \mathcal{K}} \lambda_k^{-1}\right)^{-1}, \ \Lambda_{\mathcal{R}} := \left(\sum_{k \in \mathcal{R}} \lambda_k^{-1}\right)^{-1}, \tag{3.4}$$

which can be interpreted as the aggregate risk aversion coefficients, respectively for the set of all actors and the set of all retailers. Parameter λ_k^{-1} corresponds to the risk tolerance of actor k, as defined in [10]. Our equilibrium problem is similar on each market to that faced by syndicates in [19], where an aggregate risk tolerance is defined by summing over the risk tolerances of the syndicate members, in agreement with (3.4).

We only focus on interior equilibria, i.e. equilibria where constraints $\alpha^* \in [0,1]$ and $p^* \geq 0$ are not binding, by discarding cases where some retailers in $\mathcal{R}$ have null market shares. The equilibrium is then characterized by the following proposition.

Proposition 3.1. $(p^*,\ \alpha^*) \in \mathbb{R}_+^* \times \mathrm{int}(\mathbf{A})$ *defines an equilibrium of the retail equilibrium problem without a forward market iff:*

$$\alpha_k^* \quad = \quad \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\frac{\mathrm{Cov}[\Pi^r, \Pi_I^g]}{\mathrm{Var}[\Pi^r]} - \frac{\mathrm{Cov}[\Pi^r, \Pi_k^g]}{\mathrm{Var}[\Pi^r]}\ , \tag{3.5}$$

and p^* solves the second order polynomial equation:

$$0 \quad = \quad \mathbb{E}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}}\mathrm{Cov}[(p^* - P^*)D, (p^* - P^*)D + \Pi_I^g]\ . \tag{3.6}$$

Proof. See Appendix A □

3.2 Equilibrium retail price

Having now an explicit formulation of the equilibrium, we analyse the retail price properties.

Risk neutral case and uniqueness. First, we observe that the retail price is characterized by a second order polynomial in (3.6). This equation may have several solutions or none, thus existence and uniqueness are not guaranteed a priori, as a drawback of the mean-variance analysis. We next argue that, if there exists two solutions, only one is relevant.

For this purpose, we start the analysis with the risk neutral case. If some retailer is risk neutral, i.e. $\lambda_{k_0} = 0$ for some $k_0 \in \mathcal{R}$, the equilibrium retail price reduces to:

$$p^0 \quad = \quad \frac{\mathbb{E}[P^*D]}{\mathbb{E}[D]}\ , \tag{3.7}$$

Since $P^* = C'(D)$ is a non-decreasing function of D, implying that P^* and D are positively correlated, we deduce that the risk neutral retail price is greater than the expected spot price:

$$p^0 \quad \geq \quad \mathbb{E}[P^*]\ .$$

Suppose now that all retailers are risk-averse. It seems natural to expect the equilibrium retail price to tend to the risk neutral price when the risk aversion coefficient of some retailer tends to zero. In this case, the aggregate risk aversion coefficient $\Lambda_{\mathcal{R}}$ also becomes zero. Suppose equation (3.6) has two non-negative solutions $p_- \leq p_+$. The Taylor expansion of these roots around $\Lambda_{\mathcal{R}} = 0$ reads:

$$\begin{aligned} p_- &\simeq p^0 + \frac{2\Lambda_{\mathcal{R}}}{\mathbb{E}[D]\mathrm{Var}[D]}\Big(\mathrm{Var}[D]\mathrm{Cov}[PD, PD - \Pi_I^g] - \mathrm{Cov}^2[D, PD - \frac{1}{2}\Pi_I^g] \\ &\quad + 2\mathrm{Cov}^2[D, PD - \frac{1}{2}\Pi_I^g - p^0 D]\Big) \\ p_+ &\simeq \frac{\mathbb{E}[D]}{2\Lambda_{\mathcal{R}}\mathrm{Var}[D]} \ . \end{aligned}$$

This shows that p_- tends to p^0 as $\Lambda_{\mathcal{R}}$ tends to 0, while p_+ tends to infinity. For this reason, the following analysis considers that p_- is the relevant root from the economic point of view, and the equilibrium is in fact uniquely characterized.

Impact of integration. Let us consider the impact of vertical integration on the equilibrium retail price. We prove that the presence of integrated producers has a downward impact on retail price. To this end, we denote by p_{NI}^* the smallest solution of (3.6) in the absence of integrated producers, and by p_I^* in the presence of some integrated producers. Suppose that $\mathcal{R} \cap \mathcal{P} = \emptyset$, i.e. there are no integrated producers who have both generation and supplying activities. Equation (3.6) then reduces to:

$$0 = \mathbb{E}[(p_{NI}^* - P^*)D] - 2\Lambda_{\mathcal{R}}\mathrm{Var}[(p_{NI}^* - P^*)D] \ . \tag{3.8}$$

As a consequence, it is also greater than the expected spot price.
When some actors are integrated, i.e. $\mathcal{R} \cap \mathcal{P} \neq \emptyset$, the equation that determines the equilibrium price is

$$0 = \mathbb{E}[(p_I^* - P^*)D] - 2\Lambda_{\mathcal{R}}\mathrm{Cov}[(p_I^* - P^*)D, (p_I^* - P^*)D + \Pi_I^g] \ .$$

Suppose one retailer, whom we will name i, decides to become an integrated producer. In this case, $\Lambda_{\mathcal{R}}$ is unchanged and $\Pi_I^g = \Pi_i^g$. Suppose also that Π^r and Π_I^g are negatively correlated. We then obtain:

$$0 \geq \mathbb{E}[(p_I^* - P^*)D] - 2\Lambda_{\mathcal{R}}\mathrm{Var}[(p_I^* - P^*)D] \ . \tag{3.9}$$

This last assumption concerning the correlation of Π^r and Π_I^g is natural. Since $P^* = C'(D)$, profit Π_k^g is an increasing function of D. We cannot prove that the retail profit $\Pi^r = (p^* - P^*)D$ is a decreasing function of D but as p^* is a fixed price and P^* is increasing with D, we have the intuition that Π^r will decrease with D. This justifies the negativity assumption on the covariance of Π^r and Π_I^g. The numerical application performed in Section 5 confirms this intuition.
Previous inequality (3.9), confronted to (3.8), shows that p_I^* is smaller than p_{NI}^*. Integrated firms then have a downward impact on retail price.

3.3 Equilibrium market shares

We turn to the analysis of market shares properties at equilibrium.

Risk neutral case. The equilibrium market shares of the risk averse retailers are given by:

$$\alpha_k^0 \;=\; -\frac{\mathrm{Cov}[\Pi^r, \Pi_k^g]}{\mathrm{Var}[\Pi^r]} \;, \tag{3.10}$$

while the remaining demand is split among the risk neutral retailers. In particular, a risk averse retailer who has no generation unit ends up with a null market share. Notice that, in order to satisfy the non-negativity condition of the market shares, equation (3.10) implies $\mathrm{Cov}[\Pi^r, \Pi_k^g] \leq 0$. We then have another justification of the assumption made in the previous subsection.

Impact of integration. We now argue that a supplier can increase its market share when integrating. In the absence of integrated producers and risk-neutral suppliers, the equilibrium market shares are given by:

$$\alpha_k^* \;=\; \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \;.$$

The market shares are distributed proportionally to the risk tolerances, and only depend on these parameters.
If integrated firms enter the market, the pure retailers see their equilibrium market shares move to:

$$\alpha_k^* \;=\; \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\frac{\mathrm{Cov}[\Pi^r, \Pi_I^g]}{\mathrm{Var}[\Pi^r]} \;,$$

while the integrated firms have market shares:

$$\alpha_k^* \;=\; \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\frac{\mathrm{Cov}[\Pi^r, \Pi_I^g]}{\mathrm{Var}[\Pi^r]} - \frac{\mathrm{Cov}[\Pi^r, \Pi_k^g]}{\mathrm{Var}[\Pi^r]} \;.$$

Suppose once again that Π^r is negatively correlated to Π_I^g. The pure retailers then see their equilibrium market shares decrease while the integrated actors increase their market shares. Indeed, the latter will decrease their risk by diversification if they invest more in the retail market. We also observe that, although the market shares have changed in comparison to the previous case, the relative market shares among the set of pure retailers remain unchanged:

$$\frac{\alpha_i^*}{\alpha_j^*} \;=\; \frac{\lambda_j}{\lambda_i} \;.$$

4 Equilibria with a forward market

We now turn to the characterization of the equilibrium in the presence of a forward market.

4.1 Characterization of the equilibria

We still focus on interior equilibria, when the constraints are not binding. We define the following quantity:

$$\Pi^e \;:=\; \sum_{k\in\mathcal{K}} \Pi_k(p^*, q^*, \alpha_k^*, f_k^*) \;=\; p^*D - C(D)\ ,$$

which is the aggregate profit of the whole economy. The equilibrium in the presence of a forward market is characterized by the following proposition.

Proposition 4.1. $(p^*,\ q^*,\ \alpha^*,\ f^*) \in \mathbb{R}_+^* \times \mathbb{R}_+^* \times \mathrm{int}(\mathbf{A}) \times \mathbf{F}$ *defines an equilibrium of the retail-forward equilibrium problem iff:*

$$\begin{aligned}
f_k^* &= \frac{\Lambda}{\lambda_k}\frac{\mathrm{Cov}\left[P^*, \Pi^e\right]}{\mathrm{Var}\left[P^*\right]} - \frac{\mathrm{Cov}\left[P^*, \Pi_k^g\right]}{\mathrm{Var}\left[P^*\right]} - \alpha_k^* \frac{\mathrm{Cov}\left[P^*, \Pi^r\right]}{\mathrm{Var}\left[P^*\right]} && (4.1)\\
\alpha_k^* &= \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\mathrm{Cov}[P^*, \Pi^r]}{\Delta}\mathrm{Cov}\left[P^*, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\Pi_I^g\right] - \frac{\mathrm{Var}[P^*]}{\Delta}\mathrm{Cov}\left[\Pi^r, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\Pi_I^g\right] && (4.2)\\
q^* &= \mathbb{E}[P^*] - 2\Lambda\mathrm{Cov}[P^*, p^*D - C(D)]\ , && (4.3)
\end{aligned}$$

and p^* is a root of the second order polynomial equation

$$\begin{aligned}
0 &= \mathbb{E}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}}\mathrm{Cov}[(p^* - P^*)D, (p^* - P^*)D + \Pi_I^g] && (4.4)\\
&\quad + 2\Lambda_{\mathcal{R}}\frac{\mathrm{Cov}[P^*, (p^* - P^*)D]}{\mathrm{Var}[P^*]}\mathrm{Cov}\left[P^*, (p^* - P^*)D + \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}}(p^*D - C(D))\right]\ ,
\end{aligned}$$

where

$$\Delta \;:=\; \mathrm{Var}[P^*]\mathrm{Var}[\Pi^r] - \mathrm{Cov}^2[P^*, \Pi^r]\ . \qquad (4.5)$$

Proof. See Appendix A. □

Having explicitly solved the equilibrium problem, we can analyse the properties of equilibrium prices and positions on the retail and forward markets.

4.2 Equilibrium forward price

The equilibrium forward price is given by:

$$q^* \;=\; \mathbb{E}[P^*] - 2\Lambda\mathrm{Cov}[P^*, \Pi^e]\ .$$

It is equal to the expectation of the spot price corrected by a risk premium term accounting for the correlation between spot price and global profit Π^e at equilibrium, and the aggregate risk aversion of the market. This kind of formula is typical in the mean-variance utility based equilibria, as shown first in [1]. We can also write:

$$q^* \;=\; \mathbb{E}[ZP^*] \quad \text{with} \quad Z \;:=\; 1 - 2\Lambda(\Pi^e - \mathbb{E}[\Pi^e])\ .$$

If Λ is small enough so that Z is always strictly positive, Z defines a change of probability and q^* is given by the expectation under a risk-neutral probability of P^*.
We note that the equilibrium forward price does not depend on the distribution of market shares or that of generation assets, but only on retail and spot equilibrium prices. Moreover, if some traders are risk neutral, the forward price reduces to the expected spot price:

$$q^0 \quad = \quad \mathbb{E}[P^*] \ .$$

In the context of quadratic cost functions $c_k(x) := \frac{1}{2}x(a_k x + b_k)$, $a_k, b_k > 0$, q^* has the following expression:

$$q^* \quad = \quad \mathbb{E}[P^*] - \frac{2\Lambda}{a}\mathrm{Var}[P^*](p^* - \mathbb{E}[P^*]) + \frac{\Lambda}{a}\mathrm{Var}^{\frac{3}{2}}[P^*]\mathrm{Skew}[P^*] \ ,$$

where $a^{-1} := \sum_{k\in\mathcal{P}} a_k^{-1}$, as presented in [4] in the case of electricity. The forward price increases with spot price skewness and, in the case where the equilibrium retail price is higher than the expected spot price, the forward price decreases with spot price volatility. This shows that, in the case of electricity, forward prices lower than the expected spot price are common since spot price volatility is high for electricity. Nevertheless, forward prices greater than the expected spot price can occur when spot price skewness is large and positive, i.e. when large upward peaks are possible. In addition, the equilibrium on the forward market establishes the following relationship between retail and forward prices:

$$p^* \quad = \quad \frac{\mathbb{E}[P^*] - q^*}{2\Lambda\mathrm{Cov}[P^*, D]} + \frac{\mathrm{Cov}[P^*, C(D)]}{\mathrm{Cov}[P^*, D]} \ .$$

In particular, higher forward prices correspond to lower retail prices and conversely.

4.3 Equilibrium retail price

The expression of the retail price is more complicated. Nevertheless, the equation giving p^* in the presence of a forward market is similar to that found in the absence of a forward market. An extra term

$$2\Lambda_{\mathcal{R}}\frac{\mathrm{Cov}[P^*, (p^* - P^*)D]}{\mathrm{Var}[P^*]}\mathrm{Cov}\left[P^*, (p^* - P^*)D + \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}}(p^* D - C(D))\right]$$

is added in the presence of a forward market, but its interpretation is not obvious. We can still proceed to the Taylor expansion around $\Lambda_{\mathcal{R}} = 0$ and show that only the smallest root p_-^* of this equation is relevant, ensuring the uniqueness of the equilibrium. We can also exhibit the following properties of equilibrium retail prices:

- **Risk neutral price.** If some retailers are risk neutral, the retail price reduces to:

$$p^0 \quad = \quad \frac{\mathbb{E}[P^* D]}{\mathbb{E}[D]} \ ,$$

  as in the absence of a forward market.

- **Price in a fully integrated economy.** If all producers are integrated, i.e. $\mathcal{P} \subset \mathcal{R}$, we obtain:

$$\begin{aligned} 0 &= \mathbb{E}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}}\text{Cov}[(p^* - P^*)D, p^*D - C(D)] \\ &\quad +2\Lambda_{\mathcal{R}}\left(1 - \frac{\Lambda}{\Lambda_{\mathcal{R}}}\right)\frac{\text{Cov}[P^*, (p^* - P^*)D]}{\text{Var}[P^*]}\text{Cov}[P^*, p^*D - C(D)] \ . \end{aligned}$$

  In particular, in the absence of pure traders, $\mathcal{R} = \mathcal{K}$ and $\Lambda = \Lambda_{\mathcal{R}}$, so that the above equation reduces to (3.6). This means that the forward market has no impact on the retail price in this case. There is one example where this conclusion is obvious. Suppose there are only N integrated producers, with the same cost function and the same risk aversion coefficient. By an argument of symmetry, we immediately derive $S_k^* = \frac{D}{N}$, $\alpha_k^* = \frac{1}{N}$ and $f_k^* = 0$ for all k. The actors do not take any position on the forward market. The retail price should therefore not be impacted. This conclusion highlights the symmetry between forward hedging and vertical integration. When the firms already diversify their risk by means of vertical integration, the effect of forward hedging tends to vanish. Conversely, in Subsection 5.2.3, we will see that in the presence of forward hedging, the level of integration of the actors has little impact on retail price.

- **Impact of forward trading.** In a partially integrated economy, the retail price behaviour is not obvious. We can say that $p_F^* \leq p_{NF}^*$, i.e. retail price in the presence of a forward market is smaller than in the absence of it, iff

$$0 \quad \leq \quad \left(1 - \frac{\Lambda}{\Lambda_{\mathcal{R}}}\right)\text{Cov}^2[P^*, (p_{NF}^* - P^*)D] + \text{Cov}[P^*, (p_{NF}^* - P^*)D]\text{Cov}[P^*, \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}}\Pi^g] \ .$$

  In particular, this is verified if no producer is integrated and the retail income without a forward market is negatively correlated to the spot price. This is also verified if no retailer is integrated and $\Lambda = 0$ (e.g. existence of a risk-neutral trader). In Subsection 5.2, we will see that forward hedging has always a downward impact on retail price, but that its intensity decreases with the level of integration of the actors.

- **Impact of integration.** Let p_{NI}^* be the equilibrium retail price in the absence of integration. In order to compare this price with the equilibrium retail price in the presence of integrated firms, we substitute p_{NI}^* to p^* in the right side of (4.4) and study the sign of the expression. This leads to studying the sign of:

$$\frac{\text{Cov}[P^*, (p_{NI}^* - P^*)D]}{\text{Var}[P^*]}\text{Cov}[P^*, \Pi_I^g] - \text{Cov}[(p_{NI}^* - P^*)D, \Pi_I^g].$$

  If we consider as previously the particular case of quadratic cost functions, we obtain:

$$\begin{aligned} &\frac{\text{Cov}[P^*, (p_{NI}^* - P^*)D]}{\text{Var}[P^*]}\text{Cov}[P^*, \Pi_k^g] - \text{Cov}[(p_{NI}^* - P^*)D, \Pi_k^g] \\ &= \frac{a^3}{2a_k}\left(\text{Var}[D^2] - \frac{\text{Cov}^2[D^2, D]}{\text{Var}[D]}\right) \ , \end{aligned}$$

which is always positive. This means that the right side of (4.4) is positive for p^*_{NI} and shows that the equilibrium retail price decreases in the presence of integrated firms. More generally, we will see in Subsection 5.3 that the presence of vertically integrated producers has always a downward impact on retail price. This impact is nonetheless drastically reduced in comparison to the case without a forward market.

4.4 Equilibrium positions on the forward market

In contrast with forward price, equilibrium forward positions do depend on both p^* and α^*:

$$f^*_k \;=\; \frac{\Lambda}{\lambda_k}\frac{\mathrm{Cov}\left[P^*, \Pi^e\right]}{\mathrm{Var}\left[P^*\right]} - \alpha^*_k \frac{\mathrm{Cov}\left[P^*, \Pi^r\right]}{\mathrm{Var}\left[P^*\right]} - \frac{\mathrm{Cov}\left[P^*, \Pi^g_k\right]}{\mathrm{Var}\left[P^*\right]} \;. \tag{4.6}$$

The equilibrium forward position of actor k is composed of three parts. We first note that if k is a pure speculator, then the last two terms are zero. The first term can thus be interpreted as the trading component. It is the fraction $\frac{\Lambda}{\lambda_k}$ of a constant term involving the correlation between the equilibrium global profit and the spot price. An extra supplying component is added to retailers. It is the fraction α^*_k of a constant term involving the correlation between the global retail profit and the spot price. If the retail market revenue is negatively correlated to the spot price, as we argued in the previous section, retailers will take long positions on the forward market to hedge against high spot prices. Finally, an extra generation component is added to producers, involving the correlation between their generation profit and the spot price. As generation profits are positively correlated to the spot price, producers will take short forward positions to hedge against low spot prices.

4.5 Equilibrium positions on the retail market

The equilibrium market shares are given by:

$$\alpha^*_k \;=\; \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\mathrm{Cov}[P^*, \Pi^r]}{\Delta}\mathrm{Cov}\left[P^*, \Pi^g_k - \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\Pi^g_I\right] - \frac{\mathrm{Var}[P^*]}{\Delta}\mathrm{Cov}\left[\Pi^r, \Pi^g_k - \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\Pi^g_I\right] \;.$$

In the case where $\Pi^g_k = \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\Pi^g_I$ for all $k \in \mathcal{R}$, the market shares read:

$$\alpha^*_k \;=\; \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \;,$$

as in a non-integrated economy without a forward market. This is the case when, for example, there are no integrated firms, or all producers are integrated and generation profits are proportional to risk tolerances. Another formulation for α^*_k is:

$$\alpha^*_k \;=\; \alpha^0_k + \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\left(1 - \frac{\mathrm{Cov}[P^*, \Pi^r]}{\Delta}\mathrm{Cov}\left[P^*, \Pi^g_I\right] + \frac{\mathrm{Var}[P^*]}{\Delta}\mathrm{Cov}\left[\Pi^r, \Pi^g_I\right]\right) \;,$$

where

$$\alpha_k^0 = \frac{\mathrm{Cov}[P^*, \Pi^r]}{\Delta} \mathrm{Cov}\left[P^*, \Pi_k^g\right] - \frac{\mathrm{Var}[P^*]}{\Delta} \mathrm{Cov}\left[\Pi^r, \Pi_k^g\right]$$

is retailer k's market share in the presence of a risk neutral retailer. This expression allows for an analysis of the deviation of market shares from the risk neutral equilibrium.
In Section 5, we will observe two important characteristics of market shares. First, the presence of a forward market is a means for pure retailers to corner larger market shares. Second, the higher the level of integration of an integrated producer, the higher its market share.

4.6 Utilities at equilibrium and the strong asymmetry between downstream and upstream

As mentioned in the introduction, there exists a strong asymmetry in terms of risk between retailers and producers. First, in the absence of a forward market and vertical integration, retailers have to take market share decisions under uncertainty, while producers know the realization of demand when they take their generation decision. Second, if the demand is inelastic to retail price, upstream profits are independent of retail price, while downstream revenues depend on spot price. This asymmetry is central in our analysis. The example for this is the California electricity crisis, where retailers where suffering large losses while producers were taking advantage of high spot prices.
As a consequence, a pure producer always benefits from trading forward contracts. Indeed, the generation profit Π_k^g is an exogenous random variable in this model. As a consequence, when a pure producer bids the strategy $\bar{f}_k(q) = 0$ for all forward price q, it is guaranteed to receive a utility $\mathrm{MV}_{\lambda_k}[\Pi_k^g]$, i.e. the utility in the absence of a forward market. Pure producers thus always enhance their utility in the presence of a forward market because the strategy $\bar{f} = 0$ is admissible and yields the same utility as without one. On the opposite, pure retailers have no guarantee to obtain a higher utility when forward contracts become available. Indeed, if retailer k decides to bid $\bar{f} = 0$, it will receive a utility $\mathrm{MV}_{\lambda_k}[\Pi_k^r]$. Nevertheless, the retail profit Π_k^r depends on p^*, and thus on the other actors' decisions. If the retail price p^* in the presence of forward trading is different from the retail price without forward trading, actor k's retail profit will change. Not taking position on the forward market will not guarantee it to receive the same utility as in the absence of it. In Subsection 5.2 we observe that the availability of forward contracts decreases pure retailers' utility. Forward contracts are thus not optional in the sense that when they are available, each retailer is individually better off contracting. As a consequence, all retailers trade forward contracts and are able to offer lower retail prices. Nevertheless, we observe that the decrease in expected profit offset that in variance and this risk hedging mechanism implies a decrease in utility in comparison to the case where no forward contracts are available.
The impact of vertical integration on the actors' utility is difficult to measure. First of all,

the question of the evolution of risk aversion with the level of integration has no obvious answer. One could argue that actors become more or less risk averse when integrating as the structure itself of the company is affected. This is an open question to define the risk aversion of an integrated actor knowing the risk aversions of the different subsidiaries. Wilson [19] in his theory of syndicates develops the idea that the risk tolerance of a syndicate, i.e. a group of actors, should be the sum of the actors' risk tolerance. This idea is also suggested in our model, in the way Λ and $\Lambda_{\mathcal{R}}$ are defined. But one could also argue that the resulting risk aversion should be the lowest of the subsidiaries' risk aversions. Indeed, all risky positions would be borne by the least risk averse actor. We illustrate this point in Subsection 5.3. We observe that, like forward hedging, vertical integration decreases the actors' utility because of a downward impact on retail prices. Nevertheless, for large risk aversions this effect can be opposite and the gain from hedging can be higher than the loss in expected profit. Finally, one obvious aspect of vertical integration is that it breaks the asymmetry between upstream and downstream, and producers' and retailers' utilities are impacted similarly by vertical integration.

5 Application to the electricity industry

In the previous section, we provided theoretical results which can be inferred from the explicit form of the equilibrium. In order to go further and enhance our understanding of the impact of forward trading and vertical integration, we present a serie of case studies, illustrated with historical data of the French electricity market. This market is characterized by the presence of a dominant actor, the former monopoly, and recently entered competitors. This situation is far from being compatible with the perfect competition assumption, but, as we argued above, we are only interested in studying the fundamental mechanisms of risk diversification, and do not want them to be correlated to market power effects or strategic behaviors. Nonetheless, a joint study of both components is definitely of great interest. In the following, we will often study cases involving two actors, who can be viewed respectively as the former monopoly and the set of competitors.

5.1 Methodology

In this section we aim at computing the retail and forward equilibria using data from the French market. Only spot prices and demand levels are publicly available. To have access to the global cost function of the economy, C, we need to know the details of all generation assets in the market and all unavailability plannings. As this information is not available, we choose to invert the spot price formula $P^* = C'(D)$ to derive a candidate for C. We use demand and spot price hourly data from December 2004 to March 2005 (available respectively on the web site of RTE* and Powernext**) as samples for D and P^*. The winter under consideration was generally mild, but was followed by a wave of intense cold in March. These recordings are showed on Figure 5.1-left. The circles correspond to values for March. These are interesting samples as they contain high spot prices and are therefore indicative of the high volatility observed in the market. Nonetheless they remain strongly heterogeneous since many generation units were unavailable during March's cold wave. This implies a cost function C evolving with time. We therefore processed the data from March 2005 by adding a constant to the demand sample, as suggested by the shape of the plots, to offset the unavailability effect. We then regressed the function C on these processed samples, so that $P^* = C'(D)$, according to our competitive model (Figure 5.1-right). The data for D and P^*, the risk aversion coefficients and the regressed cost

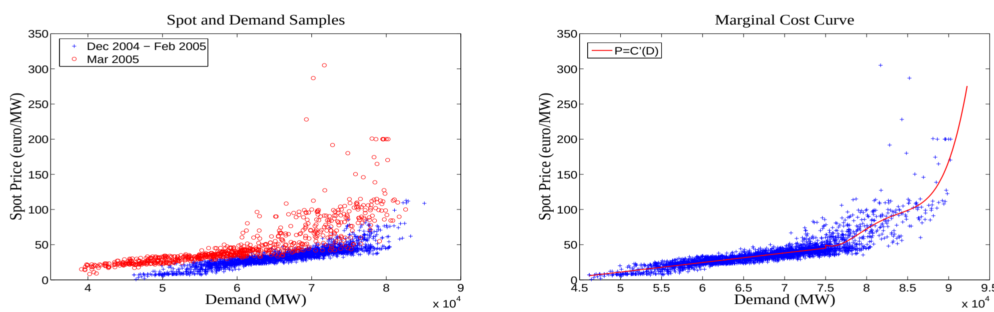

Figure 5.1: Demand and spot price samples (left). Processed and interpolated data (right).

function C allow us to compute the equilibrium.

Before commenting on the numerical application, we observe that the analysis can be led with a limited number of actors without loss of generality. The model involves a number N of actors. As mentioned in Section 2.1, there are only four types of actors: integrated producers, pure retailers, pure traders and pure producers. The equilibrium of any N-actor competition is equivalent to an equilibrium involving at most four actors of different kinds. Indeed, equations (4.1) and (4.2) are linear in λ_k^{-1} and Π_k^g while (4.4) and (4.3) only involve the aggregate risk aversion coefficients Λ and $\Lambda_{\mathcal{R}}$. Hence, N pure retailers of risk tolerances λ_k^{-1} can be aggregated into a single pure retailer of risk tolerance $\sum_k \lambda_k^{-1}$. The equilibrium prices remain unchanged because Λ and $\Lambda_{\mathcal{R}}$ are not impacted, the market share of the aggregate retailer is equal to the sum of the market shares by linearity, and the forward position of the aggregate retailer is equal to the sum of the forward positions by the same argument. Similarly, N pure producers can be aggregated into a single pure producer, N pure traders can be aggregated into a single pure trader and N integrated producers can be aggregated into a single integrated producer. This allows us to only

*www.rte-france.com
**www.powernext.fr

consider examples with a limited number of actors in the following subsections, keeping in mind that the associated equilibria are also relevant to more general cases.

5.2 Impact of the forward market

In this paragraph we focus on the impact of forward trading. Therefore, we compare the equilibria with and without a forward market for different configurations of the economy involving 2 actors:

1. actor 1 is an integrated producer only facing competition on the retail market
2. actor 1, unbundled, becomes a pure producer, while actor 2 is a pure retailer
3. actor 1 is integrated and faces the competition of a pure producer
4. actor 1 and actor 2 are integrated.

5.2.1 Unbundling of the retail activity

We consider the case where the former monopoly faces competition on the retail market. More precisely, we suppose that the former monopoly is an integrated producer owning all the generation assets, and that all the competitors are pure retailers. According to the remark of the previous paragraph regarding the aggregation of actors, it is equivalent to consider a two actor competition with one integrated retailer, denoted actor 1, and one pure retailer, actor 2, standing for all the retail competitors. This experiment addresses the question of the viability of retail competition in the presence of an integrated generation monopoly.

In the absence of a forward market, the pure retailer is forced to source on the spot market from the integrated producer in order to satisfy its demand. Figure 5.2-left shows actor 1's market share in this case as a function of the risk aversion coefficients of both actors. We observe that actor 2 is limited to a very small market share, less than 2%, whatever the values of the risk aversion coefficients of the actors. Actor 2 is in a position of high financial risk as it is forced to buy on the spot market and is therefore exposed to the high volatility of spot prices. Actor 2 has therefore very limited possibilities to enter the retail market, all the more limited as its risk aversion is large. In this context, there is little incentive for a non-integrated producer to enter the market.
Figure 5.2-right shows actor 1's market share in the presence of a forward market. Dotted mesh regions represent zones where there is no equilibrium ($q^* < 0$). In contrast to the previous case, we observe that if actor 2 is less risk averse than actor 1 then it can enter deeper in the retail market, up to 40%. The pure retailer can take advantage of its lower risk aversion to penetrate the market. We conclude from this example that the presence of the forward market allows non-integrated producers to contest the retail monopoly.

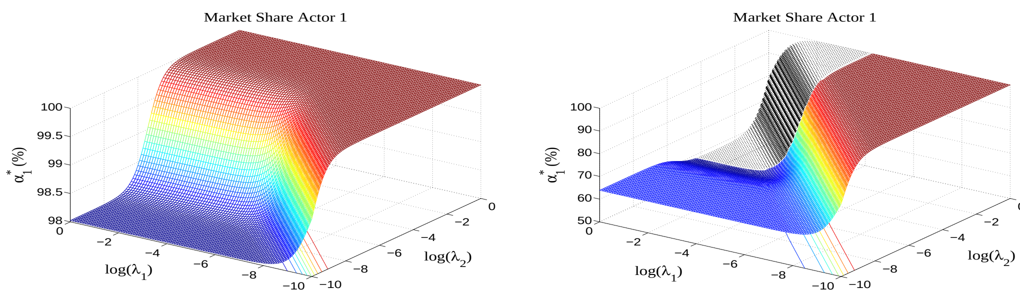

Figure 5.2: Actor 1's market share in the absence (left) and presence (right) of the forward market as functions of the logarithm of risk aversion coefficients.

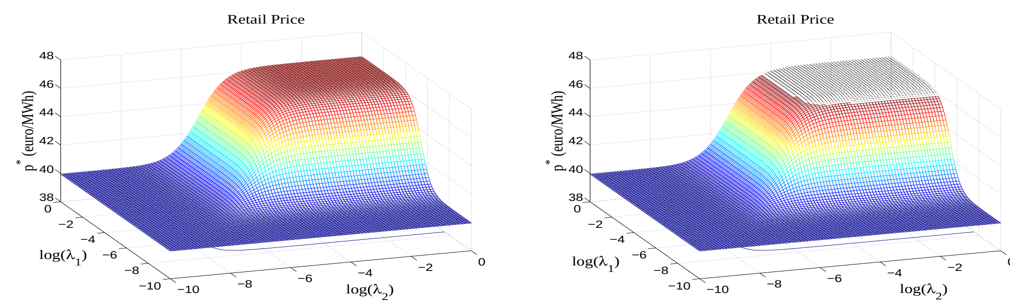

Figure 5.3: Equilibrium retail price in the absence (left) and in the presence (right) of the forward market as functions of the logarithm of risk aversion coefficients.

The equilibrium retail price remains unchanged, as we can see on Figure 5.3. This is because all producers are integrated (cf. Section 4.3). We can give an interpretation of this fact: if all retailers are pure retailers, they will hedge by buying forward contracts to producers, and will be willing to sell at a lower price on the retail market. But if the producers are also retailers, these integrated producers will take short positions on the forward market and will be willing to increase retail prices. We are then in the presence of conflicting incentives that offset each other.

We observe that the equilibrium does not always exist in the forward market (dotted mesh zones in Figure 5.4). When both actors are highly risk averse, they do not agree on exchanging forward. This aspect shows the drawback of the mean-variance utility function. Nonetheless, when the equilibrium exists, actor 2's forward position is in the range of its expected demand. More precisely, we can state that $f_2 \simeq 1.1\,\alpha_2 \mathbb{E}[D]$: actor 2 hedges its retail demand by 10% above the expected demand, whatever its risk aversion coefficient. This means that the integrated producer is better off being short even if it has to buy back part of the previously sold volumes on the spot market. We also observe that the equilibrium forward price is almost always greater than the expected spot price ($E[P^*] = 37.9518$), at least for sufficiently small risk aversion coefficients.

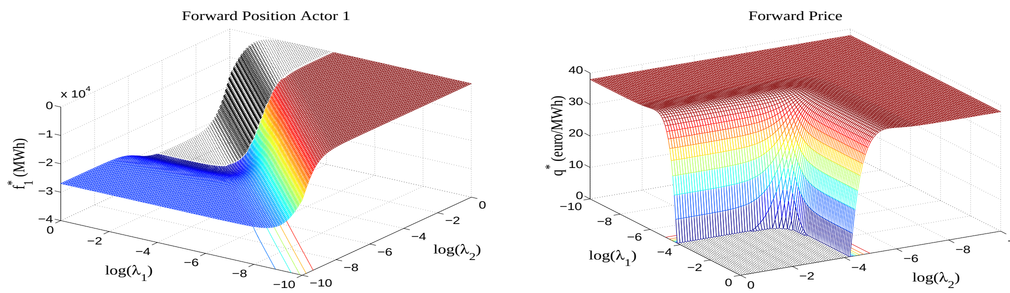

Figure 5.4: Actor 1's forward position (left) and forward price at equilibrium as functions of the logarithm of risk aversion coefficients.

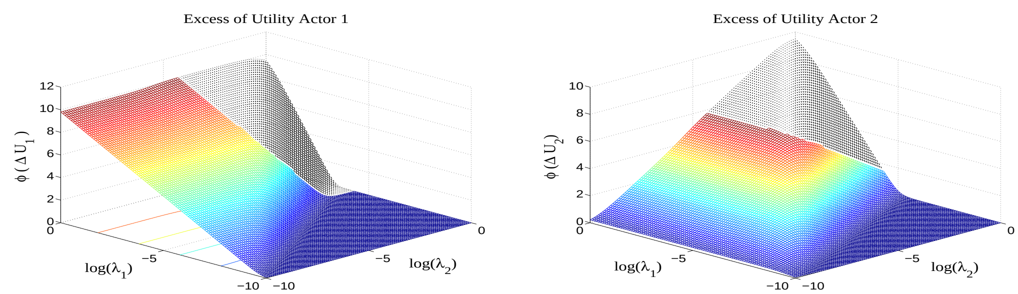

Figure 5.5: Excess of utility for actor 1 (left) and actor 2 (right) as functions of the logarithm of risk aversion coefficients.

Finally, we observe on Figure 5.5 that the utilities of both actors are higher in the presence of the forward market. This figure shows for both actors the excess of utility $\Delta U = U^F - U^{NF}$, induced by the presence of the forward market. For convenience, we plotted the monotonic transform $\phi(\Delta U)$, where $\phi(x) := \text{sgn}(x) \log(1 + |x|)$, in order to show both the logarithm of ΔU and its sign. In this case, both actors benefit from the presence of the forward market.

For a better understanding of the impact of forward hedging, Table 1 reports the relative excesses of utility, average profit and variance ("Risk") on the example $\lambda_1 = \lambda_2 = 10^{-6}$. We also computed the two following quantities. First, we indicate the ratio "excess of average profit" over "excess of risk", denoted "Profit vs. Risk", and we provide a measure of the excess of performance, the performance being defined by the ratio "expected profit" over risk. We observe that forward trading has a higher impact on actor 2's utility than on actor 1's. Actor 1 mainly uses forward contracts for hedging purposes, thus reducing the variance of its profits by 59 %. The relative loss in average profit associated to this hedging position is low, 0.67 times smaller than the variance reduction, resulting in a slight increase in utility. On the other hand, actor 2 increases both its average profit and risk by 793 %. The average profit increase being twice the risk increase, its utility also increases. From this point of view, forward trading has a higher impact on actor 2's business. Nevertheless, in terms of performance, measured by the ratio expected gain over risk, actor 1 benefits more from forward trading than actor 2.

		Actor 1	Actor 2
Utility:	$\frac{\Delta U}{\lvert U \rvert}$	$8.8\ 10^{-2}$ %	793 %
Av. Profit:	$\frac{\Delta \mathbb{E}}{\lvert \mathbb{E} \rvert}$	-0.17 %	793 %
Risk:	$\frac{\Delta \mathrm{Var}}{\mathrm{Var}}$	-59 %	793 %
Profit vs. Risk:	$\frac{\Delta \mathbb{E}}{\lambda \Delta \mathrm{Var}}$	0.67	2
Performance:	$\frac{\lambda \mathrm{Var}}{\lvert \mathbb{E} \rvert} \times \Delta \frac{\mathbb{E}}{\lambda \mathrm{Var}}$	146 %	≈ 0

Table 1: Impact of forward trading on utility, profit and risk, when $\lambda_1 = \lambda_2 = 10^{-6}$.

Conclusion. This case study shows that:

- trading forward contracts allows actors with low generating capacity to corner larger market shares
- retail price is not impacted by forward trading when all producers are integrated
- in terms of utility, both actors can benefit from trading forward contracts

5.2.2 Full unbundling

We now consider the case where the former monopoly is totally unbundled, i.e. the generation activity is separated from the retail activity. Actor 1 is now a pure producer while actor 2 is a pure retailer (we do not consider the presence of pure traders here).

In the case of an unbundled economy, we proved that the forward market has no impact on the market shares, which only depend on the risk aversion coefficients of the actors (cf. Section 4.5). Nonetheless, the retail price is impacted by the presence of the forward market. Figure 5.6 shows the equilibrium retail price as a function of the risk aversion coefficients in the absence (left) and the presence (right) of the forward market (For the sake of clarity the no-equilibrium zones are not showed). We first observe that if actor 2, the pure retailer, is too risk-averse, equilibrium does not exist even in the presence of the forward market. This is an illustration of the asymmetry between retailers and producers. Supply might not be sustainable in this model, whereas generation always is. Second, we observe a large decrease of the retail price due to the presence of the forward market.

An interesting feature of this example is the impact of the forward market on each actor's

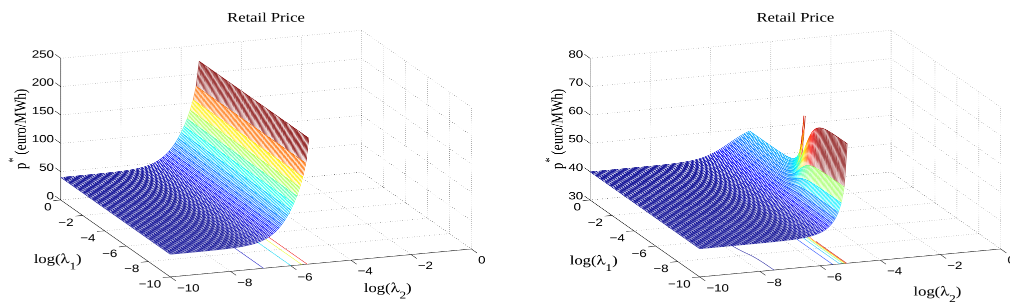

Figure 5.6: Equilibrium retail price in the absence (left) and presence (right) of the forward market as functions of the logarithm of risk aversion coefficients.

utility. Figure 5.7 shows the logarithmic transform of the excess of utility ($\phi(\Delta U)$) due to the presence of the forward market for both actors. We observe that the presence of the forward market increases the pure producer's utility, while it decreases that of the pure retailer, unlike the integrated economy of Section 5.2.1.
As argued in Section 4, the forward strategy $f_k = 0$ is always admissible, which ensures an increase of utility for producers when forward contracts are available. The situation is different for pure retailers. While the strategy $f_k = 0$ is admissible, it does not yield the profit without a forward market if the other retailers do trade forward contracts. In the absence of a collusive behaviour where all retailers avoid trading forward contracts, they are not guaranteed to increase their utility compared to the case without a forward market. In addition, not contracting forward is sub-optimal when the others do, implying that all retailers will indeed trade forward contracts. Once partially hedged on the forward market, the retailers can offer lower retail prices. In the meantime, market shares are fixed proportionally to risk tolerances, as demand is inelastic to retail price, and are thus not impacted by the presence of the forward market. There are therefore no possibilities to expand the market shares and compensate for the loss of profit. We observe that the gain from hedging is half the expected loss on the retail market induced by the fall of retail price: $U_2^F - U_2^{NF} \simeq \frac{1}{2}(p^F - p^{NF})\mathbb{E}[D]$. This mechanism explains the retail price fall and the loss of utility of the retailers.

To illustrate this argument, we computed some indicators of risk and profit as in the previous subsection. The results are presented in Table 2. We now observe that actor 1 is able to both increase its average profit by 0.88 % and decrease its risk by 99 %, hence increasing its utility by 326 %. In the meantime, actor 2 decreases its risk, average profit and utility by 98 %, the average profit reduction being twice the risk reduction, as we mentioned above.

Conclusion. This case study shows that:

- forward trading has a downward impact on retail price

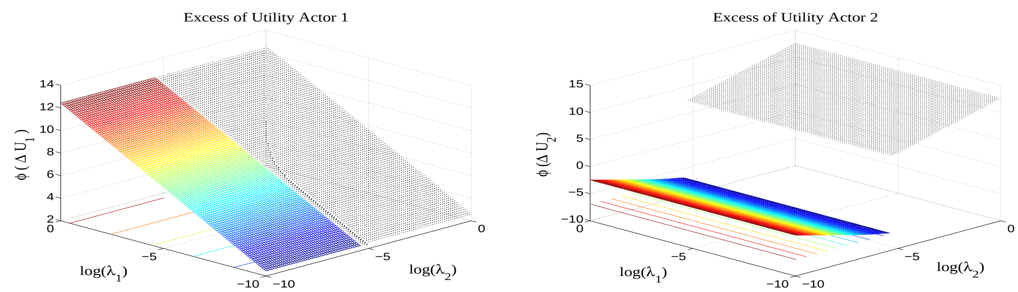

Figure 5.7: Excess of utility due to the presence of the forward market for actor 1 and 2, as functions of the logarithm of risk aversion coefficients.

		Actor 1	Actor 2
Utility:	$\frac{\Delta U}{\lvert U\rvert}$	326 %	-97 %
Av. Profit:	$\frac{\Delta \mathbb{E}}{\lvert\mathbb{E}\rvert}$	0.88 %	-97 %
Risk:	$\frac{\Delta \mathrm{Var}}{\mathrm{Var}}$	-99 %	-97 %
Profit vs. Risk:	$\frac{\Delta \mathbb{E}}{\lambda \Delta \mathrm{Var}}$	-0.62 %	2
Performance:	$\Delta \frac{\mathbb{E}}{\lambda \mathrm{Var}}$	117.63	≈ 0

Table 2: Impact of forward trading on utility, profit and risk, when $\lambda_1 = \lambda_2 = 10^{-6}$.

- there is a strong asymmetry between producers and suppliers and an equilibrium may not exist when retailers are highly risk averse, even in the presence of a forward market

- Due to the fall in retail price, increase in utility by trading forward contracts is not guaranteed for retailers

5.2.3 Unbundling of the generation activity

We now consider the case where the former monopoly is forced to share its generation assets. Actor 1 still denotes the former monopoly, while actor 2 stands for the new pure producer. We are interested in observing the impact of the forward market on retail prices in this setting. This experiment is representative of a deregulation of the generation activity.

In this case, three parameters are needed to fully describe the problem: the two risk aversion coefficients and the distribution of generation assets among the two actors. Let us first fix the risk aversion coefficients $\lambda_1 = \lambda_2 = 10^{-6}$ and let only the distribution of generation assets vary. This way, we can study the impact of the level of competition on the equilibrium. We choose three different methods to make the distribution of generation

means vary between the two actors.
The first method consists in starting with all production means owned by actor 1 and transferring progressively generation assets from actor 1 to actor 2 by decreasing merit order. Suppose there are 100 power plants in the economy, ranked by merit order. When actor 1 has 100 % of the production means, it owns them all and actor 2 is in fact a pure trader. When actor 1 has x%, $0 < x < 100$, of the production means, it owns the first x power plants of lowest marginal cost. When it has 0 %, actor 1 becomes a pure retailer. With this method actor 1 has a large advantage over actor 2 since it can always produce at a lower cost than its competitor.
Method 2 is the opposite: the generation assets are transferred from actor 1 to actor 2 by increasing merit order.
Method 3 is halfway between the previous two. If the power plant are ranked from 1 to 100 by merit order, we first transfer the power plants of even label from 2 to 100 then the power plants of odd label from 99 down to 1. This method is representative of a better balanced competition between the two actors.
Notice that the case where actor 1 has no generation capacity corresponds to the case of the previous subsection.

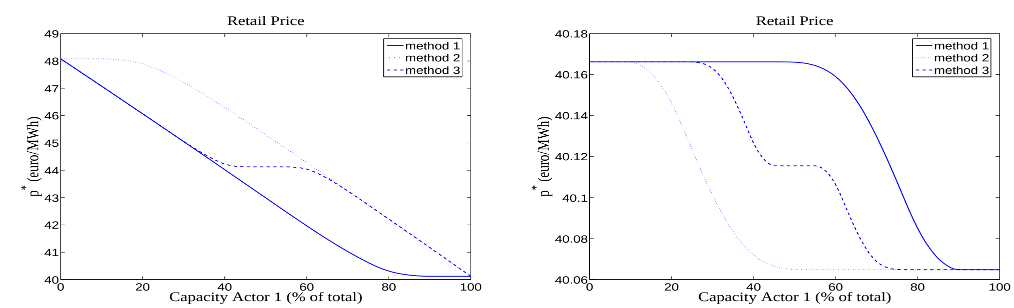

Figure 5.8: Equilibrium retail price for $\lambda_1 = \lambda_2 = 10^{-6}$, without (left) and with (right) a forward market, as a function of actor 1's proportion of total capacity.

In this setting, the equilibrium retail price decreases with actor 1's total capacity (see Figure 5.8) in both cases, with and without a forward market. It is thus minimal when actor 1 owns all the generation assets. Nonetheless, the variations of retail price in the presence of a forward market are highly reduced. In this context, unbundling of the generation monopoly has an upward impact on the retail price. In addition, we observe that the forward market has a major impact on retail price, which decreases by up to 20%.

Concerning the utility of both actors, we observe on Figure 5.9, as in paragraph 5.2.2, that the presence of the forward market always increases the utility of actor 2, the pure producer, while it always decreases that of actor 1, the integrated producer. The intensity of this decrease is nonetheless reduced with the level of integration of the actor. As soon as pure producers are present in the economy, retailers are forced to trade forward contracts and decrease the retail price, decreasing their utility in the meantime. From this point of view, there exists a strong asymmetry between producers and retailers.

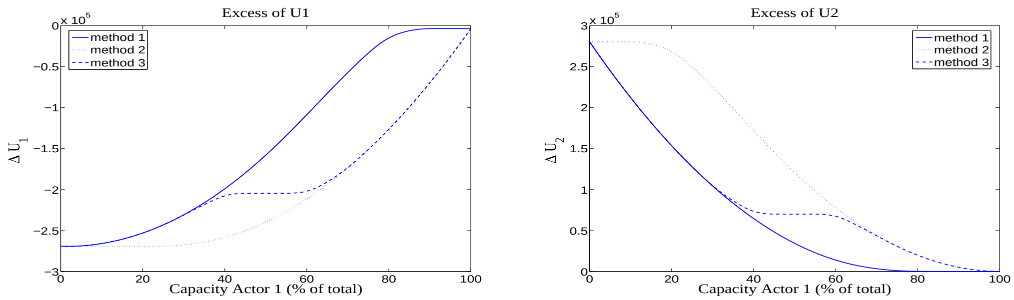

Figure 5.9: Excess of utility for actor 1 and actor 2 due to the presence of the forward market, as a function of actor 1's proportion of total capacity.

If we now fix the distribution of generation assets and vary the risk aversion coefficients in the absence of a forward market we obtain Figure 5.10. The left figure corresponds to the situation where actor 1 owns 70 % of generation capacity, and the right figure to 60 %. In comparison to Figure 5.6 we observe that an equilibrium always exists if the level of integration of actor 1, i.e. its proportion of total capacity, is high enough. This proves that vertical integration can induce better risk diversification than forward trading when retailers are highly risk averse. In this context, vertical integration is more robust to high risk aversion.

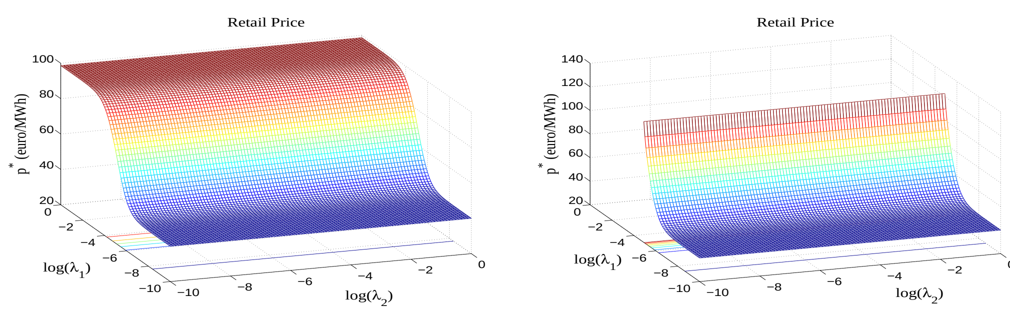

Figure 5.10: Equilibrium retail price in the absence of a forward market, when actor 1 owns 70 % (left) and 60 % (right) of production capacity, as a function of risk aversion coefficients.

Conclusion. This case study shows that:

- vertical integration and forward trading both have a downward impact on retail price

- the impact of forward trading on the actors' utility is dissymmetric between retailers and producers

- impact of vertical integration on retail price is drastically reduced in the presence of a forward market

- vertical integration is a more robust risk diversification tool in the presence of highly risk averse retailers

5.2.4 Competition between integrated producers

We finally consider the case where the former monopoly faces competition from an integrated producer. In this case the economy is fully integrated, which implies that the retail price is not impacted either by the presence of a forward market or by the distribution of the generation means. As before, we start by fixing the risk aversion coefficients $\lambda_1 = \lambda_2 = 10^{-6}$, and vary the distribution of generation assets available to each actor.

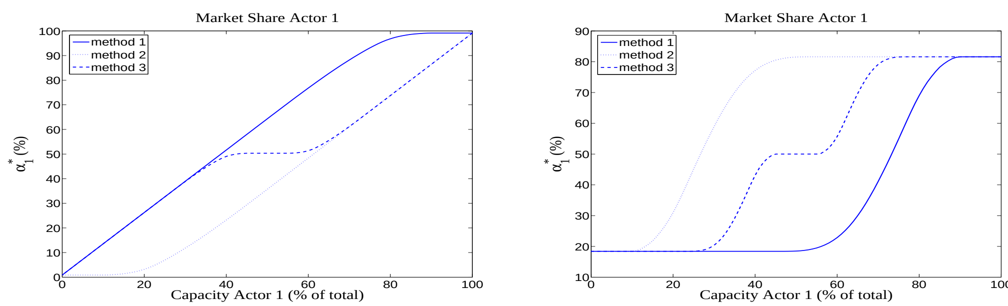

Figure 5.11: Actor 1's market share in the absence (left) and presence (right) of the forward market, for $\lambda_1 = \lambda_2 = 10^{-6}$, as a function of actor 1's proportion of total capacity.

Figure 5.11 shows actor 1's market share as a function of its total capacity available, in the absence (left) and the presence (right) of a forward market. We observe, as we did in paragraph 5.2.1, that the presence of the forward market enhances the ability of an actor owning few generation means to take a significant position on the retail market.

Utilities of both actors increase with the total capacity available, in both cases with or without a forward market, as we can see for example on Figure 5.12-left. Notice that in this case, the actors are symmetric, and thus have the same behaviour. We also observe that the presence of the forward market has an upward impact on the utility of both actors (see Figure 5.12-right), although the excess of utility is very small (in the range of 0.1%).

Conclusion. This case study shows that:

- trading forward contracts allows actors with low generating capacity to corner larger market shares

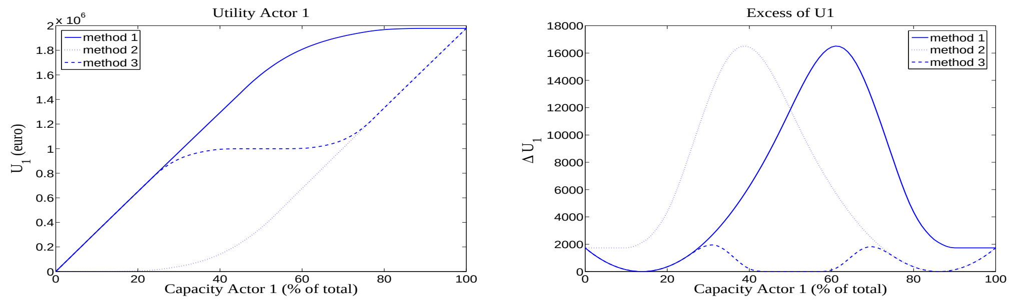

Figure 5.12: Utility in the absence of the forward market (left) and excess of utility due to the presence of the forward market (right) for actor 1 with $\lambda_1 = \lambda_2 = 10^{-6}$.

- both actors can benefit from trading forward contracts
- vertical integration breaks the asymmetry between retailers and producers

5.3 Impact of vertical integration

We now focus on the impact of the vertical organization of the actors on both retail prices and the actors' utility.

5.3.1 Methodology

We consider four actors, two pure retailers R_1, R_2, and two pure producers P_1, P_2, and compute the associated equilibrium. We then suppose that R_1 and P_1 decide to merge, leading to a situation involving an integrated producer I_1, R_2 and P_2. Finally we consider the case where R_2 and P_2 also decide to merge, leading to a situation with two integrated producers I_1 and I_2. We assume that R_1, R_2, P_1 and P_2 have the same risk aversion coefficient equal to λ.
The natural relationship between the risk aversion coefficients of actor I_1, R_1 and P_1 is difficult to identify. It might depend on the synergies resulting from the merger, the cost reductions, the risk management policy in the new structure, etc. In the absence of a forward market, the risk aversion coefficients of the producers do not impact the equilibrium. It then seems reasonable to attribute a coefficient λ to I_1 and I_2. This is no more the case in the presence of a forward market. The aggregate risk aversion coefficients appearing in the previous sections suggest one should define the risk tolerance of actor I_1 as the sum of R_1's and P_1's risk tolerances. Similarly, we pointed out in the previous subsection that an N-actor competition can be reduced to a 4-actor competition by aggregating all actors of the same type, and by summing over risk tolerances. Nonetheless, when aggregating actors of different kinds, this argument does not seem so obvious. In the absence of an

obvious answer, we choose to apply the same rule as in the absence of a forward market and attribute a coefficient λ to I_1 and I_2.
In order to evaluate the benefit that actors R_1 and P_1 would have if they merge, we compare the utility $\mathrm{MV}_\lambda[\Pi_{I_1}]$ of the resulting entity to the aggregate utility $\mathrm{MV}_\lambda[\Pi_{R_1} + \Pi_{P_1}]$ of R_1 and P_1. In the following subsections, we denote by actor 1 either the pair R_1, P_1 or I_1 if they have merged.

5.3.2 Absence of a forward market

We start by considering the equilibrium in the absence of a forward market when $\lambda = 10^{-6}$. Figure 5.13 shows the variations of retail price and actor 1's market share with actor 1's proportion of total capacity (the distribution of generation means varies according to method 1) for the three cases. We observe that the retail price is lower in the presence of vertically integrated actors. We also observe that the integrated actor I_1 has a higher market share than the pair R_1, P_1 as independent entities. Vertical integration has a downward impact on retail prices and gives an advantage in terms of market shares over pure retailers.

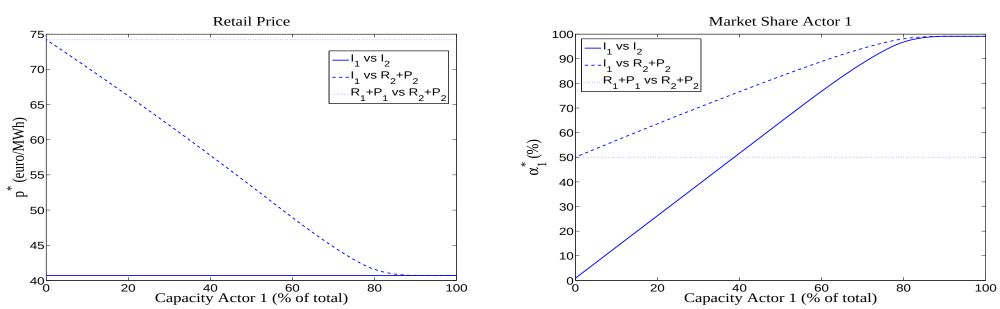

Figure 5.13: Equilibrium retail price (left) and market share of actor 1 (right) in the absence of a forward market, as functions of actor 1's proportion of total capacity.

Figure 5.14 shows that, in terms of utility, a firm is worst off being integrated, whatever the type of its competitors. We observe here the same effect as with forward hedging. Vertical integration induces such a fall in retail price that the decrease in variance is offset by the decrease in expected profit. As a result the utility decreases. Nonetheless, there is no such equilibrium effect with vertical integration that drives the actors to integrate although they suffer a decrease in utility. Here, the stable equilibrium would be that no one integrates.
The great difference between vertical integration and forward trading is that it can increase utility when actors are highly risk averse. The same computation as in Figure 5.14 with a risk aversion coefficient $\lambda = 10^{-5.1}$ gives the following results. Figure 5.15-left shows

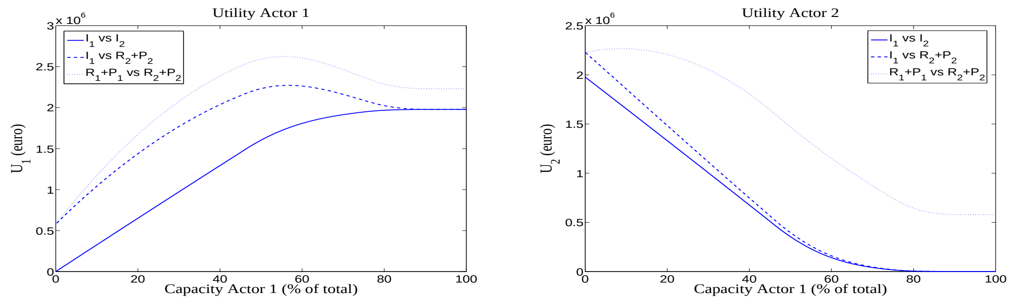

Figure 5.14: Utility of actor 1 (left) and actor 2 (right) in the absence of a forward market, as functions of actor 1's proportion of total capacity.

that, provided that R_1 owns enough generation capacity, the utility of I_1 facing $R_2 + P_2$ is higher than that of $R_1 + P_1$, hence an incentive to integrate in face of non-integrated actors. Similarly, figure 5.15-right shows that the utility of I_2 facing I_1 is always higher than that of $R_2 + P_2$, hence an incentive to integrate in face of integrated actors. The stable equilibrium in this case would be that all actors integrate.

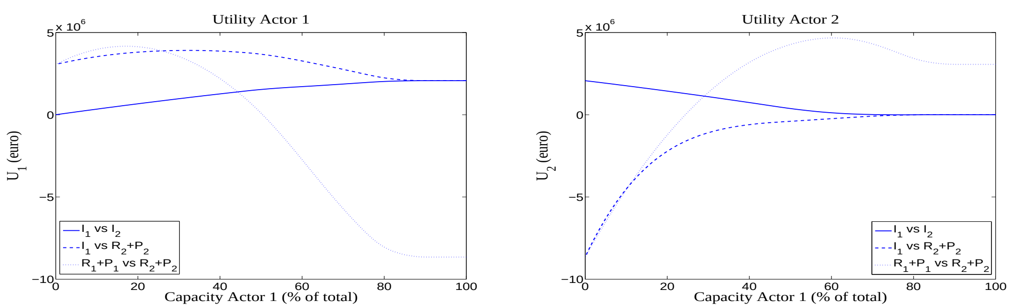

Figure 5.15: Utility of actor 1 (left) and actor 2 (right) in the absence of a forward market, as functions of actor 1's proportion of total capacity.

Conclusion. This case study shows that:

- vertical integration has a downward impact on retail price
- integrated producers can take larger market shares than pure suppliers
- in terms of utility, actors with a low risk aversion are better off not integrating and taking advantage of a higher retail price
- in terms of utility, actors with a high risk aversion are better off integrating and taking advantage of this natural hedge
- vertical integration is a better risk diversification lever in the presence of highly risk averse actors

5.3.3 Presence of a forward market

In the presence of a forward market and with risk aversion coefficient $\lambda = 10^{-6}$, the behaviour of retail price and market shares with respect to the actors' total capacity is similar as in the absence of it. Figure 5.16 shows the variation of these quantities with actor 1's total capacity. We observe that vertical integration has a downward impact on retail prices, however this impact is smaller (at most 1 %) than in the previous case. It also allows the integrated actor to gain higher market shares.

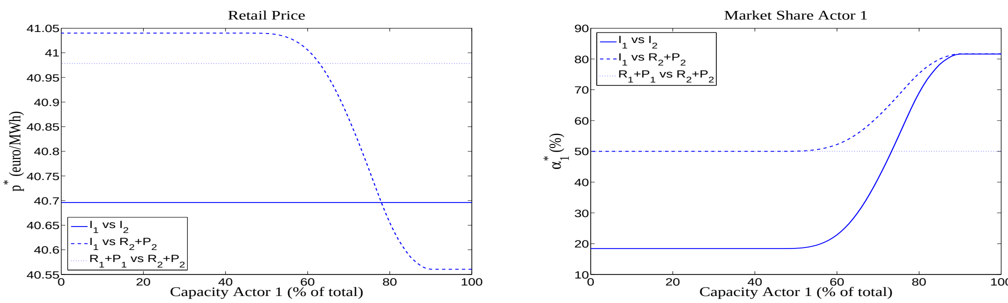

Figure 5.16: Equilibrium retail price (left) and market share of actor 1 (right) in the presence of a forward market, as functions of actor 1's proportion of total capacity.

Regarding utilities, Figure 5.17 shows that the impact of vertical integration is drastically reduced in the presence of a forward market. Being integrated or not almost leads to the same utility. In addition, even for a larger risk aversion (ex: $\lambda = 10^{-5.1}$), the incentive for vertical integration is reduced in the presence of a forward market.

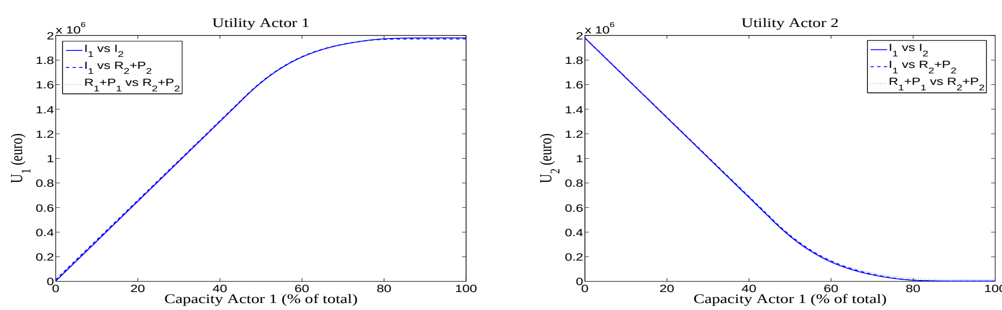

Figure 5.17: Utility of actor 1 (left) and actor 2 (right) in the presence of a forward market, as functions of actor 1's proportion of total capacity.

Conclusion. This case study shows that:

- the qualitative impact of vertical integration is not changed in the presence of a forward market

- the intensity of this impact is drastically reduced when forward contracts are available
- the incentive to integrated is drastically reduced in the presence of a forward market

6 Extensions

6.1 Extension to the case of an elastic demand

In the previous sections the model was analyzed under the hypothesis of inelastic demand. If demand is elastic to spot price but we still assume perfect competition on the spot market, the results are unchanged because the spot market equilibrium remains independent of retail and forward equilibrium. Nevertheless, if perfect competition is replaced by Cournot competition for example, the argument does not hold anymore and we are beyond the scope of our analysis.

As mentioned in Remark 2.1, the model can also be solved similarly in the presence of demand elasticity to retail price. Suppose that the demand is a random function of the retail price $D(p)$. In this case, we can solve the equilibrium problem as in Sections 3 or 4 and equations (3.5) and (4.1)-(4.2)-(4.3) remain valid. The only difference lies in the equation satisfied by p^*. In the presence of elasticity to retail prices, this equation reads:

$$0 = \mathbb{E}[(p^* - P^*(p^*))D(p^*)] - 2\Lambda_{\mathcal{R}}\mathrm{Cov}[(p^* - P^*(p^*))D(p^*), (p^* - P^*(p^*))D(p^*) + \Pi_I^g(p^*)]$$

in the absence of a forward market, and:

$$\begin{aligned} 0 &= \mathbb{E}[(p^* - P^*(p^*))D(p^*)] - 2\Lambda_{\mathcal{R}}\mathrm{Cov}[(p^* - P^*(p^*))D(p^*), (p^* - P^*(p^*))D(p^*) + \Pi_I^g(p^*)] \\ &\quad +2\Lambda_{\mathcal{R}}\frac{\mathrm{Cov}[P^*(p^*), (p^* - P^*(p^*))D(p^*)]}{\mathrm{Var}[P^*(p^*)]}\mathrm{Cov}\left[P^*(p^*), (p^* - P^*(p^*))D(p^*) + \Pi_I^g(p^*)\right] \\ &\quad -2\Lambda\frac{\mathrm{Cov}[P^*(p^*), (p^* - P^*(p^*))D(p^*)]}{\mathrm{Var}[P^*(p^*)]}\mathrm{Cov}\left[P^*(p^*), p^*D(p^*) - C(D(p^*))\right] \end{aligned}$$

in the presence of it. This non-linear equation may be hard to solve, especially if we cannot have an explicit formulation of the spot equilibrium. Nevertheless, the equation simplifies in some cases, as we show in the following subsection.

6.1.1 Particular case of quadratic cost functions

Consider the particular case of quadratic and symmetric cost functions:

$$c_k(x) = \frac{c}{2}x^2 \ , \ \forall k \in \mathcal{K} \ .$$

Suppose in addition that demand is a linear function of retail price of the form:

$$D(p) = D_0 - \mu(p - p_0) \ ,$$

where D_0 is an exogenous random variable, p_0 is some non-negative reference price and $\mu > 0$. In this setting the equilibrium on the spot market can be solved explicitly and we obtain:

$$\begin{array}{rclrcl} S_k^* &=& \frac{1}{N_P} D(p^*), & P^* &=& \frac{c}{N_P} D(p^*) \\ \Pi_k^g &=& \frac{c}{2N_P^2} D^2(p^*), & \Pi_I^g &=& \frac{cN_I}{2N_P^2} D^2(p^*) \\ \Pi^g &=& \frac{c}{2N_P} D^2(p^*) & & & \end{array} \tag{6.1}$$

where N_P is the number of producers and N_I the number of integrated producers. The retail price at equilibrium p^* is then given by the smallest root of a second order polynomial equation (cf. Appendix A).

6.1.2 Examples

To illustrate the impact of demand elasticity, we compute the equilibrium found above in two cases. First, we study the competition between one pure retailer and one pure producer, as we did in paragraph 5.2.2. Second, we examine the case of a pure retailer and an integrated producer, as in paragraph 5.2.1. To this end, we use the demand samples of the previous section. Taking expectation on both sides of the equation giving P^* in (6.1), we estimate the cost function coefficient c as:

$$c \;=\; N_P \frac{\mathbb{E}[P^*]}{\mathbb{E}[D]} \;\simeq\; N_P \times 5.6143\ 10^{-4}\ .$$

We set $p_0 = \mathbb{E}[P] \simeq 37.95$ as the expected spot price, and we express the elasticity coefficient μ in percentage of expected demand $\mathbb{E}[D]$.

Pure retailer and pure producer. In this case, actor 1 is the pure producer and actor 2 the pure retailer. We set both risk aversion coefficients to $\lambda_1 = \lambda_2 = 10^{-6}$. Figure 6.18-right shows the equilibrium retail price in the absence and presence of a forward market, as a function of μ, in percentage of $\mathbb{E}[D]$. We still observe that the presence of a forward

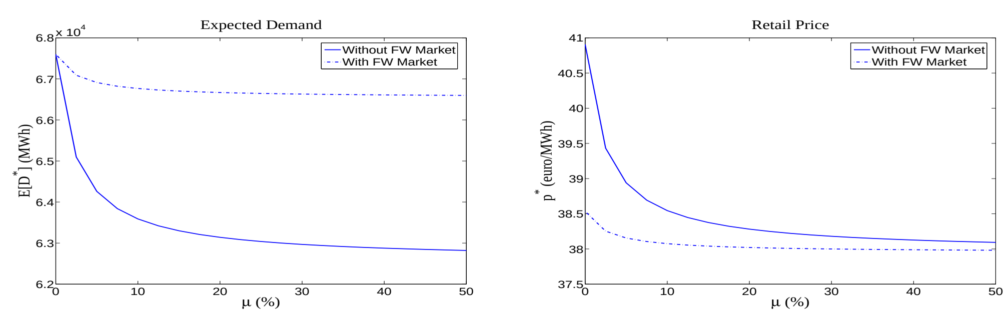

Figure 6.18: Expected demand (left) and retail price (right) at equilibrium in the absence and presence of a forward market as a function of μ.

market has a downward impact on retail price. Nonetheless, this impact tends to shrink as demand elasticity increases. We also note that, in both cases, demand elasticity decreases the retail price. As customers respond to retail price, the retailers face low demand if they set a high retail price. They are thus forced to decrease the equilibrium retail price.

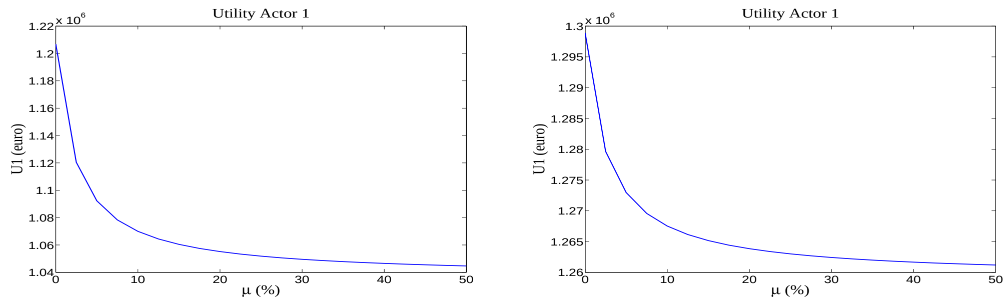

Figure 6.19: Actor 1's utility in the absence (left) and presence (right) of a forward market as a function of μ.

Figure 6.19 shows actor 1's utility with and without a forward market. We still observe that this utility increases in the presence of a forward market, but the interesting aspect here is that the pure producer's utility decreases with demand elasticity. We already mentioned the strong asymmetry between retailers and producers: producers set the equilibrium spot price and thus impact retailers' profit, while retailers cannot impact producers' profit. When demand is elastic to retail price, this asymmetry is reduced and retailers impact producers' profit via the retail price. Since the expected demand decreases with demand elasticity (see Figure 6.18-left), so does the producers' utility.

Concerning actor 2, we observe on figure 6.20 that its utility decreases with demand

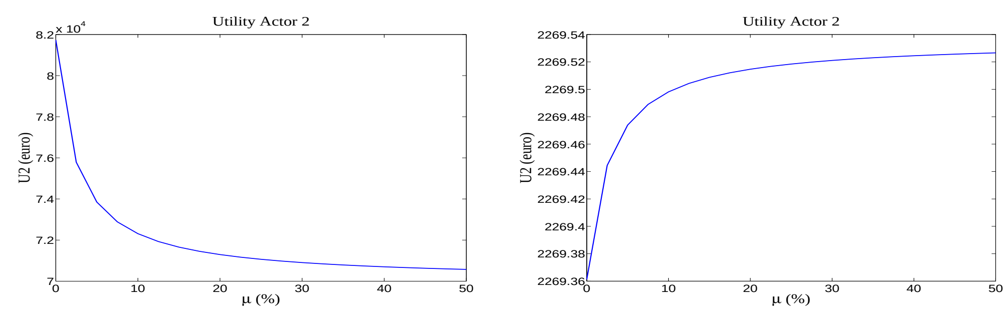

Figure 6.20: Actor 2's utility in the absence (left) and presence (right) of a forward market as a function of μ.

elasticity in the absence of a forward market but increases in its presence. In the absence of a forward market, the pure retailer is impacted by both the decrease in demand and in retail price. In the presence of a forward contract, the pure retailer is able to transfer more risk to the pure producer and takes advantage of demand elasticity. In particular the excess of utility of actor 2 due to the presence of a forward market increases with demand elasticity, as we suggested in Subsection 5.2.2.

Pure retailer and integrated producer. In order to study the impact of demand elasticity on market shares, we consider the case where an integrated producer, actor 1, competes with a pure retailer, actor 2. The risk aversion coefficients of both actors are set to $\lambda_1 = \lambda_2 = 10^{-6}$. Figure 6.21-right shows actor 1's market share in the presence and ab-

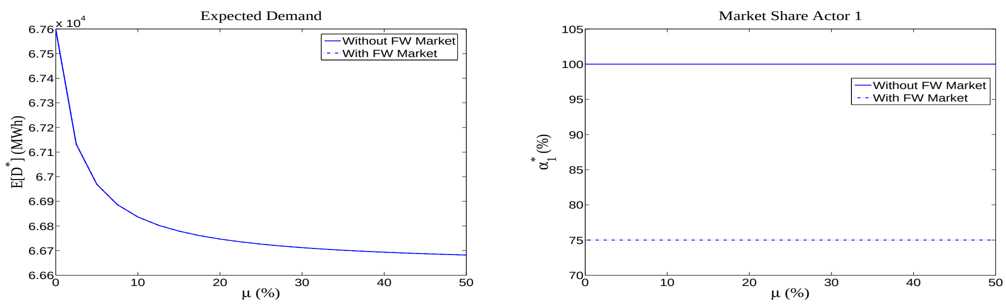

Figure 6.21: Expected demand (left) and actor 1's market share (right) at equilibrium in the absence and presence of a forward market as a function of μ.

sence of a forward market. We first observe, as in Subsection 5.2.1, that the pure retailer is not able to compete with the integrated retailer in the absence of a forward market. In this example, both actors have the same risk aversion coefficient. Actor 2 has no possibility to enter the market and has a market share almost equal to zero. In the presence of a forward market though, actor 2 is able to corner 25% of market share. We also observe an interesting effect: market shares are not affected by demand elasticity. Although retail price and

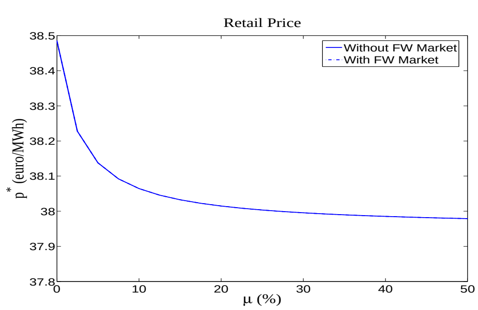

Figure 6.22: Equilibrium retail price at equilibrium as a function of μ.

expected demand decrease with demand elasticity (see Figures 6.21-left and 6.22), market shares remain unchanged. The advantage of being integrated is not enhanced nor reduced in the presence of demand elasticity.

Conclusion. The results presented above show that the assumption of demand inelasticity to retail price, assumed in the previous sections, is not restrictive regarding our objective. Our conclusions concerning the impact of vertical integration and forward trading on risk diversification remain unchanged in the presence of demand elasticity to retail price.

6.2 Other definitions of an equilibrium

In our analysis, the equilibrium was defined in Definition 2.1 as a simultaneous equilibrium on both the retail and forward markets. The following two definitions can also be considered, that introduce a sequentiality between the two markets. We use the following notation: $(\alpha^*_{-k}, \alpha_k) := (\alpha^*_1, \ldots, \alpha^*_{k-1}, \alpha_k, \alpha^*_{k+1}, \ldots, \alpha^*_{|\mathcal{R}|})$.

Definition 6.1. *A sequential equilibrium with anticipation of the forward positions is a quadruple* $(p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbf{A} \times \mathbf{F}$*, such that there exist two functions* $\bar{q} : \mathbf{A} \mapsto \mathbb{R}$ *and* $\bar{f} : \mathbf{A} \mapsto \mathbf{F}$ *verifying*

$$\bar{f}_k(\alpha) = \operatorname*{argmax}_{f_k} \mathrm{MV}_{\lambda_k} \left[\Pi_k \left(p^*, \bar{q}(\alpha), \alpha_k, f_k\right)\right], \ \forall k \in \mathcal{K} \tag{6.2}$$

$$\alpha^*_k = \operatorname*{argmax}_{\alpha_k} \mathrm{MV}_{\lambda_k} \left[\Pi_k \left(p^*, \bar{q}(\alpha^*_{-k}, \alpha_k), \alpha_k, \bar{f}_k(\alpha^*_{-k}, \alpha_k)\right)\right], \ \forall k \in \mathcal{K} \tag{6.3}$$

$$q^* = \bar{q}(\alpha^*) \tag{6.4}$$

$$f^* = \bar{f}(\alpha^*) \tag{6.5}$$

This equilibrium corresponds to a two-step equilibrium where the actors first decide their market shares. Define the set

$$\widetilde{\mathbf{A}} := \left\{(\alpha_k)_{k\in\mathcal{K}} \in \mathbb{R}^{|\mathcal{K}|} \ : \ \forall k \notin \mathcal{S}, \ \alpha_k = 0 \text{ and } \sum_{k\in\mathcal{K}} \alpha_k = 1\right\},$$

this following definition describes the situation where the actors first invest on the forward market before deciding their market shares:

Definition 6.2. *A sequential equilibrium with anticipation of the market shares is a quadruple* $(p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbf{A} \times \mathbf{F}$*, such that there exist two functions* $\bar{p} : \mathbf{F} \mapsto \mathbb{R}$ *and* $\bar{\alpha} : \mathbf{F} \mapsto \widetilde{\mathbf{A}}$ *verifying*

$$\bar{\alpha}_k(f) = \operatorname*{argmax}_{\alpha_k} \mathrm{MV}_{\lambda_k} \left[\Pi_k(\bar{p}(f), q^*, \alpha_k, f_k)\right], \ \forall k \in \mathcal{K} \tag{6.6}$$

$$f^*_k = \operatorname*{argmax}_{f_k} \mathrm{MV}_{\lambda_k} \left[\Pi_k \left(\bar{p}(f^*_{-k}, f_k), q^*, \bar{\alpha}_k(f^*_{-k}, f_k), f_k\right)\right], \ \forall k \in \mathcal{K} \tag{6.7}$$

$$p^* = \bar{p}(f^*) \tag{6.8}$$

$$\alpha^* = \bar{\alpha}(f^*) \tag{6.9}$$

We can in fact prove that the above two definitions are equivalent to Definition 2.3. This is done by deriving explicitly the expression of the equilibrium in each case and showing that the equilibrium prices and positions are exactly the same as in Proposition 4.1. It is probable that this equivalence result disappears when the markets are no longer assumed competitive.

7 Conclusion

We developed a competitive equilibrium model for retail, forward and spot markets of a non-storable good. Our aim was to understand the fundamental mechanisms of risk diversification in this kind of economy. We stated the explicit formulation of the equilibrium prices and positions on each market. Finally, we illustrated the model with case studies in the electricity sector.

We showed that vertical integration and forward hedging are two levers for achieving risk diversification, that exhibit similar properties. First, they both have a downward impact on retail price. Second, they both allow for actors with low generation capacity to corner larger market shares. Third, they both tend to decrease downstream firms' utility when upstream firms are only partially integrated. Fourth, the impact of one of these levers one retail price and utilities is drastically reduced in the presence of the other.

We also showed that they exhibit discrepancies because of the asymmetric structure of risk between upstream and downstream. First, we showed that vertical integration restores this symmetry while forward hedging does not. Second, vertical integration is more robust towards high risk aversions in the sense that it can achieve risk diversification when forward hedging fails. Third, vertical integration can also increase downstream firms' utility provided that they have sufficiently high risk aversion. Fourth, a non-integrated economy can be a stable equilibrium whereas actors always trade forward contract at equilibrium when they are available. Finally, we proved that these conclusions still prevail in the presence of demand elasticity to retail price.

Our main conclusion is that vertical integration has a positive impact, especially in terms of lower retail prices. In terms of risk diversification, it exhibits properties that linear instruments such as forward contracts can not achieve. It finally helps reducing the asymmetric risk structure between upstream and downstream. We then converge to the idea of Chao, Oren and Wilson that some level of integration is benefic even in the presence of wholesale markets. Nonetheless, our approach does not take into account the impact of market power that such large actors can exercise. Allaz showed in [1] the implications of forward trading on producers' market power. Vertical integration seems to have a different impact. It would be interesting to study the resulting impact when these two factors coexist. Future developments in the field of electricity market equilibria should integrate those elements into the risk management analysis. Our analysis could then serve as a benchmark to measure the effects of imperfect competition.

A Appendix

<u>Proof of Proposition 3.1</u>

Maximizing the mean-variance criteria over α_k yields the following first order condition for actor $k \in \mathcal{R}$:

$$0 \quad = \quad \mathbb{E}[(p^* - P^*)D] - 2\lambda_k \mathrm{Cov}[(p^* - P^*)D, (p^* - P^*)\alpha_k^* D + \Pi_k^g] \ ,$$

which is sufficient for maximality by convexity, and gives:

$$\alpha_k^* \quad = \quad \frac{\mathbb{E}[(p^* - P^*)D]}{2\lambda_k \mathrm{Var}[(p^* - P^*)D]} - \frac{\mathrm{Cov}[(p^* - P^*)D, \Pi_k^g]}{\mathrm{Var}[(p^* - P^*)D]} \ .$$

The coupling constraint $\sum_{k\in\mathcal{R}} \alpha_k^* = 1$ gives the condition on p^*:

$$0 \quad = \quad \mathbb{E}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}} \mathrm{Var}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}} \mathrm{Cov}[(p^* - P^*)D, \Pi_I^g] \ ,$$

and allows us to derive (3.6).

<u>Proof of Proposition 4.1</u>

If $k \in \mathcal{R}$, the first order condition of profit maximization reads:

$$0 \quad = \quad M \begin{bmatrix} f_k^* \\ \alpha_k^* \end{bmatrix} - \begin{bmatrix} \frac{\mathbb{E}[P^* - q^*]}{2\lambda_k} - \mathrm{Cov}[P^*, \Pi_k^g] \\ \frac{\mathbb{E}[(p^* - P^*)D]}{2\lambda_k} - \mathrm{Cov}[(p^* - P^*)D, \Pi_k^g] \end{bmatrix} \ ,$$

where M is the variance-covariance matrix of vector $[P^*, (p^* - P^*)D]$. By inverting the system, we obtain the expressions of f_k^* and α_k^* in terms of p^* and q^*. If $k \notin \mathcal{R}$, Π_k does not depend on α_k and the first order condition reads:

$$0 \quad = \quad \mathbb{E}[P^* - q^*] - 2\lambda_k \mathrm{Cov}[P^* - q^*, (P^* - q^*)f_k^* + \Pi_k^g] \ .$$

Market-clearing constraint (2.2) can be expressed as:

$$\begin{aligned} 0 \quad &= \quad \sum_{k\in\mathcal{K}} f_k^* \\ &= \quad \frac{\mathrm{Var}[(p^* - P^*)D]}{\Delta} \left(\frac{\mathbb{E}[P^* - q^*]}{2\Lambda_{\mathcal{R}}} - \mathrm{Cov}[P^*, \sum_{k\in\mathcal{R}} \Pi_k^g] \right) \\ &\quad - \frac{\mathrm{Cov}[P^*, (p^* - P^*)D]}{\Delta} \left(\frac{\mathbb{E}[(p^* - P^*)D]}{2\Lambda_{\mathcal{R}}} - \mathrm{Cov}[(p^* - P^*)D, \sum_{k\in\mathcal{R}} \Pi_k^g] \right) \\ &\quad + \frac{\mathbb{E}[P^* - q^*]}{2\mathrm{Var}[P^*]} \left(\frac{1}{\Lambda} - \frac{1}{\Lambda_{\mathcal{R}}} \right) - \frac{\mathrm{Cov}[P^*, \sum_{\mathcal{K}_{\mathcal{R}}} \Pi_k^g]}{\mathrm{Var}[P^*]} \ , \end{aligned}$$

where Δ is in fact the determinant of M, which leads to:

$$\begin{aligned} 0 \quad = \quad & \Delta \left(\frac{\mathbb{E}[P^* - q^*]}{2\Lambda} - \mathrm{Cov}[P^*, \sum_{k\in\mathcal{K}} \Pi_k^g] \right) \\ & - \mathrm{Cov}[P^*, (p^* - P^*)D]\mathrm{Var}[P^*] \left(\frac{\mathbb{E}[(p^* - P^*)D]}{2\Lambda_{\mathcal{R}}} - \mathrm{Cov}[(p^* - P^*)D, \sum_{k\in\mathcal{R}} \Pi_k^g] \right) \\ & + \mathrm{Cov}^2[P^*, (p^* - P^*)D] \left(\frac{\mathbb{E}[P^* - q^*]}{2\Lambda_{\mathcal{R}}} - \mathrm{Cov}[P^*, \sum_{k\in\mathcal{R}} \Pi_k^g] \right) \ . \end{aligned} \tag{A.1}$$

Load satisfaction constraint (2.1) reads:

$$
\begin{aligned}
1 &= \sum_{k\in\mathcal{R}} \alpha_k^* \\
&= -\frac{\mathrm{Cov}[P^*, (p^*-P^*)D]}{\Delta}\left(\frac{\mathbb{E}[P^*-q^*]}{2\Lambda_{\mathcal{R}}} - \mathrm{Cov}[P^*, \sum_{k\in\mathcal{R}} \Pi_k^g]\right) \\
&\quad + \frac{\mathrm{Var}[P^*]}{\Delta}\left(\frac{\mathbb{E}[(p^*-P^*)D]}{2\Lambda_{\mathcal{R}}} - \mathrm{Cov}[(p^*-P^*)D, \sum_{k\in\mathcal{R}} \Pi_k^g]\right) ,
\end{aligned}
$$

which yields, using (A.1):

$$
0 = \frac{\mathbb{E}[P^*-q^*]}{2\Lambda} - \mathrm{Cov}[P^*, (p^*-P^*)D + \sum_{k\in\mathcal{K}} \Pi_k^g] .
$$

As $\sum_{k\in\mathcal{K}} \Pi_k^g = P^*D - C(D)$, we obtain equation (4.3). Using this result to simplify (A.1), we derive the desired expression (4.4) for p^*. Finally, using those two equations, we can re-arrange the expressions for f_k^* and α_k^* to obtain equations (4.1) and (4.2).

Equations for the retail price under elastic demand

The retail price at equilibrium p^* is then given by the smallest root of the second order polynomial:

$$
\begin{aligned}
0 = \; & (p^*)^2 \left\{-\mu\left(1+\mu\frac{c}{N_P}\right) - 2\Lambda_{\mathcal{R}}\left[1+4\mu\frac{c}{N_P}\left(1+\mu\frac{c}{N_P}\right) - \mu\frac{c}{N_P}\frac{N_I}{N_P}\left(1+2\mu\frac{c}{N_P}\right)\right]\mathrm{Var}[D_0]\right\} \\
& + p^*\left\{\left(1+2\mu\frac{c}{N_P}\right)\mathbb{E}[D_0] + \mu p_0\left(1+2\mu\frac{c}{N_P}\right)\right\} \\
& + p^*\Lambda_{\mathcal{R}}\frac{c}{N_P}\left(4-\frac{N_I}{N_P}+4\mu\frac{c}{N_P}\left(2-\frac{N_I}{N_P}\right)\right)\left(\mathrm{Cov}[D_0, D_0^2] + 2\mu p_0 \mathrm{Var}[D_0]\right) \\
& - \frac{c}{N_P}\left(\mathbb{E}[D_0^2] + 2\mu p_0\mathbb{E}[D_0] + \mu^2 p_0^2\right) \\
& - \Lambda_{\mathcal{R}}\frac{c^2}{N_P^2}\left(2-\frac{N_I}{N_P}\right)\left(\mathrm{Var}[D_0^2] + 4\mu p_0\mathrm{Cov}[D_0, D_0^2] + 4\mu^2 p_0^2 \mathrm{Var}[D_0]\right)
\end{aligned}
$$

in the absence of a forward market, and by:

$$
\begin{aligned}
0 = \; & (p^*)^2\left\{-\mu-\mu^2\frac{c}{N_P} - 2\Lambda\left(1+3\mu\frac{c}{N_P}+2\mu^2\frac{c^2}{N_P^2}\right)\mathrm{Var}[D_0]\right\} \\
& + p^*\left\{\left(1+2\mu\frac{c}{N_P}\right)\mathbb{E}[D_0] + \mu p_0\left(1+2\mu\frac{c}{N_P}\right)\right\} \\
& + p^*\Lambda\frac{c}{N_P}\left(3+4\mu\frac{c}{N_P}\right)\left(\mathrm{Cov}[D_0, D_0^2] + 2\mu p_0\mathrm{Var}[D_0]\right) \\
& - \frac{c}{N_P}\left(\mathbb{E}[D_0^2] + 2\mu p_0\mathbb{E}[D_0] + \mu^2 p_0^2\right) - \Lambda_{\mathcal{R}}\frac{c^2}{N_P^2}\left(2-\frac{N_I}{N_P}\right)\mathrm{Var}[D_0^2] \\
& + 2\Lambda_{\mathcal{R}}\frac{c^2}{N_P^2}\left(1-\frac{\Lambda}{2\Lambda_{\mathcal{R}}}-\frac{N_I}{2N_P}\right)\frac{\mathrm{Cov}^2[D_0, D_0^2]}{\mathrm{Var}[D_0]} \\
& - 4\mu p_0\Lambda\frac{c^2}{N_P^2}\left(\mathrm{Cov}[D_0, D_0^2] + \mu p_0\mathrm{Var}[D_0]\right)
\end{aligned}
$$

in the presence of a forward market.

References

[1] B. Allaz, *Oligopoly, uncertainty and strategic forward transactions*, International Journal of Industrial Organization (1992), no. 10, 297–308.

[2] B. Allaz and Vila J.L., *Cournot competition, forward markets and efficiency*, Journal of Economic Theory **59** (1993), no. 1, 1–16.

[3] K.J. Arrow, *Vertical integration and communication*, Bell Journal of Economics (1975), no. 6, 175–183.

[4] H. Bessembinder and M.L. Lemmon, *Equilibrium pricing and optimal hedging in electricity forward markets*, The Journal of Finance **LVII** (2002), no. 3, 1347–1382.

[5] P. Bolton and M.D. Whinston, *Incomplete contracts, vertical integration and supply assurance*, The Review of Economic Studies **60** (1993), no. 1, 121–148.

[6] D.W. Carlton, *Vertical integration in competitive markets under uncertainty*, The Journal of Industrial Economics **27** (1979), no. 3, 189–209.

[7] H. Chao, S. Oren, and R. Wilson, *Restructured electricity markets: A risk management approach*, Tech. report, EPRI Technical Paper, July 2005.

[8] ______, *Restructured electricity markets: Reevaluation of vertical integration and unbundling*, Tech. report, EPRI Technical Paper, July 2005.

[9] W Emons, *Good times, bad times, and vertical upstream integration*, International Journal of Industrial Organization **14** (1996), 465–484.

[10] C. Gollier, *The economics of risk and time*, MIT Press, Cambridge, MA, 2004.

[11] J.K. Green, *New developments in the analysis of market structure*, MIT Press, Cambridge, 1986, Vertical Integration and Assurance of Markets.

[12] G. Hendrikse and H. Peters, *A note on partial vertical integration*, Economics Letters (1989), no. 30, 249–252.

[13] P. Joskow, *Handbook of new institutional economics*, Kluwer Academic Publishing, 2003, Vertical Integration.

[14] P.L. Joskow, *Vertical integration and long-term contracts: The case of coal-burning electric generating plants*, Journal of Law **1** (1985), no. 1, 33–80.

[15] B. Klein, *Vertical integration as organizational ownership: The fisher body-general motors relationship revisited*, Journal of Law **4** (1988), no. 1, 199–213.

[16] H.M. Markowitz, *Approximating expected utility by a function of mean and variance*, American Economic Review (1979), no. 69, 308–317.

[17] M.K. Perry, *Handbook of industrial organization*, Elsevier, 1989, Vertical Integration: Determinants and Effects.

[18] O.E. Williamson, *The vertical integration of production: Market failure considerations*, The American Economic Review **61** (1971), no. 2, 112–123.

[19] R. Wilson, *The theory of syndicates*, Econometrica **36** (1968), no. 1, 119–132.