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Optimal Electricity Demand Response Contracting with Responsiveness Incentives

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Abstract. Demand response programs in retail electricity markets are very popular. However, despite their success in reducing average consumption, the random responsiveness of consumers to price events makes their efficiency questionable to achieve the flexibility needed for electric systems with a large share of renewable energy. This paper aims at designing demand response contracts that allow to act on both the average consumption and its variance. The interaction between a risk-averse producer and a risk-averse consumer is modelled as a principal–agent problem, thus accounting for the moral hazard underlying demand response contracts. The producer, facing the limited flexibility of production, pays an appropriate incentive compensation to encourage the consumer to reduce his average consumption and to enhance his responsiveness. We provide a closed-form solution for the optimal contract in the linear case. We show that the optimal contract has a rebate form where the initial condition of the consumption serves as a baseline and where the consumer is charged a price for energy and a price for volatility. The first-best price for energy is a convex combination of the marginal cost and the marginal value of energy, where the weights are given by the risk-aversion ratios, and the first-best price for volatility is the risk-aversion ratio times the marginal cost of volatility. The second-best price, for energy and volatility, is a decreasing nonlinear function of time inducing decreasing effort. The price for energy is lower (respectively, higher) than the marginal cost of energy during peak-load (respectively, off-peak) periods. We illustrate the potential benefits issued from the implementation of an incentive mechanism on the responsiveness of the consumer by calibrating our model with publicly available data.

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Keywords: moral hazard • demand response • retail electricity markets

1. Introduction

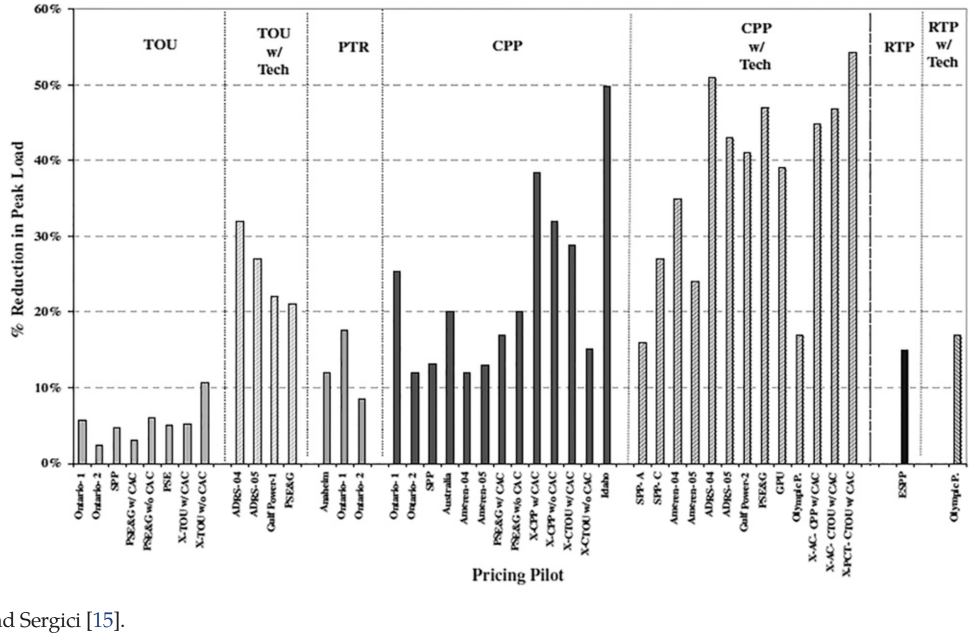
The massive development of renewable energy sources, such as wind farms and solar panels, to reduce the carbon emissions of electricity generation induced an urgent need for flexibility to cope with their intermittency. An important source of flexibility lies in the consumption itself. Demand-side management techniques based on the direct control of deferrable loads are actively studied and currently being developed (see Taylor and Mathieu [36] for a survey). Nonetheless, because of growing concerns about the information a power utility can get on consumers' daily lives, alternative methods based on a contractual framework between a utility and a pool of consumers have also seen a revival. This paper studies the use of demand response contracts to achieve flexible power systems. In its general form, a demand response contract is an incentive mechanism that allows the electricity provider to impact customers' demand. For instance, a demand response contract may offer a cheaper electricity tariff at the expense of much higher prices during some periods, at the discretion of the producer. Such periods, which typically correspond to high load days, are called *price events*. An alternative form consists in giving a cash premium at the beginning of the year and then subtracting the value of the energy consumed during price events (see Jessoe and Rapson [23] for an example). For industrial customers, the contract is typically indexed on the difference between the energy consumed and the energy they would have consumed had they not received any signal, that is, the baseline consumption. This last form is referred to as peak-time rebate.

Demand response mechanisms are a long-dated subject; see Tan and Varaiya [35] for a seminal model of interruptible contracts for a pool of consumers. The idea that a suitable combination of contracts can perform an optimal consumption reduction is also at the heart of Kamat and Oren's [24] model of exotic options to control consumption reduction, and of the thorough study by Baldick et. al. [4] of the valuation and usage of different types of interruptible contracts to control load. More recently, Adelman and Uçkun [1] developed an equilibrium model aiming at finding the optimal dynamic price that would take into account the consumer's utility from electricity consumption and studied different alternative dynamic pricing strategies. Although demand response mechanisms resort to contract theory, this framework has been used, to our knowledge, only to formulate the enrolment of customers in demand response programs as an adverse selection problem. This idea can be traced back to the work of Fahrioğlu and Alvarado [14] on incentive compatible demand response programs. It has been recently employed in the work of Crampes and Léautier [11] to design suitable baseline consumption reference, and by Aizenberg et. al. [2] and Alasseur et al. [3] for peak-load pricing.

The first objective of demand response programs is to incite consumption reduction during peak-load periods. Significant experiments have been conducted worldwide to assess the efficiency of such programs. Faruqui and Sergici [15] document more than 30 trials that find positive effects of demand response incentives. Figure 1, extracted from their paper, shows the average consumption reduction. It can be observed that demand response mechanisms do reduce average consumption in peak load but still exhibit a large variance, with an efficiency ranging from 10% to 50%. With the development of renewable energy generation and its significant intermittency, the capacity of demand response programs to achieve a consumption reduction with the appropriate regularity is being questioned because of the limited *responsiveness* of consumers to price events.

The success of a demand response mechanism depends on the responsiveness of the pool of enrolled consumers, that is, whether consumers do react to price events or not. But, when receiving a price signal, households may give up their consumption reduction commitments because of some everyday life constraints (see Torriti [38] for a sociological analysis of households' energy consumption behaviour). Because of a need for more efficient and targeted responses to demand reduction, the recent literature on demand response pays more attention to the variance of consumer responses. Experiments report that the level of reduction during peak-time events is significant, but suffers from a large heterogeneity of consumer responses, thus inducing a large variance. In the recent Low Carbon London (LCL) pricing trial, Carmichael et al. [8, section 4.3, figure 4.9] report consumers' reactions to a high price event that ranged from –200 W to +200 W for a consumption of an order of magnitude of 1,000 W. Mathieu et al. [31, table 6] report that a furniture store with a peak demand of 1,300 kW reduced, on average, its consumption by 78 kW, with a standard deviation of 30 kW. Furthermore, we see the development of demand response programs aiming at providing a targeted demand reduction with a given probability of success (see Kuppannari et al. [27], Kwac and Rajagopal [28]).

Figure 1. Household response to dynamic pricing of electricity.



Source. Faruqui and Sergici [15].

Our starting point is to relate the large variance observed in demand response experiments to a *moral hazard* problem. Because the consumption during price events is random, it is not possible to know whether the observed consumption is the result of a truly devoted effort by the consumers or whether it is the effect of pure chance.

Whereas all of the existing literature on demand response program analyses some particular form of contract, we address in the present paper the important issue of their optimal design in a situation of moral hazard, and we examine their efficiency. We formulate the mechanism design of demand response programs as a problem of optimal contracting between a producer and one single consumer as a representative of a pool of homogeneous customers.[1] Both are possibly risk averse. We focus on one price event with a fixed duration. The producer has a constant marginal cost of energy generation θ and is also subject to a constant marginal cost of the consumption variability h. This direct cost of volatility captures the impact of intermittency on generation costs. It can be seen as a continuous-time version of the Tsitsiklis and Xu [39] model in discrete time, where the authors consider not only the generation cost of energy but also the cost of variation of generation between two time steps. The consumer has also a constant marginal value of energy κ. He can reduce the average level of his consumption during the demand response event by curtailing some electricity usages at the expense of some convex costs, differentiated per usage. Besides, the representative consumer can also decrease the uncertainty on the reduction by providing more regular responses to the solicitations of the producer. Following our agent-based justification of Section 2, the regularity of the responses within the pool of agents is naturally measured by the quadratic variation of the representative agent consumption. The producer observes the total consumption trajectory but has no access to the consumer's actions, efforts, or consumption per usage. It aims at finding the optimal contract that best serves its utility criterion, given the optimal consumer's response to the proposed contract. As common in the principal–agent framework, the producer's problem is subject to the consumer's participation constraint.

We solve the optimal contracting problem following the dynamic programming technique of Sannikov [33] and Cvitanić et al. [13]. A remarkable feature of our model is that the linear specification of the energy cost functions leads to a completely explicit optimal contract, which turns out to be of the peak-time rebate form where the initial level of the consumption serves as a baseline. The optimal contract uses a price for energy and a price for volatility. Both prices are potentially functions of time. In the limiting case where the consumer is risk neutral, the second-best contract achieves the first-best optimum, as is usual in optimal contract theory with moral hazard. In this case, the consumer is charged for the energy at the marginal cost of energy and for the volatility at the direct marginal cost of volatility. Furthermore, the price charged for volatility is simply a fraction of the direct cost of volatility, where the fraction is given by the risk ratio between the producer and the consumer. Except for the limiting risk-neutral consumer case, the risk-sharing process leads to charging the consumer at energy and volatility prices different from their marginal costs.

Our explicit solution reveals more details about the contract. For instance, in contrast with off-peak periods, during peak periods ($\kappa < \theta$), the second-best contract fails to achieve the first-best optimum. Moreover, the producer induces efforts to reduce the average consumption and to increase responsiveness. The second-best price for energy is a nonlinear decreasing function of time, thus inducing a decreasing effort through time.

As another surprising effect, we find that under the optimal contract, the resulting consumption volatility may decrease as required, but it may also increase depending on the risk aversion parameters of both actors. This can be explained by the consumer's risk aversion, which already implies a reduction of the consumption volatility and illustrates how the risk-sharing process between the producer and the consumer allows the electric system to bear more risk.

We illustrate the potential benefit from such a mechanism design by using the Low Carbon London data to calibrate the parameters of our model. We interpret this experiment as the implementation of the optimal contracting model under no responsiveness incentives, that is, the consumer is incentivised only to reduce the average level of his consumption. We find a potential important consumption volatility reduction and a significant increase of the producer's benefit for a small increase in the cost of effort from the consumer.

We finally examine the robustness of this result to the assumption of constant marginal cost and value of energy. We implement a numerical approximation of the solution of the optimal contracting problem for a concave consumer energy value function, and we perform the comparison with the corresponding linear contract approximation. We find that the concavity of the energy value function of the consumer has a downward effect on the benefit of the producer. The linear approximation of the energy value function exacerbates this downward effect for strongly concave functions.

This paper is organised as follows. Section 2 describes the model. Section 3 provides the first-best and second-best optimal contracts and the optimal contract when no incentives on responsiveness are provided. Section 4 provides empirical results. Section 5 concludes.

2. The Model

2.1. Representative Agent of a Pool of Consumers

We assume that the consumers are partitioned into different homogeneous pools of consumers, and we focus on one of them. For instance, we may think of the residential consumers pool, the small firms pool, or the large industries pool.

Our objective is to describe a demand response experiment within this pool of consumers, $i = 1, \ldots, I$, during one single price event of duration T. The price event period $[0, T]$ can be situated during a peak period or an off-peak period of the day. Individual consumptions are observed in discrete time $0 = t_0 < \cdots < t_N = T$. Given the current smart metering technologies, we may think of the time step $\Delta t_n := t_n - t_{n-1}$ as very small, for example, 30 minutes in the LCL experiment cited above.

The increments of individual consumptions is assumed to be described by the model

$$\Delta X^i_{t_j} := X^i_{t_j} - X^i_{t_{j-1}} = -\sum_{k=1}^d \alpha^k_{t_{j-1}} \eta^{i,k}_{t_j} + \sum_{k=1}^d \rho^k_0 \varepsilon^{i,k}_{t_j}, \tag{2.1}$$

where $k = 1, \ldots, d$ indicates d different electricity usages; the random variables $\eta^{i,k}_{t_j}$ are independent and identically distributed (i.i.d.) as Bernoulli variables with parameter $p^k_{t_{j-1}}$; the noise random variables $\varepsilon^{i,k}_{t_j}$ are i.i.d. independent of the $\eta^{i,k}_{t_j}$'s with zero mean and unit variance; and $\rho^k_0 > 0$ are given noise intensity parameters. Here $\alpha^k_{t_{j-1}}$ indicates the intended effort of each agent of the pool, and $\eta^{i,k}$ takes the value one or zero depending on whether agent i indeed follows the intended effort; thus, $p^k_{t_{j-1}}$ represents the willingness to follow the intended effort and is another decision variable under full control of the agent.

Direct manipulation provides the equivalent formulation with decision variables for the agent's effort $a^k_{t_{j-1}} \geq 0$ and $\rho^k_{t_{j-1}} > 0$:

$$\Delta X^i_{t_j} = -\sum_{k=1}^d a^k_{t_{j-1}} + \sum_{k=1}^d \rho^k_{t_{j-1}} \bar{\varepsilon}^{i,k}_{t_j}, \text{where } a^k_{t_{j-1}} := p^k_{t_{j-1}} \alpha^k_{t_{j-1}}, \Big(\rho^k_{t_{j-1}}\Big)^2 := (\rho^k_0)^2 + \Big(\alpha^k_{t_{j-1}}\Big)^2 p^k_{t_{j-1}} \Big(1 - p^k_{t_{j-1}}\Big), \tag{2.2}$$

and $\bar{\varepsilon}^{i,k}_{t_j} := (\rho^k_{t_{j-1}})^{-1} [\rho^k_0 \varepsilon^{i,k}_{t_j} + \alpha^k_{t_{j-1}} (p^k_{t_{j-1}} - \eta^{i,k}_{t_j})]$ are i.i.d. random variables with mean zero and unit variance.

The last formulation is parameterised by the vector decision variables $a_{t_{j-1}}$ and $\rho_{t_{j-1}}$, which represent the average decrease and the variance of the consumption, respectively. The objective of the demand response program, which is to reduce consumption with high probability, can now be achieved by an action on the mean consumption $a_{t_{j-1}}$ and its variance $|\rho_{t_{j-1}}|^2$. The first action addresses the level of consumption, whereas the second one addresses the responsiveness within the pool. The latter can be understood as the variability of responses within the pool of homogeneous independent agents, as it may be computed by the interagent variance

$$\frac{1}{I} \sum_{i=1}^I \left(\Delta X^i_{t_j} - \frac{1}{I} \sum_{\ell=1}^I \Delta X^\ell_{t_j} \right)^2 = \frac{1}{I} \sum_{i=1}^I \left(\sum_{k=1}^d \rho^k_{t_{j-1}} \bar{\varepsilon}^{i,k}_{t_j} \right)^2 \underset{I \to \|infty}{\longrightarrow} |\rho_{t_{j-1}}|^2, \quad \text{a.s.}, \tag{2.3}$$

by the law of large numbers. The second action modelled by the coefficient ρ is induced by the randomisation through the binomial random variables $\eta^{i,k}$, and thus represents the regularity of response of each individual agent to the solicitations of the producer, an interpretation that will prevail throughout this paper, as we shall summarise the action of the homogenous pool by that of a single representative agent. This action of the consumer consists in the choice of an amount of effort to be exerted together with the probability of taking on such an effort.

Notice that the components of $\rho_{t_{j-1}}$ cannot be identified from the observation of the total consumption of each individual agent; only its norm $|\rho_{t_{j-1}}|$ is accessible.

We shall determine an optimal incentive scheme by modelling the interaction between the producer and the representative consumer as a principal–agent moral hazard problem. To do this, we consider the continuous-time version of (2.1) with $a = \alpha \Delta t$ and $\rho = \sigma \sqrt{\Delta t}$:

$$\mathrm{d}X_t = -\alpha_t \cdot \mathbf{1} \mathrm{d}t + \sum_{k=1}^d \sigma^k_t \mathrm{d}W^k_t.$$

The continuous-time setting is natural in the current problem, given the availability of high-frequency smart meter data, and is in fact crucial for obtaining an explicit solution of the optimal contract. More importantly, the continuous-time setting offers a natural interpretation of the incentive instrument for the responsiveness incentive due to the well known quadratic variation property

$$\langle X\rangle_t = \lim_{\Delta t\searrow 0}\sum_{t_j\le t}\left|X^i_{t_j} - X^i_{t_{j-1}}\right|^2 = \int_0^t |\sigma_s|^2 \mathrm{d}s,$$

where the limit is in the probability sense. Notice that in contrast with (2.3), which expresses the responsiveness in terms of the interagent variance of consumption, the last formula expresses the responsiveness in terms of the intertemporal variance of consumption of a representative agent. Thus, the quadratic variation offers to the producer access to the responsiveness of the pool $|\sigma_t|^2$, as its computation is based on the observation of the individual total consumption. Hence, it gives a tool to implement incentives on the responsiveness of consumers: by answering with high probability the solicitations of the producer, the consumer induces a consumption with low quadratic variation. This highlights a highly relevant feature of our model:

- the representative consumer chooses the responsiveness effort σ_t by fixing the regularity of effort;
- the producer has no access to this information in the discrete-time model (2.1) as it has no information on the distribution induced by the unobservable agent effort;
- in the continuous-time version of the model, the quadratic variation of an individual consumption allows one to identify the intensity $|\sigma_t|^2$ of the responsiveness effort.

This explains why the quadratic variation plays a central role in the design of contracts with responsiveness incentives.

Remark 2.1. It is important to notice that there is a natural subdivision of the noise in the individual agent consumption into an idiosyncratic part, fundamentally related to the habits of a given consumer, and a part that is common to all consumers, for instance, linked to weather conditions. Though it is true that the idiosyncratic part should be averaged out for a large group of consumers, we would like to stress that we consider here a *representative* agent of a pool of homogeneous consumers. For this reason, there is no averaging out of noises, and the decomposition of noises between idiosyncratic and common is not directly relevant for our model. We could of course have added a common factor correlating the different Brownian motions W^k, but we did not pursue this direction so as to not overcomplicate the model and distract from its inherent ability to improve responsiveness in demand response programs.

2.2. The Consumer

As in the previous section, the (representative) agent aggregate consumption process is denoted by $X = \{X_t, t\in[0,T]\}$ and is the canonical process of the space Ω of scalar continuous trajectories $\omega:[0,T]\to\mathbb{R}$, that is, $X_t(\omega) := \omega(t)$ for all $(t,\omega)\in[0,T]\times\Omega$. We denote by $\mathbb{F} := \{\mathcal{F}_t, t\in[0,T]\}$ the corresponding filtration. The agent allocates his electricity consumption between d different usages, for example, refrigerator, heating, air conditioning, etc.

We denote by $\mathcal{U}$ the collection of all control processes $\nu := (\alpha,\beta)$, that is to say, $\mathbb{F}$-adapted processes, which take values in appropriate subsets $A\subset\mathbb{R}^d_+$ and $B\subset[0,1]^d$, respectively, inducing the controlled stochastic differential equation[2] driven by a d-dimensional Brownian motion W:

$$X_t = X_0 - \int_0^t \alpha_s\cdot\mathbf{1}\mathrm{d}s + \int_0^t \sigma(\beta_s)\cdot \mathrm{d}W_s,\ t\in[0,T],\ \text{with}\ \sigma(b) := \left(\sigma_1\sqrt{b_1},\ldots,\sigma_d\sqrt{b_d}\right)^\top, \tag{2.4}$$

for some initial condition $X_0\in\mathbb{R}$ and given parameters $\sigma_1,\ldots,\sigma_d > 0$.

All of our utility criteria depend only on the distribution $\mathbb{P}^\nu$ of the state process X corresponding to the effort process ν. Let $\mathcal{P}$ be the collection of all such measures $\mathbb{P}^\nu$. The agent enjoys a value $f(X) = \kappa X$ from the consumption X.[3] Moreover, his effort on the level and the responsiveness is costly. For the sake of simplicity, we shall consider the following separable cost function:

$$c(\nu) := c_1(\alpha) + \frac{1}{2}c_2(\beta),\ \text{with}\ c_1(a) := \frac{1}{2}\sum_{k=1}^d \frac{a_k^2}{\mu_k},\ \text{and}\ c_2(b) := \sum_{k=1}^d \frac{\sigma_j^2}{\lambda_k}\left(b_k^{-1} - 1\right),$$

for some $\mu_k, \lambda_k > 0$, $k = 1,\ldots,d$. Notice that c is convex, increasing in a, and decreasing in b as the responsiveness effort consists in reducing the volatility, thus reproducing the requested effects of increasing the marginal cost of effort. Moreover, $c_1(0) = c_2(1) = 0$ captures the fact that there is no cost for making no effort.

For technical reasons, we need to consider bounded efforts

$$A := [0,\mu_1 A_{\max}]\times\cdots\times[0,\mu_d A_{\max}]\ \text{and}\ B := [\varepsilon, 1]^d,$$

for some constants $A_{\max} := \varepsilon^{-1} > 0$ and $\varepsilon > 0$. Propositions 3.1 and 3.2 below provide an explicit value $\bar{\varepsilon}$ so that all of our results are independent of ε as long as $\varepsilon \le \bar{\varepsilon}$.

The execution of the contract starts at $t = 0$. The consumer receives the value ξ from the producer at time T. The value ξ can be positive (payment) or negative (charge). The producer has no access to the consumer's actions and does not observe the consumer's different usages of electricity. It observes only the overall consumption X. Consequently, the compensation ξ can only be contingent on X; that is, ξ is $\mathcal{F}_T$-measurable. We denote by $\mathcal{C}_0$ the set of $\mathcal{F}_T$-measurable random variables. The objective function of the consumer is then defined for all $(\nu, \xi) \in \mathcal{U} \times \mathcal{C}_0$ by

$$J_{\mathrm{A}}(\xi, \mathbb{P}^\nu) := \mathbb{E}^{\mathbb{P}^\nu}\left[U_{\mathrm{A}}\left(\xi + \int_0^T (f(X_s) - c(\nu_s))\mathrm{d}s\right)\right], \text{ where } U_{\mathrm{A}}(x) := -\mathrm{e}^{-rx}, \tag{2.5}$$

for some constant risk-aversion parameter $r > 0$. It is implicitly understood that the limiting case where r tends to zero corresponds to a risk-neutral consumer. The consumer's problem is

$$V_{\mathrm{A}}(\xi) := \sup_{\mathbb{P}^\nu \in \mathcal{P}} J_{\mathrm{A}}(\xi, \mathbb{P}^\nu), \tag{2.6}$$

that is, maximising utility from consumption subject to the cost of effort. A control $\mathbb{P}^{\widehat{\nu}} \in \mathcal{P}$ will be called optimal if $V^{\mathrm{A}}(\xi) = J_{\mathrm{A}}(\xi, \mathbb{P}^{\widehat{\nu}})$. We denote by $\mathcal{P}^\star(\xi)$ the collection of all such optimal responses $\mathbb{P}^{\widehat{\nu}}$. We finally assume that the consumer has a reservation utility $R_0 \in \mathbb{R}_-$, and we denote by $L_0 := -\frac{1}{r}\log(-R_0)$ the corresponding certainty equivalent.

2.3. The Producer

The producer (the principal) provides electricity to the consumer, and thus faces the generation cost of the produced energy, and the cost induced by the variation of production. Its performance criterion is defined by

$$J_{\mathrm{P}}(\xi, \mathbb{P}^\nu) := \mathbb{E}^{\mathbb{P}^\nu}\left[U\left(-\xi - \int_0^T g(X_s)\mathrm{d}s - \frac{h}{2}\langle X\rangle_T\right)\right], \text{ with } U(x) := -\mathrm{e}^{-px}. \tag{2.7}$$

Here, $p > 0$ is the constant absolute risk-aversion parameter, and g is taken to be linear, $g(x) = \theta x$.[4] The parameter h is a positive constant representing the *direct marginal cost* induced by the quadratic variation $\langle X\rangle$ of the consumption. The higher the volatility of the consumption, the more costly it is for the producer to follow the load curve. Note that because of risk aversion, even if $h = 0$, the producer bears also a disutility from volatility.

An $\mathcal{F}_T$-measurable random variable ξ will be called a *contract*, which we denote by $\xi \in \mathcal{C}$, if it satisfies the additional integrability property

$$\sup_{\mathbb{P}^\nu \in \mathcal{P}} \mathbb{E}^{\mathbb{P}^\nu}[e^{-rm\xi}] + \sup_{\mathbb{P}^\nu \in \mathcal{P}} \mathbb{E}^{\mathbb{P}^\nu}[e^{pm\xi}] < +\infty, \text{ for some } m > 1. \tag{2.8}$$

This integrability condition guarantees that the consumer's criterion, (2.5), and the principal's, (2.7), are well defined. Throughout this paper, we shall consider the two following standard contracting problems:

• *First-best contracting* corresponds to the benchmark situation where the producer has full power to impose a contract to the consumer and to dictate the agent's effort

$$V^{\mathrm{FB}} := \sup_{(\xi, \mathbb{P}^\nu) \in \mathcal{C} \times \mathcal{P}} \{J_{\mathrm{P}}(\xi, \mathbb{P}^\nu) : J_{\mathrm{A}}(\xi, \mathbb{P}^\nu) \ge R_0\}. \tag{2.9}$$

• *Second-best contracting* allows the consumer to respond optimally to the producer's offer. We follow the standard convention in the principal–agent literature in the case of multiple optimal responses in $\mathcal{P}^\star(\xi)$, that the consumer implements the optimal response that is the best for the producer. This leads to the second-best contracting problem

$$V^{\mathrm{SB}} := \sup_{\xi \in \Xi} \sup_{\mathbb{P}^\nu \in \mathcal{P}^\star(\xi)} J_{\mathrm{P}}(\xi, \mathbb{P}^\nu), \text{ where } \Xi := \{\xi \in \mathcal{C} : V_{\mathrm{A}}(\xi) \ge R_0\}, \tag{2.10}$$

with the convention $\sup \emptyset = -\infty$, thus restricting the contracts that can be offered by the producer to those $\xi \in \mathcal{C}$ such that $\mathcal{P}^\star(\xi) \ne \emptyset$.

2.4. Consumer's Optimal Response and Reservation Utility

We collect here some calculations related to the consumer's optimal response that will be useful throughout this paper. Following Cvitanić et al. [13], we introduce the consumer's Hamiltonian:

$$H(z, \gamma) := H_{\mathrm{m}}(z) + H_{\mathrm{v}}(\gamma), \quad (z, \gamma) \in \mathbb{R}^2, \tag{2.11}$$

where H_{m} and H_{v} are the components of the Hamiltonian corresponding to the instantaneous mean and volatility, respectively,

$$H_{\mathrm{m}}(z) := -\inf_{a\in A}\{c_1(a) + a\cdot \mathbf{1}z\} \text{ and } H_{\mathrm{v}}(\gamma) := -\frac{1}{2}\inf_{b\in B}\Big\{c_2(b) - \gamma|\sigma(b)|^2\Big\}. \tag{2.12}$$

Here, z represents the payment rate for a decrease of the consumption, and γ represents the payment rate for a decrease of the volatility of the consumption. Both payment rates can be positive or negative. Given these payments, the consumer maximises the instantaneous rate of benefit given by the Hamiltonian to deduce the optimal response $\widehat{a}(z)$ on the drift and $\widehat{b}(\gamma)$ on the volatilities. The following result collects the closed-form expression of the optimal responses. We denote $x^- := 0\vee(-x)$, $x\in\mathbb{R}$.

Proposition 2.1 (Consumer's Best Response). *The optimal response of the consumer to an instantaneous payment rate (z,γ) is*

$$\widehat{a}_j(z) := \mu_j(z^-\wedge A_{\max}) \text{ and } \widehat{b}_j(\gamma) := (1\wedge(\lambda_j\gamma^-)^{-\frac{1}{2}})\vee\varepsilon,\ j=1,\dots,d,$$

so that, with $\bar\mu := \mu\cdot\mathbf{1}$, $\widehat{\sigma}(\gamma) := \sigma(\widehat{b}(\gamma))$, $\widehat{c}_1(z) := c_1(\widehat{a}(z))$, and $\widehat{c}_2(\gamma) := c_2(\widehat{b}(\gamma))$,

$$H_{\mathrm{m}}(z) = \frac{1}{2}\bar\mu(z^-\wedge A_{\max})^2 \text{ and } H_{\mathrm{v}}(\gamma) = -\frac{1}{2}\Big(\widehat{c}_2(\gamma) - \gamma|\widehat{\sigma}(\gamma)|^2\Big).$$

The payment z induces an effort of the consumer on all usages to reduce the average consumption deviation, and this effort is inversely proportional to its cost $1/\mu_i$. The payment γ induces an effort only on the usages whose cost $1/\lambda_j$ is lower than the payment. We hereafter let $\bar\lambda := \max_j\lambda_j$.

Because at this stage the intuition behind the payment rates z and Γ may not be clear, let us explain here concisely their origin. The general methodology of Cvitanić et al. [13], based on Sannikov [33], allows us to prove that there is no loss of generality in considering the subclass of contracts $\xi = Y_T^{y_0,Z,\Gamma}$, where

$$Y_t^{y_0,Z,\Gamma} := y_0 + \int_0^t Z_s\mathrm{d}X_s + \frac{1}{2}\int_0^t(\Gamma_s + rZ_s^2)\mathrm{d}\langle X\rangle_s - \int_0^t(H(Z_s,\Gamma_s) + f(X_s))\mathrm{d}s,\ t\in[0,T],$$

for a given pair of processes (Z,Γ) and constant $y_0\in\mathbb{R}$. Though we refer the reader to Section A.3 of the appendix for mode details on this specific form of contract, it is that theoretical result which justifies the introduction of the payment rates.

We next provide the following characterisation of the consumer's behaviour without contract.

Proposition 2.2 (Consumer's Behaviour Without a Contract). *Let $f(x) = \kappa x$, $x\in\mathbb{R}$, for some $\kappa\geq 0$, and $\varepsilon < \bar\varepsilon_0 := 1\wedge(\kappa T\sqrt{r\bar\lambda})^{-1}$. Then, the consumer's value function without a contract is independent of ε and is given by $V_A(0) = U_A(\ell_0)$, where*

$$\ell_0 = \kappa X_0 T + E_0(T) \text{ and } E_0(T) := \int_0^T H_{\mathrm{v}}(\gamma(t))\mathrm{d}t,\ \gamma(t) := -r\kappa^2(T-t)^2.$$

The consumer's optimal effort on the drift and on each volatility usage are, respectively,

$$a^0 = 0 \text{ and } b_j^0(t) := 1\wedge\frac{1}{\kappa(T-t)\sqrt{r\lambda_j}},\ j=1,\dots,d,$$

thus inducing an optimal $\mathbb{P}^0$ under which the deviation process follows the dynamics $\mathrm{d}X_t = \widehat{\sigma}(b^0(t))\cdot\mathrm{d}W_t$, for some $\mathbb{P}^0$-Brownian motion W.

Proof. See the appendix, Section A.1.

Notice that the consumer may optimally exert effort to reduce the volatility of his consumption even without any contract compensation. This is consistent with his utility maximisation criterion. Notice also that we do not here assume that the consumer will indeed produce efforts without a contract offered. The point is that his optimal utility without a contract provides us with his reservation utility level: he would not accept a contract that would provide him at least as much utility as the amount he could get by himself without a contract.

3. Main Results

We consider the case where

$$(f-g)(x) := \delta x, \ x \in \mathbb{R}, \tag{3.1}$$

for some constant parameter $\delta := \kappa - \theta$, hereafter called *energy value discrepancy*. The case $\delta \geq 0$ corresponds to off-peak hours, whereas negative δ corresponds to peak-load hours.

3.1. First-Best Contract

Proposition 3.1 (First-Best Contract). *Let* $(f-g)(x) = \delta x$ *and* $\varepsilon < \bar{\varepsilon}_1$, *where* $\bar{\varepsilon}_1 := 1 \wedge (\sqrt{\bar{\lambda}(h + \rho\delta^2 T^2)})^{-1} \wedge (\delta^- T)^{-1}$. *Then, the first-best contracting problem is independent of* ε. *Moreover,*

i. *the first-best value function* $V^{\mathrm{FB}} = U(\bar{v}(0, X_0) - L_0)$, *where*

$$\bar{v}(t,x) = \delta(T-t)x + \int_t^T m_{\mathrm{FB}}(s)\mathrm{d}s \ \text{and} \ m_{\mathrm{FB}}(t) := \frac{1}{2}\bar{\mu}(\delta^-)^2(T-t)^2 + H_{\mathrm{v}}(-h - \rho\delta^2(T-t)^2), \quad \rho := \frac{rp}{r+p};$$

ii. *the consumer's optimal effort is given by*

$$a_{\mathrm{FB}}(t) := \mu\delta^-(T-t) \ \text{and} \ b_{\mathrm{FB}}(t) := 1 \wedge \frac{1}{\sqrt{\lambda_j(h + \rho\delta^2(T-t)^2)}},$$

thus inducing an optimal distribution $\mathbb{P}^{\mathrm{FB}}$ *under which the deviation process follows the dynamics* $\mathrm{d}X_t = -\mu\delta^-(T-t)\cdot \mathbf{1}\mathrm{d}t + \sigma(b_{\mathrm{FB}}(t))\cdot \mathrm{d}W_t$, *for some* $\mathbb{P}^{\mathrm{FB}}$*-Brownian motion* W; *and*

iii. *the optimal first-best contract is given by*

$$\xi_{\mathrm{FB}} = L_0 - \kappa X_0 T + \int_0^T c(\nu_t)\mathrm{d}t + \int_0^T \pi^{\mathrm{E}}_{\mathrm{FB}}(X_0 - X_t)\mathrm{d}t - \frac{1}{2}\int_0^T \pi^{\mathrm{V}}_{\mathrm{FB}}\mathrm{d}\langle X\rangle_t,$$

where

$$\pi^{\mathrm{E}}_{\mathrm{FB}} := \frac{r}{r+p}\kappa + \frac{p}{r+p}\theta, \ \pi^{\mathrm{V}}_{\mathrm{FB}} := \frac{p}{r+p}h.$$

Proof. See the appendix, Section A.2.

The first-best contract admits the form of a rebate contract. It can be decomposed in two parts:

$$\xi^{\mathrm{F}}_{\mathrm{FB}} := L_0 - \kappa T X_0, \ \xi^{\mathrm{V}}_{\mathrm{FB}} := \int_0^T c(\nu_t)\mathrm{d}t + \int_0^T \pi^{\mathrm{E}}_{\mathrm{FB}}(X_0 - X_t)\mathrm{d}t - \frac{1}{2}\int_0^T \pi^{\mathrm{V}}_{\mathrm{FB}}\mathrm{d}\langle X\rangle_t.$$

The term $\xi^{\mathrm{F}}_{\mathrm{FB}}$ provides the consumer with the certainty equivalent of his reservation decreased by the average value of energy if no efforts are undertaken. The variable term $\xi^{\mathrm{V}}_{\mathrm{FB}}$ consists in first paying the exact amount of his efforts that are here observable and enforceable and then providing a positive payment if the consumption X_t is lower than its initial value X_0 or if volatility decreases.

If the consumer is risk neutral, the first-best contract transfers all the uncertainty of the generation cost to the consumer, as is standard in the moral hazard optimal contract framework. The first-best price for energy $\pi^{\mathrm{E}}_{\mathrm{FB}}$ is simply the sum of the energy values for the consumer and the producer, weighted by their risk aversion. The first-best price for the responsiveness $\pi^{\mathrm{V}}_{\mathrm{SB}}$ is a constant fraction of the direct cost of volatility if the consumer is risk averse, and it is constant. As a consequence, we see that a contract that would be indexed only on the information of the cost function of the producer, as proposed by the Federal Energy Regulatory Commission in Order 719, is optimal only in the case when the consumer is risk neutral (for discussions of this topic, see Brown and Sappington [7], Chao [9], Hogan [19, 20]). The economic intuition that the marginal cost of generation triggers a socially optimal response is correct only if the consumer is risk neutral. If not, the consumers needs a compensation payment for the risk he takes in the contract.

Regarding the induced behaviour of the consumer on the volatilities, reduction is performed only on those usages for which the marginal cost of effort measured by $1/\lambda_j$ is lower than the marginal cost of volatility for the producer measured by $h + \rho\delta^2(T-t)^2$. Responsiveness is triggered by the mere comparison of h and the λ_j in two cases: either one of the actors is risk neutral or they agree on the energy value ($\delta = 0$).

3.2. Second-Best Contract

Proposition 3.2 (Second-Best Contract). *Let $(f-g)(x)=\delta x$, and assume in addition that $\varepsilon<\bar\varepsilon_2:=1\wedge(\sqrt{\bar\lambda}(h+r\delta^2T^2))^{-1}\wedge(\delta^-T)^{-1}$. Then, the second-best contracting problem is independent of ε. Moreover, we have the following:*

i. *The producer's second-best value function is $V^{\mathrm{SB}}=U(v(0,X_0)-L_0)$, with certainty equivalent function*

$$v(t,x)=\delta(T-t)x+\int_t^T m_{\mathrm{SB}}(s)\mathrm{d}s,$$

where

$$m_{\mathrm{SB}}(t):=\frac{1}{2}\bar\mu\delta^2(T-t)^2-\inf_{z\in\mathbb{R}}\Big\{\bar\mu(z^-+\delta(T-t))^2-H_{\mathrm{v}}(-q(z))\Big\}\text{ and }q(z):=h+rz^2+p(z-\delta(T-t))^2.$$

ii. *The optimal payment rates are the deterministic functions*

$$z_{\mathrm{SB}}(t):=\operatorname{Arg}\min_{z\in\mathbb{R}}\Big\{\bar\mu(z^-+\delta(T-t))^2-H_{\mathrm{v}}(-q(z))\Big\}\text{ and }\gamma_{\mathrm{SB}}(t):=-q(z_{\mathrm{SB}}(t)).$$

iii. *The second-best optimal contract is given by $\xi_{\mathrm{SB}}=\xi^{\mathrm{F}}_{\mathrm{SB}}+\xi^{\mathrm{V}}_{\mathrm{SB}}$, where*

$$\xi^{\mathrm{F}}_{\mathrm{SB}}:=L_0-\kappa TX_0-\int_0^T H(z_{\mathrm{SB}},\gamma_{\mathrm{SB}})(t)\mathrm{d}t,\ \xi^{\mathrm{V}}_{\mathrm{SB}}:=\int_0^T\pi^{\mathrm{E}}_{\mathrm{SB}}(t)(X_0-X_t)\mathrm{d}t-\frac{1}{2}\int_0^T\pi^{\mathrm{V}}_{\mathrm{SB}}(t)\mathrm{d}\langle X\rangle_t,$$

and

$$\pi^{\mathrm{E}}_{\mathrm{SB}}(t):=\kappa+z'_{\mathrm{SB}}(t),\ \pi^{\mathrm{V}}_{\mathrm{SB}}(t):=h+p(z_{\mathrm{SB}}(t)-\delta(T-t))^2.$$

iv. *The induced dynamics of the consumption reduction is*

$$\mathrm{d}X_t=-\widehat{a}(z_{\mathrm{SB}}(t))\mathrm{d}t+\widehat{\sigma}(\gamma_{\mathrm{SB}}(t))\cdot\mathrm{d}W_t,$$

for some $\mathbb{P}^{\mathrm{SB}}$-Brownian motion W.

Proof. See the appendix, Section A.3.

The second-best optimal contract has the form of a rebate contract as in the first-best case. But now the producer can no longer charge in the variable part of the contract the real cost of the consumer's effort, which is not observable. Instead it charges in the fixed part of the contract the net benefit of his optimal efforts, given by the integral of the consumer's Hamiltonian for optimal incentives $(z_{\mathrm{SB}},\gamma_{\mathrm{SB}})$. If the consumer is risk neutral, it holds that

$$\xi^{\mathrm{V}}_{\mathrm{SB}}=\int_0^T\theta(X_0-X_t)\mathrm{d}t-\frac{1}{2}\int_0^T h\mathrm{d}\langle X\rangle_t.$$

The optimal prices are the marginal costs of energy and volatility. Apart from this limiting case, the optimal price for energy is a decreasing nonlinear function of time, thus inducing a decreasing effort (see Section 4 for illustration).

Demand response contracts written in rebate form are prone to baseline manipulation.[5] If we define baseline manipulation as an artificial increase of the baseline consumption to obtain a higher utility, then the optimal contract given in Proposition 3.2 does not suffer from this drawback. In this case, whatever the initial consumption level X_0 used as a baseline, the consumer gets no more than the requested participation level defined by the certainty equivalent L_0.

Remark 3.1. We observe that the contract ξ_{sb} exhibits an incentive to responsiveness through the term $\int_0^t\pi^V_{SB}(t)\mathrm{d}\langle X\rangle_t$, which involves the quadratic variation of the consumption of the representative agent. We recall from the discussion at the end of Section 2.1 that the quadratic variation $\langle X\rangle_t$ captures the same information as the variance of responses among the pool of consumers; see (2.3). We recall that this is a consequence of our assumption of independence between the actions of the individual consumers on one hand and of the independence of the action of each single consumer at each time step on the other hand. An important consequence is the possible implementability of the second-best contract by contracting on the basis of the individual's level of consumption and the interagent variance of consumption, or, equivalently, the intertemporal variance of consumption.

In peak periods, the second-best payment rates are obtained up to a scalar optimisation (Proposition 3.2ii). During off-peak periods, it is possible to obtain more explicit results.

Corollary 3.1. *Suppose $(f-g)' = \delta \geq 0$ and $\varepsilon < \bar{\varepsilon}_2$. Then we have the following:*

i. *The optimal payment rates are deterministic functions of time given by*

$$z_{\mathrm{SB}}(t) = \frac{p}{r+p}\delta(T-t) \text{ and } \gamma_{\mathrm{SB}}(t) = -h - \rho\,\delta^2(T-t)^2,$$

and the second-best optimal contract of Proposition 3.2iii is defined by the constant prices

$$\pi^{\mathrm{E}}_{\mathrm{SB}} := \frac{r}{r+p}\kappa + \frac{p}{r+p}\theta \text{ and } \pi^{\mathrm{V}}_{\mathrm{SB}} := h + \rho\frac{r}{r+p}\delta^2(T-t)^2.$$

ii. *The consumer's optimal effort on the drift and the volatility of the consumption deviation are*

$$\widehat{a}(z_{\mathrm{SB}}(t)) = 0 \text{ and } \widehat{b}_j(\gamma_{\mathrm{SB}}(t)) = 1 \wedge \frac{1}{\sqrt{\lambda_j(h+\rho\delta^2(T-t)^2)}},\ \ t \in [0,T],$$

so that the optimal response of the consumer induces the optimal probability distribution $\mathbb{P}^{\mathrm{SB}}$ such that $\mathrm{d}X_t = \widehat{\sigma}(\gamma_{\mathrm{SB}})\cdot \mathrm{d}W_t$, for some $\mathbb{P}^{\mathrm{SB}}$-Brownian motion W.

Proof. This is a direct application of Proposition 3.2 by observing that the minimiser z_{SB} can be computed explicitly when $\delta \geq 0$. Indeed, z_{SB} is obtained by $\min\{M_{(+)}, M_{(-)}\}$, where

$$M_{(+)} := \min_{z\geq 0} \bar{\mu}s^2 - H_{\mathrm{v}}(-h - rz^2 - p(z-s)^2),\ M_{(-)} := \min_{z\leq 0} \bar{\mu}(s-z)^2 - H_{\mathrm{v}}(-h - rz^2 - p(z-s)^2)$$
$$= \min_{z\geq 0} \bar{\mu}(z+s)^2 - H_{\mathrm{v}}(-h - rz^2 - p(z+s)^2),$$

and $s := \delta(T-t)$. Notice that for $s, z \geq 0$, we have $(z+s)^2 \geq s^2 \geq (z-s)^2$. As H_{v} is increasing, this implies that $M_{(+)} \leq M_{(-)}$, and therefore z_{SB} is the minimiser of $M_{(+)}$. By using the fact that H_{v} is increasing, we see that z_{SB} is the minimiser of $rz^2 + p(z-s)^2 = (r+p)(z - \frac{rs}{r+p})^2 - \frac{r^2s^2}{r+p}$ on $\{z \geq 0\}$, thus leading to $z_{\mathrm{SB}} = \frac{rs}{r+p}$. Q.E.D.

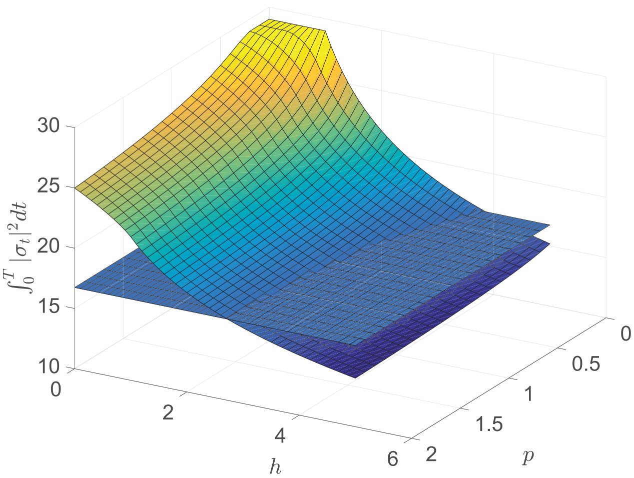
During off-peak periods, the consumer is not incentivised to reduce his consumption because the price for energy $\pi^{\mathrm{E}}_{\mathrm{SB}}$ is lower than its value for the consumer κ. But, the consumer is still required to reduce his consumption volatility. The price of volatility $\pi^{\mathrm{V}}_{\mathrm{SB}}$ is still a decreasing nonlinear function of time inducing decreasing effort and is potentially higher than the direct marginal cost of volatility for the producer at the initial time. Thus, there is an interest for responsiveness incentives even in off-peak periods. This is consistent with the observation of negative prices in off-peak hours of day-ahead markets that translates the need of producers for a more regular consumption to cope with the intermittency of renewable energy.

Furthermore, the risk-sharing process between the producer and the consumer may lead to an increase in the observed volatility of consumption. Consider the limiting case where $\delta = 0$ and $h = 0$. The optimal payment γ_{SB} is thus null. The producer does not provide any incentive for responsiveness, even if both players are risk averse. In this situation, the observed volatility is equal to the nominal volatility. But, it might happen that this quantity is greater than the observed volatility before contracting. If the consumer is sufficiently risk averse, he might have performed some efforts to reduce the nominal volatility. In fact, there is a large set of parameter values for which the producer is taking the volatility risk for the consumer. This phenomenon is illustrated in Figure 2. From a risk-sharing point of view, this means that optimal contracting allows the system to bear more risk. This remark takes all its meaning in a context where more consumers are holding local electricity generation sources, like solar panels. Demand response contracts providing incentives for responsiveness may allow consumers with local intermittent electricity generation to renounce to the installation of load smoothing devices such as batteries.

3.3. Comparisons

To assess the benefits from responsiveness incentives, we study the case where the producer provides incentives only on the average consumption. We consider the problem (2.10) of the producer where the contracts are limited to incentives indexed on the observed consumption and not on its volatility. The proof of Proposition 3.3 in Section A.4 of the appendix gives a precise meaning of this subclass of contracts. We denote by V^{SB_m} the value of the producer's problem in this case. In this situation, the behaviour of the consumer without contracting is still as given by Proposition 2.2.

Figure 2. (Color online) Total volatility of consumption deviation under the optimal contract as a function of h and p, compared with the total volatility without a contract (flat surface). Parameter values are as follows: two usages, $T = 1$, $\mu = (2, 5)$, $\sigma = (5, 2)$, $\lambda = (0.5, 0.1)$, $\kappa = 5$, and $\delta = 3$.



Proposition 3.3 (Second-Best Without Responsiveness Incentives). *Assume that $f - g = \delta x$. Then, we have the following:*

i. $V^{\mathrm{SB}_m} = U(w(0, X_0) - L_0)$ *where*

$$w(t, x) = \delta(T - t)x + \int_t^T m_{\mathrm{SB}_m}(s)\mathrm{d}s,$$

and

$$m_{\mathrm{SB}_m}(t) := \frac{1}{2}\bar{\mu}\delta^2(T - t)^2 - \frac{1}{2}(q(z_{\mathrm{SB}_m})|\sigma|^2 + \bar{\mu}(z^-_{\mathrm{SB}_m} + \delta(T - t))^2),$$

with $q(z) := h + rz^2 + p(z - \delta(T - t))^2$.

ii. *The second-best optimal payment rate is $z_{\mathrm{SB}_m}(t) = \Lambda\delta(T - t)$ with $\Lambda := \frac{p|\sigma|^2 + \bar{\mu}\mathbf{1}_{\{\delta<0\}}}{(p+r)|\sigma|^2 + \bar{\mu}\mathbf{1}_{\{\delta<0\}}}$.*

iii. *The second-best optimal contract is given by $\xi_{\mathrm{SB}_m} = \xi^{\mathrm{F}}_{\mathrm{SB}_m} + \xi^{\mathrm{V}}_{\mathrm{SB}_m}$, where*

$$\xi^{\mathrm{F}}_{\mathrm{SB}_m} = L_0 - \kappa T X_0 + \frac{1}{2}\int_0^T r z^2_{\mathrm{SB}_m}(t)|\sigma|^2\mathrm{d}t - \int_0^T H_{\mathrm{m}}(z_{\mathrm{SB}_m}(t))\mathrm{d}t,\ \xi^{\mathrm{V}}_{\mathrm{SB}_m} = \int_0^T \pi^{\mathrm{E}}_{\mathrm{SB}_m}(X_0 - X_t)\mathrm{d}t,$$

and the price for energy, $\pi^{\mathrm{E}}_{\mathrm{SB}}$, and the price for volatility, $\pi^{\mathrm{V}}_{\mathrm{SB}_m}$, are given by

$$\pi^{\mathrm{E}}_{\mathrm{SB}_m} := (1 - \Lambda)\kappa + \Lambda\theta,\ \pi^{\mathrm{V}}_{\mathrm{SB}_m} := 0.$$

Proof. A PDE characterisation of the principal's problem in the uncontrolled volatility setting is provided in Section A.4 of the appendix. Our explicit solution is directly obtained by verifying that this partial differential equation is uniquely solved by a function of the form $U(A(t)x + B(t))$ with functions A and B in closed form. Q.E.D.

The price for energy $\pi^{\mathrm{E}}_{\mathrm{SB}_m}$ is still a weighted sum of the value of energy for the consumer and the producer. Besides, it holds that

$$\pi^{\mathrm{E}}_{\mathrm{FB}} - \pi^{\mathrm{E}}_{\mathrm{SB}_m} := \underbrace{\left(\Lambda - \frac{p}{r + p}\right)}_{\geq 0}\delta.$$

During the off-peak period ($\delta > 0$), the second-best price is lower than the first-best optimum. During the peak-load period, the producer pays or charges a price greater than the social optimum.

We turn now to the measure of the welfare loss due to the information rent of the consumer because of his hidden actions. Define the informational rent as $I := -\frac{1}{p}\log\left(\frac{V^{\mathrm{FB}}}{V^{\mathrm{SB}}}\right)$. The proof of the following proposition giving the value of the information is found in the appendix, Section A.5.

Table 1. Nominal values of the parameters.

T (h)	h (p/kW^{2}h)	κ (p/kWh)	θ (p/kWh)	p (p^{-1})	r (p^{-1})	σ (W/h$^{1/2}$)	μ (kW^{2}h^{-1}p^{-1})	λ (p^{-1} kW^{2}h)
5.5	4.0 10^{-4}	11.76	67.2	0.6 10^{-2}	0.57 10^{-2}	85	9.3 10^{-5}	2.8 10^{-2}

Proposition 3.4 (Comparisons). *Assume that $f - g = \delta x$ and $\varepsilon \le \min\{\bar\varepsilon_1; \bar\varepsilon_2\}$. Then we have the following:*

i. *If $\delta \ge 0$, then there is no informational rent, $I = 0$ and $\xi_{\mathrm{SB}} = \xi_{\mathrm{FB}}$.*

ii. *When $\delta \le 0$ and $h + r\delta^2 T^2 \le \frac{1}{\bar\lambda}$, the informational rent is*

$$I = \frac{\delta^2 T^3 r^2}{6(p+r)} \frac{1}{\frac{1}{|\sigma|^2} + \frac{p+r}{\bar\mu}}.$$

During the off-peak period, there is no information rent. The optimal contract implements the same efforts as would the social planner, leading to the same reduction of volatility. In this situation, the producer's objective is to reduce only the volatility and not the expected consumption. He can achieve this objective with the two payment rates z_{SB} and γ_{SB}. During peak periods, this is no longer the case. The two payment rates are constrained by one another because the payment rate on the volatility is a deterministic function of the payment rate on the drift. Note that the information rent remains positive in the case ii even if there is only one usage. The intuition is that if there is only one usage, the producer can recover the effort by observing the quadratic variation and thus enforce optimal effort on the volatility. Nevertheless, this is not enough to enforce the first best, as the efforts on the drift remain unobservable to the producer.

4. Numerical Illustration

The purpose of this section is to illustrate the potential gains from the implementation of a responsiveness incentive mechanism, both in terms of increase in the consumer's response to price events and of benefits for the producer. We calibrate our model on the publicly available data set of the Low Carbon London pricing trial to get a consistent order of magnitude for our model parameters. We concentrate on a peak period of the day. We make the hypothesis that the LCL experiment corresponds to the implementation of an optimal contract without responsiveness incentives with one usage per consumer. All parameters can then be estimated at the exception of the marginal costs of volatility reduction ($1/\lambda_j$), for which no experiment has ever been designed to allow for its estimation. We define thus a conservative nominal value by maximising the cost of effort to reduce volatility. The details of the calibration process that leads to the nominal value of the model parameters in Table 1 are provided in the appendix (see Section A.6 and Table A.1).

Figure 3. (Color online) Results of a simulation for a peak period of the day of $T = 5.5$ hours. The left panel shows prices for energy: first best, $\pi^{\mathrm{E}}_{\mathrm{FB}}$; second best without responsiveness incentive, $\pi^{\mathrm{E}}_{\mathrm{SB}_m}$; and second best, $\pi^{\mathrm{E}}_{\mathrm{SB}}(t)$. The right panel shows prices for volatility: first best, $\pi^{\mathrm{V}}_{\mathrm{FB}}$, and second best, $\pi^{\mathrm{V}}_{\mathrm{SB}}(t)$.

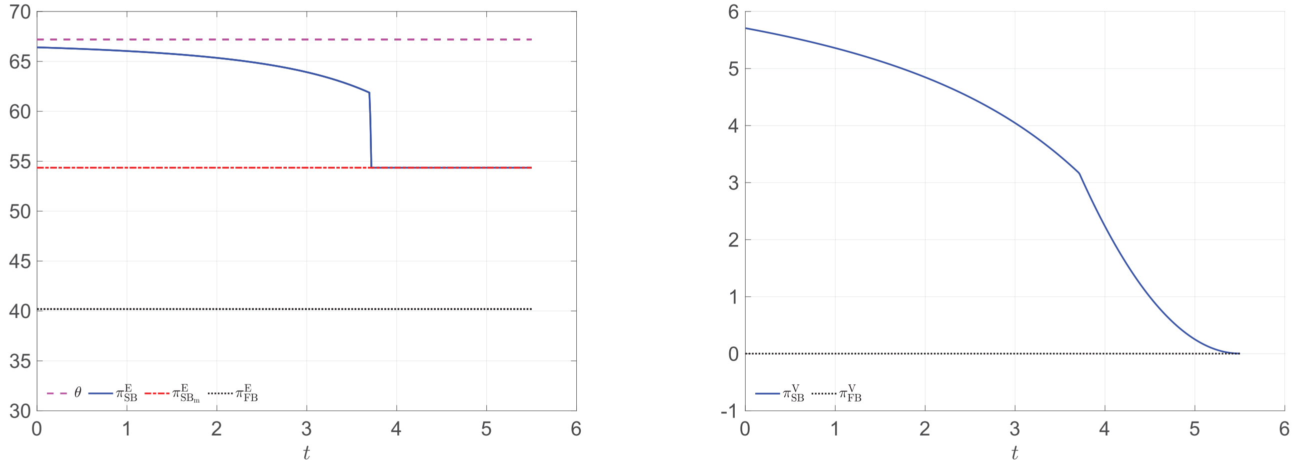


Table 2. Costs in pence and consumption and standard deviation in watts.

	First best	Second best with responsiveness	Second best without responsiveness
Cost of effort c_1	5.97	5.97	4.68
Cost of effort c_2	0.40	0.59	0
Total cost of effort	6.37	6.56	4.68
Producer's certainty equivalent benefit	6.76	6.21	5.40
Average consumption reduction	52.15	45.17	40.00
Standard deviation of reduction	46.49	39.61	85.06

Figure 3 illustrates the corresponding different energy and volatility prices defined in Section 3 for a peak period of the day. The second-best prices for energy and volatility are decreasing nonlinear functions of time, which significantly differs from the marginal costs of energy θ and volatility h and also from their first-best counterparts.

4.1. Responsiveness Incentive Estimated Benefit

Table 2 gives the results of our model for the nominal values of the parameters in three cases: the first best, the second best with responsiveness incentive, and the second best without responsiveness incentive. The second best without responsiveness incentive achieves approximately 75% of the potential consumption reduction (40 W compared with the first-best optimum of 52.15 W) and 80% of the potential producer certainty equivalent benefit (5.4 pence compared with the first best of 6.76 pence). But, the second best without responsiveness incentive exhibits a standard deviation in responses that is nearly the double that of the first best.

When implementing responsiveness incentives, the second-best contract leads to a small but significant improvement of the average consumption reduction (12.5%) and of the producer's certainty equivalent benefit (15%). Its main success consists in dividing by more than two the standard deviation of responses. This effect is the result of a 40% increase in the effort of the consumer (6.56 compared with 4.68 pence).

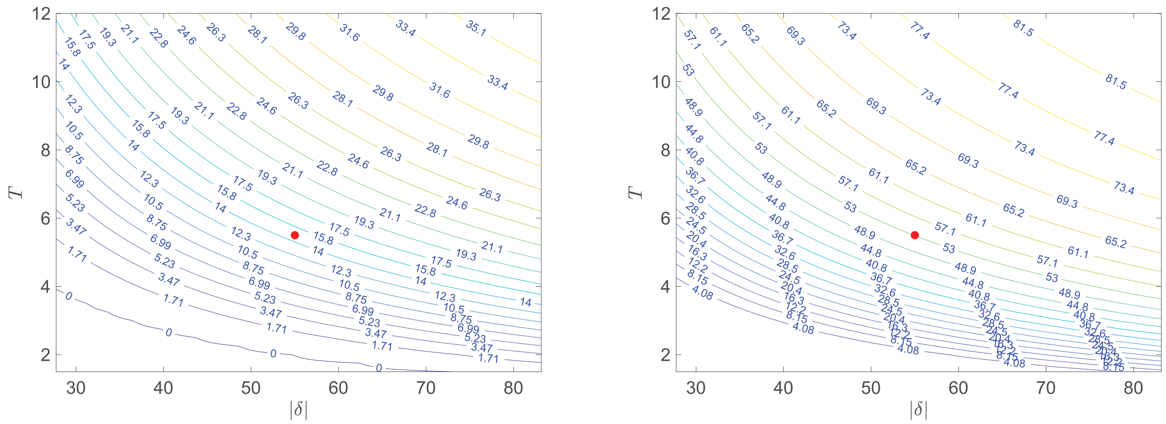
Finally, we notice that it would be socially optimal to reduce less the volatility and more the average consumption than what the second-best contract achieves.

Figure 4 presents a sensitivity analysis of the producer's certainty equivalent benefit and of the volatility reduction induced by the responsiveness incentives as functions of the duration of the price event T and the energy value discrepancy $|\delta| = \theta - \kappa$. The producer certainty equivalent benefit gain is given by $(v(0,0) - w(0,0))/w(0,0)$, where we have normalised the initial level of consumption to zero. The large dot represents the nominal situation, the results for which are given in Table 2.

We observe that there is a threshold of values of energy value discrepancy and price event duration under which no benefit should be expected from the responsiveness incentives (Figure 4, left panel). The lower the energy value discrepancy, the longer the price event should last to ensure a significant benefit of responsiveness control. The incentive on volatility needs time or a large energy value discrepancy to show its benefits. On the other hand, the reduction of volatility is less prone to this dependence on the energy value discrepancy. Even modest values of $|\theta|$ can induce substantial reduction of volatility for a price event of eight hours (Figure 4, right panel).

We conclude these numerical illustrations by showing the influence of the duration of the price event T on the decomposition of the payment between the fixed part and the random part as defined in Propositions 3.2 and 3.3. Figure 5 gives the certainty equivalent of the second-best payments ξ_{SB}, $\xi^{\mathrm{F}}_{\mathrm{SB}}$, $\xi^{\mathrm{V}}_{\mathrm{SB}}$ and their counterparts without responsiveness incentive ξ_{SB_m}, $\xi^{\mathrm{F}}_{\mathrm{SB}_m}$, $\xi^{\mathrm{V}}_{\mathrm{SB}_m}$. The total payment with or without responsiveness incentive is positive and increases when the duration of the effort becomes large (Figure 5, left). As expected, the payment with responsiveness control is larger because it requires more effort from the consumer. The interesting result comes from the decomposition of the contract between its fixed and its variable parts. The producer charges a larger fixed part when implementing a responsiveness incentive (Figure 5, middle), but provides a higher variable part (Figure 5, right). The implementation of regular responses from the consumer starts by charging him a lot more but also by rewarding him a lot more in the case of an appropriate result. The longer the consumer is asked to make an effort, the larger the difference between the payments of the two possible contracts. As a general

Figure 4. (Color online) (Left) Producer's benefit from responsiveness control $(v(0,0) - w(0,0))/w(0,0)$ and (right) the percentage of volatility reduction between the second-best and the second-best without responsiveness incentive as a function of the price event duration T and the absolute value of the energy value discrepancy δ. All values are in percentages.



principle, we may say that if a principal wants to induce regular results from an agent on a long-term basis, she should first reduce his income compared with his peers, but then pay him much more in the case of success.

4.2. Robustness Analysis

We study in this section the robustness of the hypothesis of a linear value of energy for the consumer $f(x) = \kappa x$. We consider the following specification of the function f:

$$f(x) = \kappa \frac{1 - e^{-k_1 x}}{k_1}, \tag{4.1}$$

so that, for small values of κ_1, we recover the linear case with $f(x) \approx \kappa x$, and for increasing k_1, the generation cost functions becomes more concave.

Furthermore, we consider the situation of two and four usages instead of a single one. In this case, we split the parameters μ, λ, and σ of the nominal situation provided by Table 1 with the vector of weight $(1/4, 3/4)$ for the two-usage case and $(1/8, 1/8, 1/2, 1/4)$ for the four-usage case. The choice of the vector of weights is guided by the idea of making a contrasted difference between usages.

We compute the numerical solution of the PDE of the second-best optimal contract with responsiveness incentive and nonlinear energy value and generation given in Proposition A.3i. The PDE was solved using a standard finite difference method together with an implicit Euler scheme. Figure 6 provides the resulting producer's certainty equivalent benefit (dotted lines) compared with the benefit obtained when sending to the consumer the

Figure 5. (Color online) (Left) Total payments ξ_{SB} (solid line) and ξ_{SB_m} (dashed line), (middle) fixed parts ξ^F_{SB} (solid line) and $\xi^F_{SB_m}$ (dashed line), and (right) variable parts ξ^V_{SB} (solid line) and $\xi^F_{SB_m}$ (dashed line) as functions of the price event duration T in pence.

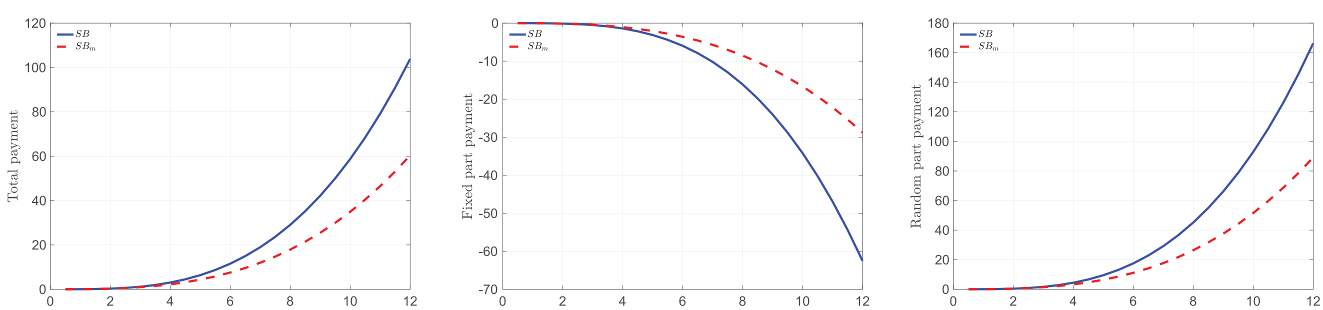
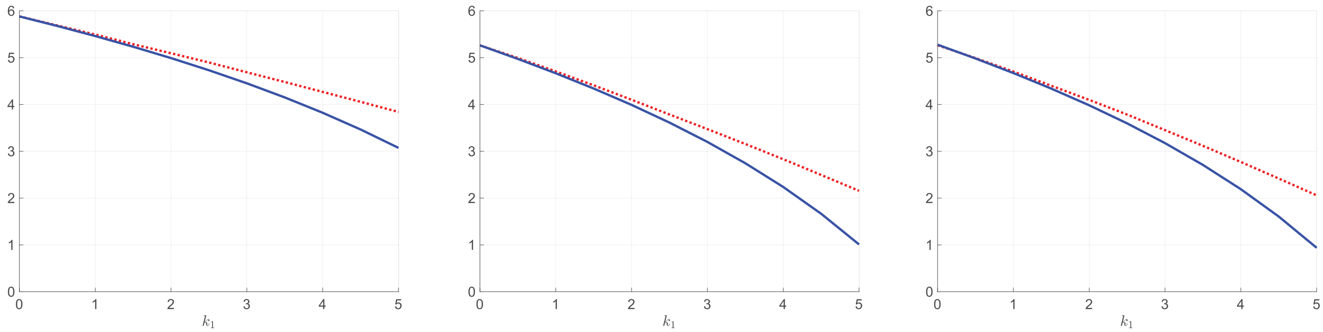


Figure 6. (Color online) Producer certainty equivalent benefit with one (left), two (middle), and four (right) usages with the linear approximation of f (solid lines) and with the nonlinear f (dotted lines).



second-best contract with the linear approximation of the energy value function (4.1) and given by Proposition 3.2i (solid lines). In both cases, the initial condition of the contract is given by the reservation utility of the consumer given by Proposition A.1, with f being given by relation (4.1). Thus, Figure 6 measures the loss of benefit issued from the linear approximation of the energy value function f of the consumer.

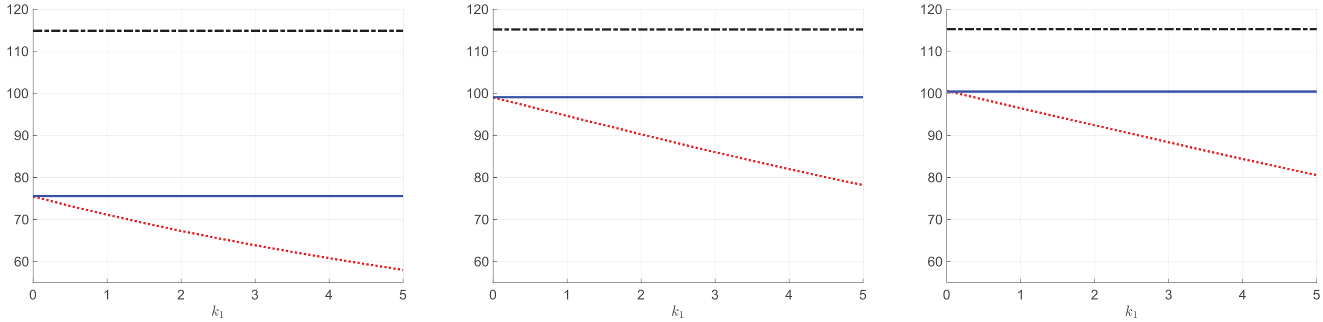
Without surprise, we observe that the greater the concavity of the energy value function, the more the linear approximation induces a loss of benefit for the producer. In the case of one usage only, a fivefold multiplication of the concavity of f leads to a loss of one pence out of four. The introduction of more usages has two effects: a general decrease of benefit independent of the linearisation of the energy value function f and an effect induced by the linearisation. The linear approximation of concavity can reduce by half the benefit of the producer.

Figure 7 shows the total volatilities under the second-best contract with the nonlinear energy value function f and the second-best optimal contract with its linear approximation. We observe that concavity increases the gap between the reduction of volatility that could be obtained with the nonlinear energy value function and its linear approximation. Nevertheless, the second-best contract with the linear approximation of f still succeeds in achieving a significant decrease of the volatility before contracting, even with an increasing number of usages.

5. Conclusion

We presented in this paper a new point of view on the demand response contract using the moral hazard problem in a principal–agent framework and showed how it makes it possible to reduce average consumption while improving the responsiveness of the consumer. We provided a closed-form expression for the optimal contract in the case of constant marginal cost and value of energy. We showed that the optimal contract has a rebate form and that the prices for energy and volatility differ from their marginal costs or values. We also showed how the optimal contract allows the system to bear more risk. The calibration of our model to pricing trial data predicts that making the consumers aware that regular responses have a value for the producer will lead to more efficient

Figure 7. (Color online) Volatilities (watts) with one (left), two (middle), and four (right) usages with the linear approximation of f (solid lines) and with the nonlinear f (dotted lines). The black dashed line gives the volatility of the consumption without a contract.



demand response programs. These predictions are testable. If our claim is true, the indexing of the payment to consumer on their regularity of consumption across price events should deeply enhance the efficiency of demand response programs.

Acknowledgments

The authors thank Hung-Po Chao, Ivar Ekeland, Sean Meyn, and Stéphane Villeneuve for their comments.

Appendix. Technical Proofs

A.1. Consumer's Behaviour Without Contract

In order to prove Proposition 2.2, we start with the following characterisation of the consumer's reservation utility.

Proposition A.1. *Assume that f is concave, nondecreasing, and Lipschitz. Then the following hold:*

i. *The consumer reservation utility is concave in X_0 and is given by $R_0 = -\mathrm{e}^{-rE(0,X_0)}$, where the corresponding certainty equivalent E is a viscosity solution of the Hamilton-Jacobi-Bellman (HJB) equation*

$$\partial_t E + H_{\mathrm{v}}(E_{xx} - rE_x^2) + f = 0, \text{ on } [0,T)\times\mathbb{R}, \text{ and } E(T,x) = 0,\ x\in\mathbb{R}, \tag{A.1}$$

with growth controlled by $|E(t,x)| \le C(T-t)|x|$, for some constant C.

ii. *Assume that the PDE (A.1) has a $C^{1,2}$ solution E, with growth controlled by $|E(t,x)| \le C(T-t)|x|$, for some constant C. Then the optimal effort of the consumer is defined by the feedback controls*

$$a^0 := 0 \text{ and } b_j^0 := 1 \wedge (\lambda_j(E_{xx} - rE_x^2))^{-\frac{1}{2}} \vee \varepsilon,\ j = 1,\dots,d.$$

Proof. For part i, because f is increasing, the consumer has no reason to make an effort on the drift of consumption deviation, as no compensation is offered for this costly effort. However, this argument does not apply to the effort on the volatility, because of the consumer's risk aversion. As the consumer has constant risk-aversion utility with parameter r, her reservation utility reduces to

$$R_0 := \sup_{\mathbb{P}^{(0,\beta)}\in\mathcal{P}} \mathbb{E}^{\mathbb{P}^{(0,\beta)}}\left[-\mathrm{e}^{-r\int_0^T\left(f(X_t) - \frac{1}{2}c_2(\beta_t)\right)\mathrm{d}t}\right]. \tag{A.2}$$

The concavity of R_0 in the initial data X_0 is a direct consequence of the concavity of f and the convexity of c_2. By standard stochastic control theory, it follows that $R_0 = R(0,X_0)$, where the function R is the dynamic version of the reservation utility, with final value $R(T,x) = -1$, and is a viscosity solution of the corresponding HJB equation

$$0 = \partial_t R - rfR + \sup_{b\in(0,1]^d} \frac{1}{2}(|\sigma(b)|^2 R_{xx} + rc_2(b)R) = \partial_t R - rfR - rRH_{\mathrm{v}}\left(\frac{R_{xx}}{-rR}\right).$$

Denote by $X_s^{t,x} := x + X_{s-t}$ the shifted canonical process started from initial data $(t,\ x)$. As f is Lipschitz, notice that

$$\begin{aligned} R(t,x) &\ge \mathbb{E}^{\mathbb{P}^{0,1}}\left[-\mathrm{e}^{-r\int_t^T f(X_s^{t,x})\mathrm{d}s}\right] \ge -\mathbb{E}^{\mathbb{P}^{0,1}}\left[\mathrm{e}r|f'|_\infty \int_t^T |X_s^{t,x}|\,\mathrm{d}s - r(T-t)f(0)\right] \\ &\ge -\mathbb{E}^{\mathbb{P}^{0,1}}\left[\mathrm{e}r|f'|_\infty((T-t)|x| + \int_t^T |X_s^{t,0}|\mathrm{d}s) - r(T-t)f(0)\right] \ge -C_1 e^{r|f'|_\infty(T-t)|x|}, \end{aligned}$$

where

$$C_1 := \mathbb{E}^{\mathbb{P}^{0,1}}\left[\mathrm{e}^{r|f'|_\infty\int_0^T |X_s^{t,0}|\mathrm{d}s) - r(T-t)f(0)}\right] < \infty,$$

because $X_s^{t,0}$ is a centred Gaussian random variable for all $s\in[t,T]$. As $c_2 \ge 0$, we also have

Table A.1. Nominal values for model parameters.

T (h)	h (p/kW^{2}h)	δ (p/kWh)	p (p^{-1})	r (p^{-1})	σ (W/h$^{1/2}$)	μ (kW^{2}h^{-1}p^{-1})	λ (p^{-1} kW^{2}h)
5.5	4.0 10^{-4}	−55.44	0.6 10^{-2}	0.57 10^{-2}	85	9.3 10^{-5}	2.8 10^{-2}

$$R(t,x) \leq \sup_{\mathbb{P}^{(0,\beta)}\in\mathcal{P}} \mathbb{E}^{\mathbb{P}^{0,\beta}}\left[-\mathrm{e}^{-r\int_t^T f(X_s^{t,x})\mathrm{d}s}\right] = \mathbb{E}^{\mathbb{P}^{0,1}}\left[-\mathrm{e}^{-r\int_t^T f(X_s^{t,x})\mathrm{d}s}\right] \leq -C_2\mathrm{e}^{-r|f'|_\infty(T-t)|x|},$$

where

$$C_2 := \mathbb{E}^{\mathbb{P}^{0,1}}\left[e^{-r|f'|_\infty\int_0^T |X_s^{t,0}|\mathrm{d}s)-r(T-t)f(0)}\right] < \infty,$$

by the same argument as before.

Then, the certainty equivalent function E, defined by $R =: -\mathrm{e}^{-rE}$, satisfies the PDE (A.1), and has growth controlled by $E(t,x) \leq (C_1 \vee C_2)(T-t)|x|$.

To prove part ii, we now assume that the PDE (A.1) has a $C^{1,2}$ solution E with growth controlled by $|E(t,x)| \leq C(T-t)|x|$. Then $\widehat{R} := -\mathrm{e}^{-rE}$ is also $C^{1,2}([0,T]\times\mathbb{R})$. Letting $K_t^\beta := \mathrm{e}^{-r\int_0^t\left(f(X_s)-\frac{1}{2}c_2(\beta_s)\right)\mathrm{d}s}$ and $T_n := \inf\{t>0 : |X_t - X_0| \geq n\}$, we compute by Itô's formula that for all $\mathbb{P}^{(0,\beta)}\in\mathcal{P}$,

$$\begin{aligned}\widehat{R}(0,X_0) &= \mathbb{E}^{\mathbb{P}^{0,\beta}}\left[K_{T_n}^\beta\widehat{R}(T_n,X_{T_n}) - \int_0^{T_n} K_t^\beta\left(\partial_t\widehat{R} + \frac{1}{2}|\sigma(\beta_t)|^2E_{xx} - r\left(f-\frac{1}{2}c_2(\beta_t)\right)\widehat{R}\right)(t,X_t)\mathrm{d}t\right]\\ &\geq \mathbb{E}^{\mathbb{P}^{0,\beta}}\left[K_{T_n}^\beta\widehat{R}(T_n,X_{T_n})\right]\\ &\underset{n\to+\infty}{\longrightarrow} \mathbb{E}^{\mathbb{P}^{0,\beta}}\left[K_T^\beta\widehat{R}(T,X_T)\right] = \mathbb{E}^{\mathbb{P}^{0,\beta}}\left[-K_T^\beta\right],\end{aligned}$$

where the local martingale part verifies that $\mathbb{E}^{\mathbb{P}^{0,\beta}}\left[\int_0^{T_n}K_t^\beta\widehat{R}_x(t,X_t)\sigma(\beta_t)\mathrm{d}W_t\right] = 0$, by the fact that $\widehat{R}_x$ is bounded on $[0,T_n]$ and $\sigma(\beta)$ is bounded. The second inequality follows from the PDE satisfied by $\widehat{R}$, and the last limit is obtained by using the control on the growth of R together with the final condition $\widehat{R}(T,.) = -1$. By the arbitrariness of $\mathbb{P}^{(0,\beta)}\in\mathcal{P}$, this implies that $\widehat{R}(0,X_0) \geq R_0$.

To prove equality, we now observe from Proposition 2.1 that by choosing $b^0(t,x) := \widehat{b}(E_{xx}-E_x^2)(t,x)$, as defined in Proposition 2.1, the (unique) inequality in the previous calculation is turned into an equality provided that the stochastic differential equation $dX_t = \sigma(b(t,X_t))\mathrm{d}W_t$ has a weak solution. This, in turn, is implied by the fact that the function $\mathring{\sigma}b$ is bounded and continuous; see Karatzas and Shreve [25, theorem 5.4.22 and remark 5.4.23]. Consequently, $\widehat{R}(0,X_0) = R_0$, and (a^0,b^0) are optimal feedback controls. Q.E.D.

Proof of Proposition 2.2. The value $V^A(0)$ is concave in X_0 and is given by $V_A(0) = -\mathrm{e}^{-rE(0,X_0)}$, where the corresponding certainty equivalent E is a viscosity solution of the HJB equation

$$-\partial_t E = f + H_{\mathrm{v}}(E_{xx} - rE_x^2) \text{ on } [0,T)\times\mathbb{R}, \text{ and } E(T,x) = 0,\ x\in\mathbb{R}. \tag{A.3}$$

By directly plugging the guess $E(t,x) = C(t)x + E_0(t)$ in the PDE (A.3), we obtain

$$C'(t)x + E_0'(t) + H_{\mathrm{v}}(-rC^2(t)) + \kappa x = 0, \text{ with } C(T) = E_0(T) = 0.$$

This entails $C(t) = \kappa(T-t)$ and $E_0(t) = \int_t^T H_{\mathrm{v}}(-rC^2(s))\mathrm{d}s$, $0\leq t\leq T$. Finally the expression of the maximiser b^0 follows from Proposition 2.1. Because this smooth solution of the PDE has the appropriate linear growth, we conclude from Proposition A.1ii that it is indeed the value function inducing $V_A(0)$. Furthermore, because $b_j^0(t) = 1\wedge\frac{1}{\kappa(T-t)\sqrt{r\lambda_j}}$, we have that $b_j^0(t) \geq 1\wedge\frac{1}{\kappa T\sqrt{r\lambda}}$. Q.E.D.

A.2. First-Best Contract

We now prove Proposition 3.1, which addresses the situation where the producer chooses both the contract ξ and the level of effort of the consumer ν, under the constraint that the consumer's satisfaction is above the reservation utility. Introducing a Lagrange multiplier $\ell \geq 0$ to penalise the participation constraint, and applying the classical Karush–Kuhn–Tucker method, we can formulate the producer's first-best problem as

$$V^{\mathrm{FB}} = \inf_{\ell\geq 0}\left\{-\ell R + \sup_{(\xi,\mathbb{P}^\nu)}\mathbb{E}^{\mathbb{P}^\nu}[U(-\xi-\mathcal{G}_T^\nu) + \ell U_{\mathrm{A}}(\xi+\mathcal{K}_T^\nu)]\right\}, \tag{A.4}$$

where $\mathcal{G}_T^\nu := \int_0^T g(X_s)\mathrm{d}s + \frac{h}{2}\langle X\rangle_T$ and $\mathcal{K}_T^\nu := \int_0^T (f(X_s) - c(\nu_s))\mathrm{d}s$. The first-order conditions in ξ are

$$-U'(-\xi_\ell - \mathcal{G}_T^\nu) + \ell U_{\mathrm{A}}'(\xi_\ell + \mathcal{K}_T^\nu) = 0.$$

In view of our specification of the utility functions, this provides the optimal contract payment for a given Lagrange multiplier ℓ,

$$\xi_\ell := \frac{1}{p+r}\ln\left(\frac{r\ell}{p}\right) - \frac{p}{p+r}\mathcal{G}_T^\nu - \frac{r}{p+r}\mathcal{K}_T^\nu. \tag{A.5}$$

Substituting this expression in (A.4), we see that the principal's first-best problem reduces to

$$V^{\mathrm{FB}} = \inf_{\ell\geq 0}\left\{\ell\left(-R + \left(1+\frac{r}{p}\right)\left(\frac{r\ell}{p}\right)^{\frac{-r}{r+p}}\bar{V}\right)\right\},\text{ with } \bar{V} := \sup_{\mathbb{P}^\nu}\mathbb{E}^{\mathbb{P}^\nu}\left[-\mathrm{e}^{-\rho\left(\int_0^T ((f-g)(X_t)-c(\nu_t))\mathrm{d}t - \frac{h}{2}\langle X\rangle_T\right)}\right],$$

and $\frac{1}{\rho} := \frac{1}{r} + \frac{1}{p}$. Notice that $\bar{V}$ does not depend on the Lagrange multiplier ℓ. Then direct calculations lead to the optimal Lagrange multiplier and first-best value function

$$\ell^\star := \frac{p}{r}\left(\frac{\bar{V}}{R}\right)^{1+\frac{p}{r}},\text{ so that } V^{\mathrm{FB}} = R\left(\frac{\bar{V}}{R}\right)^{1+\frac{p}{r}}. \tag{A.6}$$

This leads to the following proposition.

Proposition A.2. *Assume that $f-g$ is Lipschitz continuous and $\varepsilon < \bar{\varepsilon}_1$, where $\bar{\varepsilon}_1 := 1 \wedge (\sqrt{\bar{\lambda}(h+\rho\delta^2T^2)})^{-1}$. Then, the first-best contracting problem is independent of ε. Moreover, we have the following:*

i. *$\bar{V} = -\mathrm{e}^{-\rho\bar{v}(0,X_0)}$, where $\bar{v}$ has growth $|\bar{v}(t,x)| \leq C(T-t)|x|$, for some constant $C>0$, and is a viscosity solution of the PDE*

$$-\partial_t\bar{v} = (f-g) + H_{\mathrm{m}}(\bar{v}_x) + H_{\mathrm{v}}(\bar{v}_{xx} - \rho\bar{v}_x^2 - h),\ \text{on } [0,T)\times\mathbb{R},\ \text{and } \bar{v}(T,.) = 0, \tag{A.7}$$

so that, by (A.6), the first-best value function $V^{\mathrm{FB}} = U(\bar{v}(0,X_0) - L_0)$.

ii. *If, in addition, $\bar{v}$ is smooth, the optimal efforts to induce a reduction of the consumption deviation and of its volatility are given by*

$$a_{\mathrm{FB}}(t,X_t) := \widehat{a}(z_{\mathrm{FB}}(t,X_t)) \text{ and } b_{\mathrm{FB}}(t,X_t) := \widehat{b}(\gamma_{\mathrm{FB}}(t,X_t)),\ t\in[0,T], \tag{A.8}$$

where

$$z_{\mathrm{FB}}(t,x) := \bar{v}_x(t,x),\ \gamma_{\mathrm{FB}}(t,x) := \bar{v}_{xx}(t,x) - \rho\bar{v}_x^2(t,x) - h,\ (t,x)\in[0,T]\times\mathbb{R}.$$

iii. *Denoting $\nu_{\mathrm{FB}} := (a_{\mathrm{FB}}, b_{\mathrm{FB}})$, the optimal first-best contract can be written as*

$$\xi_{\mathrm{FB}} = L_0 - \frac{p}{p+r}\bar{v}(0,X_0) - \frac{r}{p+r}\int_0^T (f(X_t) - c(\nu_{\mathrm{FB}}(t,X_t)))\mathrm{d}t - \frac{p}{p+r}\int_0^T g(X_t)\mathrm{d}t + \frac{h}{2}\langle X\rangle_T.$$

Proof. By standard stochastic control theory, $\bar{V}$ can be characterised by means of the corresponding HJB equation:

$$\begin{cases}\partial_t\bar{V} - \rho(f-g)\bar{V} + \sup_{a\in A}\{-a\cdot\mathbf{1}\bar{V}_x + \rho c_1(a)\bar{V}\} + \frac{1}{2}\sup_{b\in B}\left\{|\sigma(b)|^2(\bar{V}_{xx} + \rho h\bar{V}) + \rho c_2(b)\right\} = 0,\\ \bar{V}(T,.) = -1.\end{cases}$$

Setting $\bar{V}(t,x) = -e^{-\rho\bar{v}(t,x)}$, we obtain by direct substitution the PDE satisfied by $\bar{v}$:

$$\begin{cases}-\partial_t\bar{v} = (f-g) - \inf_{a\in A}\{a\cdot\mathbf{1}\bar{v}_x + c_1(a)\} - \frac{1}{2}\inf_{b\in B}\{c_2(b) - |\sigma(b)|^2(\bar{v}_{xx} - \rho\bar{v}_x^2 - h)\},\\ \bar{v}(T,x) = 0,\end{cases} \tag{A.9}$$

which coincides with the PDE in the proposition statement, by definition of the consumer's Hamiltonian, (2.11). We next prove the control on the growth of v. First, as the cost function c is nonnegative, we have

$$\bar{V} \leq \sup_{\mathbb{P}^\nu}\mathbb{E}^{\mathbb{P}^\nu}\left[-\mathrm{e}^{-\rho(\int_0^T (f-g)(X_t)\mathrm{d}t)}\right] \leq \sup_{\mathbb{P}^\nu}\mathbb{E}^{\mathbb{P}^\nu}\left[-\mathrm{e}^{-\rho((f-g)(X_0) + |f'-g'|_\infty\int_0^T |X_t|\mathrm{d}t)}\right] < -\infty.$$

On the other hand, as the cost of no effort $c(0,1) = 0$, it follows that

$$\bar{V} \geq \mathbb{E}^{\mathbb{P}^{\nu,1}}\left[-\mathrm{e}^{-\rho(\int_0^T (f-g)(X_t)\mathrm{d}t)}\right] \geq \sup_{\mathbb{P}^\nu}\mathbb{E}^{\mathbb{P}^\nu}\left[-\mathrm{e}^{-\rho((f-g)(X_0) - |f'-g'|_\infty\int_0^T |X_t|\mathrm{d}t)}\right] > -\infty.$$

This shows that $e^{C_1T|X_0|} \leq \bar{V} \leq e^{C_2T|X_0|}$, for some constants $C_1, C_2 > 0$, and therefore $|E(0,X_0)| \leq (C_1 \vee C_2)|x|T$. By homogeneity of the problem, we deduce the announced control on growth by simply removing the time origin to any $t < T$.

Under smoothness assumptions, we follow the line of the verification argument of the proof of Proposition A.1 to prove that the optimal consumer's response derived in Proposition 2.1 is an optimal feedback control for the problem $\bar{V}$. Using the fact that $V^{\mathrm{FB}} = R_0\Big(\frac{1}{R_0}e^{-\rho(\bar{v}(0,X_0))}\Big)^{\frac{p+r}{r}}$ and $L_0 = -\frac{1}{r}\log(-R_0)$, one gets

$$V^{\mathrm{FB}} = -e^{-p(\bar{v}(0,X_0)-L_0)}.$$

Finally, the expression of ξ_{FB} follows by direct substitution of the optimal Lagrange multiplier (A.6) in (A.5). Q.E.D.

In the case when $(f-g)(x) = \delta x$, we make the guess that $\bar{v}(t,x) = \delta(T-t)x + \int_0^t \bar{m}(s)\mathrm{d}s$ satisfies the PDE (A.7) where

$$m_{\mathrm{FB}}(t) = H_{\mathrm{m}}(\delta(T-t)) + H_{\mathrm{v}}(-h-\rho\delta^2(T-t)^2).$$

The value function $\bar{v}$ satisfies the hypothesis of Proposition A.2 and the PDE (A.7). Rearranging the terms in the expression of the first-best contract iii of Proposition A.2 leads to the form of the contract in Proposition 3.1. Furthermore, because $b_{\mathrm{FB}}(t) = 1 \wedge \frac{1}{\sqrt{\lambda_j(h+\rho\delta^2(T-t)^2)}}$, we have that $b_{\mathrm{FB}}(t) \geq 1 \wedge \frac{1}{\kappa T\sqrt{\bar{\lambda}(h+\rho\delta^2T^2)}}$, and this control is indeed in B. Furthermore, because $a_{\mathrm{FB}} = \mu\delta^-(T-t)$ and $\varepsilon \leq (\delta^- T)^{-1}$, we also have that $a_{\mathrm{FB}} \in A$.

A.3. Proof of Proposition 3.2

As the volatility induced by responsiveness effort is uniformly bounded above zero, and the level reduction effort is bounded, we may follow the general methodology of Cvitanić et al. [13], based on Sannikov [33]. Let $\mathcal{V}$ be the collection of all pair processes (Z,Γ) and constants $y_0 \in \mathbb{R}$, inducing the subclass of contracts $\xi = Y_T^{y_0,Z,\Gamma}$, where

$$Y_t^{y_0,Z,\Gamma} := y_0 + \int_0^t Z_s\mathrm{d}X_s + \frac{1}{2}\int_0^t(\Gamma_s + rZ_s^2)\mathrm{d}\langle X\rangle_s - \int_0^t(H(Z_s,\Gamma_s)+f(X_s))\mathrm{d}s,\ t\in[0,T]. \tag{A.10}$$

We recall from Cvitanić et al. [13] that $U_A(Y_t^{y_0,Z,\Gamma})$ represents the agent's continuation utility so that $Y_t^{y_0,Z,\Gamma}$ is the time t value of the agent's certainty equivalent. This contract is affine in the level of consumption deviation X and the corresponding quadratic variation $\langle X\rangle$, with linearity coefficients Z and Γ. The constant part $\int_0^T(H(Z_s,\Gamma_s)+f(X_s))\mathrm{d}s$ represents the certainty equivalent of the utility gain of the consumer that can be achieved by an optimal response to the contract, and is thus subtracted from the principal's payment, in agreement with the usual principal–agent moral hazard type of contract (see Laffont and Martimort [29, chapter 4]). Furthermore, in the present setting, the risk aversion of the agent implies that the infinitesimal payment $Z_t\mathrm{d}X_t$ must be compensated by the additional payment $\frac{1}{2}rZ_t^2\mathrm{d}\langle X\rangle_t$, so that, formally, the resulting impact of the payment $Z_t\mathrm{d}X_t$ on the agent's expected utility is

$$\exp\left(-r\left(Z_t\mathrm{d}X_t + \frac{1}{2}rZ_t^2\mathrm{d}\langle X\rangle_t\right)\right) - 1 \sim 1 - r\left(Z_t\mathrm{d}X_t + \frac{1}{2}rZ_t^2\mathrm{d}\langle X\rangle_t\right) + \frac{1}{2}r^2Z_t^2\mathrm{d}\langle X\rangle_t - 1 \sim -rZ_t\mathrm{d}X_t.$$

Under the optimal response of the consumer, the dynamics of the consumption deviation and the certainty equivalent of the consumer are given by

$$X_t^{Z,\Gamma} := X_0 - \int_0^t \widehat{a}(Z_s)\cdot\mathbf{1}\mathrm{d}s + \int_0^t \widehat{\sigma}(\Gamma_s)\cdot\mathrm{d}W_s,$$
$$Y_t^{Y_0,Z,\Gamma} = Y_0 + \int_0^t\Big(c(\widehat{a}(Z_s),\widehat{b}(\Gamma_s)) - f(X_s^{Z,\Gamma}) + \frac{1}{2}rZ_s^2|\widehat{\sigma}(\Gamma_s)|^2\Big)\mathrm{d}s + \int_0^t Z_s\widehat{\sigma}(\Gamma_s)\cdot\mathrm{d}W_s,$$

so that the average rate of payment consists in paying back the consumer her costs minus benefit $c-f$ and an additional compensation for the risk taken by the consumer for bearing the volatility of consumption deviation. Note that the average rate of payment can be positive or negative.

Remark A.1.

i. Assume that the producer proposes the contract ξ^0 defined by $y_0 = -\frac{1}{r}\ln(-R_0)$, $Z = \Gamma \equiv 0$, that is, $\xi^0 = -\frac{1}{r}\ln(-R_0) - \int_0^T f(X_t^\nu)\mathrm{d}t$, as the Hamiltonian satisfies here $H_{\mathrm{v}}(0) \equiv 0$. Then, the optimal response of the consumer is obtained by solving the utility maximisation problem

$$V_A(\xi^0) = \sup_{\mathbb{P}^\nu\in\mathcal{P}}\mathbb{E}\left[U_{\mathrm{A}}\left(-\int_0^T c(\nu_t)\mathrm{d}t\right)\right].$$

As the cost function c is nondecreasing in the effort, at the optimum, the consumer makes no effort, neither on the drift nor on the volatility. Comparing with Proposition A.1, this shows that the absence of a contract is different from the above contract ξ^0 with zero payment rates.

ii. We may also examine the case where the producer offers a contract with payment $\xi = 0$ to the consumer. This is achieved by choosing $Z_t = E_x(t,X_t)$ and $\Gamma_t = (E_{xx} - E_x^2)(t,X_t)$, where the certainty equivalent reservation E is as defined in Proposition A.1. From the point of view of the consumer, such a contract is clearly equivalent to the no contracting setting, and thus induces a positive effort on the volatility in the consumer's optimal response.

By the main result of Cvitanić et al. [13], we may reduce the principal's problem to the optimisation over the class of contracts $Y_T^{y_0,Z,\Gamma}$, where $y_0 \geq L_0$ and $(Z,\Gamma) \in \mathcal{V}$. By the obvious monotonicity in y_0, this leads to the following standard stochastic control problem:

$$V^{\mathrm{SB}} = \sup_{Z,\Gamma} \mathbb{E}[U(-L_T^{Z,\Gamma})], \text{ with } L_t^{Z,\Gamma} := Y_t^{Z,\Gamma} + \int_0^t g(X_s^{Z,\Gamma})\mathrm{d}s + \frac{h}{2}\mathrm{d}\langle X^{Z,\Gamma}\rangle_s,\ t \in [0,T],$$

and starting point $y_0 = L_0$. The state variable L represents the loss of the producer under the optimal response of the consumer and is defined by the dynamics

$$\mathrm{d}L_t^{Z,\Gamma} = \frac{1}{2}(2(g-f)(X_t^{Z,\Gamma}) + \widehat{c}_1(Z_t) + f_0(rZ_t^2 + h, \Gamma_t))\mathrm{d}t + Z_t\widehat{\sigma}(\Gamma_t) \cdot \mathrm{d}W_t,\ t \in [0,T),$$

where

$$f_0(q,\gamma) := q|\widehat{\sigma}(\gamma)|^2 + \widehat{c}_2(\gamma). \tag{A.11}$$

The function $f_0(q,\gamma)$ measures the total cost the producer incurs from the volatility, when the unit cost of volatility is q and the rate of payment for the volatility reduction is γ. The term $q|\widehat{\sigma}(\gamma)|^2$ is the instantaneous cost of volatility, whereas the term $\widehat{c}_2(\gamma)$ is the cost of effort incurred by the consumer. This last cost will be paid by the producer, and enters thus in the evaluation of the cost of volatility. The producer aims at making the term $|\widehat{\sigma}(\gamma)|^2$ as small as possible. To achieve this objective, a sufficiently large γ should be paid to reduce $|\widehat{\sigma}(\gamma)|^2$, but this can be done only at the expense of an increasing cost $\widehat{c}_2(\gamma)$.

Lemma A.1. *Let $F_0(q) := \inf_{\gamma \leq 0} f_0(q,\gamma)$. Then F_0 is nondecreasing, and we have $F_0(q) = f_0(q,-q) = -2H_{\mathrm{v}}(-q)$.*

Proof. Recall that $f_0(q,\gamma) = q|\widehat{\sigma}(\gamma)|^2 + \widehat{c}_2(\gamma)$, where

$$\widehat{c}_2(\gamma) = \sum_{j=1}^d \frac{\sigma_j^2}{\lambda_j}(\widehat{b}_j(\gamma)^{-1} - 1),\ |\widehat{\sigma}(\gamma)|^2 = \sum_{j=1}^d \sigma_j^2 \widehat{b}_j(\gamma) \text{ and } \widehat{b}_j(\gamma) := 1 \wedge (\lambda_j\gamma^-)^{-\frac{1}{2}} \vee \varepsilon.$$

If $\lambda_j\gamma^- \leq 1$, then $\widehat{b}_j(\gamma) = 1$, and thus $\widehat{b}_j'(\gamma) = 0$. Similarly, if $\lambda_j\gamma^- \geq \varepsilon^{-(1+1)}$, then $\widehat{b}_j(\gamma) = \varepsilon$, and thus $\widehat{b}_j'(\gamma) = 0$. Finally, if $\varepsilon^{-(1+1)} > \lambda_j\gamma^- > 1$, then $\widehat{b}_j(\gamma) = (-\lambda_i\gamma)^{-\frac{1}{2}}$, and thus

$$\widehat{b}_j'(\gamma) = \frac{1}{2}\lambda_j^{-\frac{1}{2}}(-\gamma)^{-\frac{1}{2}-1} = -\frac{1}{2\gamma}\widehat{b}_j(\gamma).$$

Define now

$$f_j(\gamma) := \frac{\sigma_j^2}{\lambda_j}\left(\widehat{b}_j(\gamma)^{-1} - 1\right) + q\sigma_j^2\widehat{b}_j(\gamma).$$

We have

$$f_j'(\gamma) = \frac{\sigma_j^2}{\lambda_j}\left(-\widehat{b}_j(\gamma)^{-2}\widehat{b}_j'(\gamma)\right) + q\sigma_j^2\widehat{b}_j'(\gamma).$$

If $-\varepsilon^{-(1+1)} < \lambda_j\gamma < -1$, one has $\widehat{b}_j(\gamma)^{-2} = -\lambda_j\gamma$, and thus

$$f_j'(\gamma) = \sigma_j^2(\gamma + q)\widehat{b}_j'(\gamma) = -\left(1 + \frac{q}{\gamma}\right)\frac{\sigma_j^2}{2}\widehat{b}_j(\gamma)\mathbf{1}_{\{-\varepsilon^{-2} < \lambda_j\gamma < -1\}}.$$

Hence,

$$\frac{\partial f_0}{\partial \gamma}(q,\gamma) = -\left(1 + \frac{q}{\gamma}\right)\sum_{i=1}^d \frac{\sigma_i^2}{2}\widehat{b}_i(\gamma)\mathbf{1}_{\{-\varepsilon^{-2} < \lambda_i\gamma < -1\}}.$$

Therefore, the minimum of $f_0(q,\gamma)$ over negative γ is reached at $\gamma = -q$ when there is at least one index $i \in \{1,\dots,d\}$ such that $q \in [1/\lambda_i, \varepsilon^{-2}/\lambda_i]$, or f_0 does not depend on q. Hence, we can consider that the minimiser is always $\gamma = -q$, and thus $F_0(q) = f_0(q,-q)$.

Direct computations now show that

$$f_j(-q) = \frac{\sigma_j^2}{\lambda_j}\left(\mathbf{1}_{\{\lambda_j q \leq 1\}}\lambda_j q + \mathbf{1}_{\{\varepsilon^{-2} > \lambda_j q > 1\}}(2(\lambda_j q)^{\frac{1}{2}} - 1\right) + \mathbf{1}_{\{\varepsilon^{-2} \leq \lambda_j q\}}(\lambda_j\varepsilon q + \varepsilon^{-1} - 1)),$$

so that by adding all the terms,

$$F_0(q) = \sum_{j=1}^d \frac{\sigma_j^2}{\lambda_j}\left(\mathbf{1}_{\{\lambda_j q \leq 1\}}\lambda_j q + \mathbf{1}_{\{\varepsilon^{-2} > \lambda_j q > 1\}}(2(\lambda_j q)^{\frac{1}{2}} - 1\right) + \mathbf{1}_{\{\varepsilon^{-2} \leq \lambda_j q\}}(\lambda_j\varepsilon q + \varepsilon^{-1} - 1)),$$

from which it is clear that F_0 is nondecreasing. Q.E.D.

The value function of the second-best problem can be characterised as follows.

Proposition A.3 (Second-Best Contract). *Assume that $f-g$ is Lipschitz continuous and $\varepsilon < \bar{\varepsilon}_2 := 1 \wedge \frac{1}{\sqrt{\bar{\lambda}(h+r\delta^2T^2)}}$. Then, the second-best optimal contraction problem is independent of ε. Moreover, we have the following:*

i. $V^{\mathrm{SB}} = -\mathrm{e}^{-p(v(0,X_0)-L_0)}$, *where v has growth $|v(t,x)| \le C(T-t)|x|$, for some constant $C>0$, and is a viscosity solution of the PDE*

$$\begin{cases} -\partial_t v = f - g + \frac{1}{2}\bar{\mu}\, v_x^2 - \frac{1}{2}\inf_{z\in\mathbb{R}}\Big\{F_0(q(v_x,v_{xx},z)) + \bar{\mu}\big((z^- + v_x)^2 + \eta_A(v_x,z)\big)\Big\}, \text{ on } [0,T)\times\mathbb{R}, \\ v(T,.) = 0, \end{cases} \tag{A.12}$$

with $q(v_x,v_{xx},z) := h - v_{xx} + rz^2 + p(z-v_x)^2$ and $\eta_A(v_x,z) := (v_x + (z^- - A)^+)^2 - v_x^2 \longrightarrow 0$, as $A \nearrow \infty$.

ii. *If, in addition, v is smooth, the optimal payment rate γ_{SB} to incentivise the agent responsiveness is*

$$\gamma_{\mathrm{SB}}(t,X_t) := -h + v_{xx}(t,X_t) - rz_{\mathrm{SB}}^2(t,X_t) - p(z_{\mathrm{SB}}(t,X_t) - v_x(t,X_t))^2,\ t\in[0,T], \tag{A.13}$$

and the optimal payment rate for the consumption deviation reduction is the minimiser z_{SB} in (A.12), satisfying, for large A,

$$z_{\mathrm{SB}} \in \left(v_x, \frac{p}{r+p}v_x\right),\ \textit{when } v_x \le 0, \textit{ and } z_{\mathrm{SB}} = \frac{p}{r+p}v_x,\ \textit{when } v_x \ge 0.$$

iii. *The second-best optimal contract is given by*

$$\xi_{\mathrm{SB}} := \frac{-\log(-R)}{r} + \int_0^T z_{\mathrm{SB}}(t,X_t)\mathrm{d}X_t + \frac{1}{2}(\gamma_{\mathrm{SB}} + rz_{\mathrm{SB}}^2)(t,X_t)\mathrm{d}\langle X\rangle_t - (H(z_{\mathrm{SB}},\gamma_{\mathrm{SB}}) + f)(t,X_t)\mathrm{d}t.$$

Remark A.2. Consider the case of a risk-neutral consumer $r=0$. As F_0 is nondecreasing by Lemma A.1, we see that the minimum in the PDE (A.12) is attained at $z_{\mathrm{SB}} = v_x$, thus reducing the PDE (A.12) to

$$-\partial_t v - (f-g) - \frac{1}{2}\bar{\mu}\,(v_x^-)^2 = -\frac{1}{2}F_0(h - v_{xx}) = H_{\mathrm{v}}(v_{xx} - h),$$

where the last equality follows from Lemma A.1. Notice that this is the same PDE as the first-best characterisation given in Proposition A.2i, because $\rho = 0$ in the present setting.

In particular, the producer's value function is independent of its risk-aversion parameter p. The optimal payment rates for the effort on the drift and the volatility are given by $z_{\mathrm{SB}} = v_x$ and $\gamma_{\mathrm{SB}} = -h + v_{xx}$, so that the resulting optimal contract is also independent of the producer's risk-aversion p. This is consistent with the findings of Hölmstrom and Milgrom [22] in the context of a risk-neutral agent, where the optimal effort of the agent is independent of the principal risk-aversion parameter.

Proof. By standard stochastic control theory, the dynamic version of the value function of the principal, denoted by $V(t,x,\ell) := V^{\mathrm{SB}}(t,x,\ell)$, is a viscosity solution, with appropriate growth at infinity, of the corresponding HJB equation

$$\partial_t V = (f-g)V_\ell - \frac{1}{2}\sup_{(z,\gamma)\in\mathbb{R}^2}\Big\{|\widehat{\sigma}(\gamma)|^2[(h+rz^2)V_\ell + V_{xx} + z^2V_{\ell\ell} + 2zV_{x\ell}] - \bar{\mu}(2z^-V_x - (z^-)^2V_\ell) + \widehat{c}_2(\gamma)V_\ell\Big\},$$

with terminal condition $V(T,x,\ell) = U(-\ell)$, for $(x,\ell)\in\mathbb{R}^2$. Under the constant relative risk-aversion specification of the utility function of the producer, it follows that

$$-p\partial_t v = p(f-g) - \frac{1}{2}\inf_{z,\gamma}\Big\{|\widehat{\sigma}(\gamma)|^2((h+rz^2)p - pv_{xx} + p^2(v_x)^2 + z^2p^2 - 2zp^2v_x) + \bar{\mu}(2z^-pv_x + p(z^-)^2) + p\widehat{c}_2(\gamma)\Big\},$$

which reduces to the PDE (A.7).

The control on the growth of the function v is deduced from the control on the growth of V by following the same line of argument as in the proof of Proposition A.2, using the Lipschitz feature of the difference $f-g$. Similarly, under the smoothness condition, the same verification argument leads to the optimal feedback controls, defined as the maximisers of the second-best producer's Hamiltonian, which determine the optimal payment rates.

We finally verify that the additional properties of the optimal payment rates hold. First, if $v_x \ge 0$, the map $z \longmapsto F_0(h - v_{xx} + rz^2 + p(z-v_x)^2) + \bar{\mu}(z^- + v_x)^2\}$ is nonincreasing for $z \le \frac{p}{r+p}v_x$ because F_0 is a nondecreasing function. Thus, the minimum of the map is reached for the minimum of $q(v_x,v_{xx},z)$, which is $z_{\mathrm{SB}} = \frac{p}{r+p}v_x$. Second, if $v_x \le 0$, the preceding map is no longer monotonic on the interval $(v_x, \frac{p}{r+p}v_x)$. But, it is nonincreasing for $z \le v_x$ and nondecreasing for $z \ge \frac{p}{r+p}v_x$, making its infimum lie between v_x and $\frac{p}{r+p}v_x$. In both cases, the optimiser with respect to γ can be deduced from Lemma A.1 and is given by (A.13). Q.E.D.

In the case when $(f-g)(x)=\delta x$, we make the guess that $v(t,x)=\delta(T-t)x+\int_0^t m_{\mathrm{SB}}(s)\mathrm{d}s$ satisfies the PDE (A.7), where

$$m_{\mathrm{SB}}(t)=\frac{1}{2}\bar{\mu}\delta^2(T-t)^2-\inf_{z\in\mathbb{R}}\Big\{\bar{\mu}(z^-+\delta(T-t))^2-H_{\mathrm{v}}(-q(z))\Big\}.$$

The value function v satisfies the hypothesis of Proposition A.3 and the PDE (A.12). Rearranging the terms in the expression of the second-best contract of Proposition A.3iii leads to the form of the contract in Proposition 3.2. Furthermore, $\gamma_{\mathrm{SB}}(t)=-q(z_{\mathrm{SB}}(t))$ is bounded. Indeed, we know from Proposition A.3ii that $z_{\mathrm{SB}}(t)=\frac{p}{r+p}\delta(T-t)$ if $\delta>0$ or $z_{\mathrm{SB}}(t)\in(\delta(T-t),\frac{p}{r+p}\delta(T-t))$ if $\delta\le 0$. Thus, $|\gamma_{\mathrm{SB}}(t)|\le h+r\delta^2T^2$. Hence, for $\varepsilon<\bar{\varepsilon}_2:=1\wedge\frac{1}{\sqrt{\bar{\lambda}(h+r\delta^2T^2)}}\vee(\delta^-T)^{-1}$, the second-best optimal contraction problem is independent of ε.

A.4. Proof of Proposition 3.3

We isolate here the solution of the second-best optimal contracting problem when the agent is only allowed to control the drift, that is, the diffusion coefficient is fixed to $\sigma=\sigma(1)$. Similar to the controlled diffusion setting, we now prove that the optimal contracting problem is independent of ε for sufficiently small ε. We recall that this result is used as the benchmark situation in order to use the London low carbon experiment data for our parameters calibration.

Proposition A.4 (Second-Best Uncontrolled Responsiveness). *Assume $f-g=\delta x$. Then, we may find $\varepsilon_3>0$ such that, for $\varepsilon\le\varepsilon_3$, the second-best optimal contracting problem with uncontrolled responsiveness is independent of ε, with the optimal contract given by*

$$\xi^0_{\mathrm{SB}}=\frac{-\log(-R)}{r}+\Lambda\delta X_0+\frac{1}{2}\int_0^T rz^2_{\mathrm{SB}}(t)|\sigma|^2\mathrm{d}t-\int_0^T H_{\mathrm{m}}(z_{\mathrm{SB}}(t))dt-\int_0^T(\kappa-\Lambda\delta)X_t\mathrm{d}t,$$

where $z_{\mathrm{SB}}=\Lambda\delta(T-t)$, with $\Lambda:=\frac{p|\sigma|^2+\bar{\mu}\mathbf{1}_{\{w_x<0\}}}{(p+r)|\sigma|^2+\bar{\mu}\mathbf{1}_{\{w_x<0\}}}$.

Proof. When $\Gamma\equiv 0$, we have $F_0(q)=q|\sigma|^2$, and the PDE of Proposition A.3i reduces to

$$-\partial_t w=(f-g)+\frac{1}{2}\bar{\mu}\,w_x^2-\frac{1}{2}\inf_{z\in\mathbb{R}}\Big\{q(w_x,w_{xx},z)|\sigma|^2+\bar{\mu}(z^-+w_x)^2\Big\},\ \text{on } [0,T)\times\mathbb{R},\ \text{and } w(T,.)=0, \tag{A.14}$$

with $q(w_x,w_{xx},z)=h-w_{xx}+rz^2+p(z-w_x)^2$. We again find an explicit solution of the form $w(t,x)=A(t)x+B(t)$. The minimizer is obtained directly by writing first-order conditions, and the optimal contract follows. The form of the contract is obtained by applying integration by part to the term $\int_0^T z_{\mathrm{SB}}(t)\mathrm{d}X_t$ of the general form of the contract (A.10). Q.E.D.

A.5. Proof of Proposition 3.4

From Proposition A.2, we have $V^{\mathrm{FB}}=U(\bar{v}(0,X_0)+\frac{1}{r}\log(-R))$. When $(f-g)(x)=\delta x$, we have

$$\bar{v}(0,X_0)=\delta TX_0-\frac{1}{2}\int_0^T F_0(-\gamma_{\mathrm{FB}}(t))\mathrm{d}t,$$

because $z_{\mathrm{FB}}(t)\ge 0$, and thus $\widehat{c}_1(z_{\mathrm{FB}}(t))=0$. Furthermore, recall from Corollary 3.1 that when $\delta\ge 0$, we have $V^{\mathrm{SB}}=U(v(0,X_0)+\frac{1}{r}\log(-R))$ with

$$v(0,X_0)=\delta TX_0-\frac{1}{2}\int_0^T F_0(-\gamma_{\mathrm{SB}}(t))\mathrm{d}t.$$

In this case, we also have $\gamma_{\mathrm{SB}}=\gamma_{\mathrm{FB}}$. Thus, the certainty equivalent of the value function in the first best and in the second best are equal, and $I=0$. Furthermore, the equality of the certainty equivalent of the first best and the second best implies that the payments ξ_{FB} and ξ_{SB} are equal because the actions of the consumer are the same in both cases.

When $\delta<0$ and $h+r\delta^2T^2\le\frac{1}{\bar{\lambda}}$, we know that $z_{\mathrm{SB}}(t)=\Lambda\delta(T-t)$, so that

$$\inf_{z\in\mathbb{R}}\{F_0(h+rz^2+p(z-A(t))^2)+\bar{\mu}(z^-+A(t))^2\}=(h+r\Lambda\delta^2(T-t)^2)|\sigma|^2,$$

and therefore,

$$\psi(t)=\frac{1}{2}\int_t^T\bar{\mu}\delta^2(T-t)^2\mathrm{d}t-\frac{1}{2}\int_0^T(h+r\Lambda\delta^2(T-t)^2)|\sigma|^2\mathrm{d}t.$$

Hence, in this case,

$$\begin{aligned}\frac{\log(-V^{\mathrm{SB}})}{p}&=L_0-\delta TX_0-\psi(0)\\&=\pi+\frac{1}{2}\int_0^T(\gamma_s|\widehat{\sigma}(\gamma_s)|^2-\widehat{c}_2(\gamma_s)-\bar{\mu}\delta^2(T-s)^2)\mathrm{d}s+\frac{1}{2}\int_0^T(h+r\Lambda\delta^2(T-t)^2)|\sigma|^2\mathrm{d}t.\end{aligned}$$

In addition, in this setting, $c(\nu^{\mathrm{FB}}) = c_1(a_{\mathrm{FB}})$ and $-\gamma_{\mathrm{FB}} = h + \rho\delta^2(T-t)^2 \le \frac{1}{\lambda}$ because $\rho < r$, and thus $\widehat{b}_j(\gamma_{\mathrm{FB}}) = 1$, and we have $c_1(a_{\mathrm{FB}}(t)) = \frac{1}{2}\delta^2(T-t)^2$. Thus, we get

$$I = \frac{1}{2}\int_0^T \gamma_{\mathrm{FB}}(t)|\widehat{\sigma}(b_{\mathrm{FB}}(t))|^2 \mathrm{d}t + \frac{1}{2}\int_0^T (h + r\Lambda\delta^2(T-t)^2)|\sigma|^2 \mathrm{d}t = \frac{1}{2}|\sigma|^2(r\Lambda - \rho)\int_0^T \delta^2(T-t)^2 \mathrm{d}t,$$

where we used the fact that $-\gamma_{\mathrm{FB}} = h + \rho\delta^2(T-t)^2$. The required expression follows. Q.E.D.

A.6. Data and Model Calibration

The Low Carbon London demand-side response (DSR) trial performed in 2012–2013 was conducted at the initiative of the UK energy regulator (Ofgem) in partnership with both industrial players and academic institutions, among which two have to be cited in our study, Imperial College London, who treated the data of the experiment, and EDF Energy who acted as the energy provider and enrolled the consumers. The data comprise a set of 5,567 London households whose consumption was measured at half-hourly time steps from February 2011 to February 2014. For the dynamic time-of-use (dToU) tariff trial, the population was divided in two groups. One group of approximately 1,117 households was enrolled by EDF Energy in the dToU tariff, whereas the remaining 4,500 households were not subject to this dynamic tariff. The dToU was applied during the year 2013 (January 1 to December 31). Tariffs were sent to the households on a day-ahead basis using a home display or a text message to the customer's mobile phone. Prices had three levels: high (67.20 pence/kWh), normal (11.76 pence/kWh), and low (3.99 pence/kWh). The standard tariff is a flat tariff of 14.228 pence/kWh.

The precise description of the dToU trial performed in 2013 is given in Tindemans et al. [37, chapter 3]. The total numbers of events (high and low) were 93 to deal with supply events (shortage of generation) and 21 for distribution network events. In our study, we are interested in only the high-price events. There were 69 such events of high prices (45 for supply reasons and 24 for network reasons). The duration of an event could be 3, 6, 12, or 24 hours. The Low Carbon London demand-side response trial was designed to be as close as possible to a random trial experiment, while accounting for the operational constraints related to the enrolment of a large set of customers within the portfolio of given UK utility (EDF Energy). The events were randomly placed over the trial period while targeting the highest peaks of demand in the year.

The data collected by the Low Carbon London DSR trial performed in 2012–2013 can be downloaded freely at the London DataStore website (https://data.london.gov.uk) under the section "Smart Meter Energy Consumption Data in London Households." The demand response trial is extensively described in a series of reports, among which the reports Tindemans et al. [37] and Schofield et al. [34] are the most relevant for our study. Out of this data set, we eliminated all consumers for which data were not complete or exhibited outliers. The resulting sample comprises 880 consumers in the control group and 250 consumers in the dToU group.

As mentioned in Section 3.2, the optimal contract is the sum of a constant term plus a term proportional to the consumption. Thus, the optimal contract has the same form as the LCL pricing trial contract: a fixed premium to get enrolled plus a term proportional to the consumption. This provides the rationality for the calibration of the optimal contract without responsiveness control to the data of the LCL pricing trial. Thus, our strategy to calibrate our model is to use Proposition 3.3 to answer the question, What should be the parameter values of the consumer's behaviour model μ that would lead to the observed consumption reduction of the LCL pricing trial? Furthermore, because our model relies on a simplifying assumption regarding the pattern of daily consumption, we fix the initial condition of the consumption to be zero ($X_0 = 0$), making X_t directly the observed reduction of consumption.

A.6.1. Duration of the Price Event *T*. In the LCL pricing trial, there were 69 high-price events for a total of 778 half hours. The events could last 3, 6, 12, or 24 hours. Only one exceptional event lasted a full day (24 hours). Removing this outlier, we find an average duration of price event of 5.44 hours. We set $T = 5.5$ hours.

A.6.2. Energy Value Parameters κ and θ. We have seen in Proposition 3.2 that κ should be lower than θ to justify an average consumption reduction. Thus, we set the marginal value of electricity of the consumer to $\kappa = 11.76$ pence/kWh, which is the price the consumer enrolled in the dToU group pays in normal situation, and we set the marginal cost of electricity generation to $\theta = 67.2$ pence/kWh. This setting clearly refers to a high-peak-demand situation when the producer has a strong interest in avoiding costly generation.

A.6.3. Nominal Volatility σ. As pointed out in the introduction, there is significant noise in the observed reduction for the consumers enrolled in the dToU tariff. For an average reduction of consumption of 40 W for a consumption of a magnitude of 1 kW, the estimation of the responses ranges between −200 W and +200 W. No direct estimation of the standard deviation of the reductions is reported in the LCL pricing trial reports. We thus performed an estimate of the volatility of the consumption of the control group during a price event using the fact that, given our model,

$$\mathrm{Var}\left[\frac{1}{T}\int_0^T X_t \mathrm{d}t\right] = \frac{1}{3}T\sigma^2,$$

where the variance is computed under the no-effort distribution $\mathbb{P}^{(0,1)}$. We estimate an average volatility of $\sigma = 85\ \mathrm{W}\cdot\mathrm{h}^{\frac{1}{2}}$.

A.6.4. Producer's Risk Aversion *p*. Letting S denote the spot price of electricity for a given hour and F the forward price quoted the day before, one has $\mathbb{E}[e^{-pS}] \approx e^{-p(\mathbb{E}[S]-\frac{1}{2}p\sigma_S^2)}$, and by equating the certainty equivalent with the forward price, we obtain the risk premium $\mathrm{RP} := F - \mathbb{E}[S] = \frac{1}{2}p\sigma_S^2$. The risk premium electricity utilities are ready to pay to avoid the day-ahead spot price risk has been extensively analysed and estimated in the financial economics literature. Bessembinder and Lemon [6] followed by Longstaff and Wang [30], Benth et al. [5], and Viehmann [41] estimated the relation between the risk premium on each hour of delivery and the variance of the spot price on this hour. They found consistent and convergent estimations on the sign of the risk premium (negative for off-peak hours and positive for peak hours). If one focuses on the highest peak hour of the day (typically 7:00 p.m. or 8:00 p.m.), Viehmann [41, table 5, hour 20] found that dependence of the risk premium on the variance of the spot price is 0.31, which makes $p = 0.62$; Benth et al. [5, p. 14] estimated p to be no lower than 0.421; and Longstaff and Wang [30, p. 1895, table VI, hour 20] found a dependence of the risk premium on the variance of the spot price of 0.29, which makes $p = 0.58$. Bessembinder and Lemon [6] estimated risk premia not for day-ahead spot price risk but for monthly prices, which is less relevant in our context. Thus, we take as a nominal value for the risk-aversion parameter of the producer $p = 0.6$ per pound.

A.6.5. Consumer's Risk Aversion *r*. There is a large and not necessarily consensual economic literature on the relevant estimation of the consumer's risk-aversion parameters, in particular when using the constant absolute risk aversion utility function (see Gollier's [16] monograph). Nevertheless, in the context of the LCL pricing trial, the consumers were facing a small variation of their electricity bill, which is itself a fraction of their expenses, making the approximation of independence of the decision with respect to wealth sustainable. Furthermore, it is possible to provide an estimate of the risk-aversion parameter r of the population who accepted to enroll in the dynamic ToU tariff. Indeed, the enrolled consumers were paid £100 at the beginning of the trial and £50 more if they completed the trial. Besides, we estimate the financial risk taken by consumers adopting the dynamic ToU tariff. We computed for each consumer of the control group the electricity bill with the two possible tariffs, the standard flat tariff and the dynamic ToU tariff. We found that the consumers were facing a risk with a statistically significant standard deviation of £23 at the 5% level. Using the relation between the risk premium (£150) and the risk level (£23) in the relation, giving the certainty equivalent of a risk of known standard deviation for an exponential utility function, we estimated an absolute risk aversion of $r = 0.56$ per pound, which is very close to the producer's risk-aversion parameter.

A.6.6. Consumption Variation Cost *h*. This parameter is related to the flexibility of the producer's generation capacities. The higher the flexibility of the generation, the lower the variance of consumption-induced costs. With the development of intermittent energy sources, the quantification of the flexibility of a given power system has attracted the attention of researchers. For a review of this topic, we refer to Hirth et al. [18]. The value for h depends on the whole electric system, and not only on the capacity of a single power plant. There is a difference in flexibility between the electric system of Norway, which relies only on hydraulic generation, and an electric system based on wind generation and coal-fired plants. Nevertheless, if we focus on peak period of the day where flexibility is provided by gas-fired plants, we can make use of the estimations that exist for the cost of flexibility provided by power plants (see Kumar et al. [26], Oxera [32, table 3.2, p. 8], Van den Bergh and Delarue [40, table IV]). Estimates find consistent values of an order of magnitude of €25/MW$^2\cdot$h to €42/MW$^2\cdot$h for gas-fired plants, which is in general the technology used in peak period of the day. Thus, we choose a nominal value of h = €40/MW$^2\cdot$h to consider a not-so-flexible system in which there may be room for flexibility exchange.

A.6.7. Costs of Effort on Average Consumption μ. Because we do not have access to data at the usage level, we consider a single average usage. In order to fix an estimate of μ, we interpret the LCL experiment as the implementation of our demand-side model when there is no control of responsiveness or volatility (see Proposition 3.3). The conclusion of Schofield et al. [34] provides an estimate for the realised average consumption reduction of 40 W. According to Proposition 3.3, the absolute value of the average consumption deviation is given by

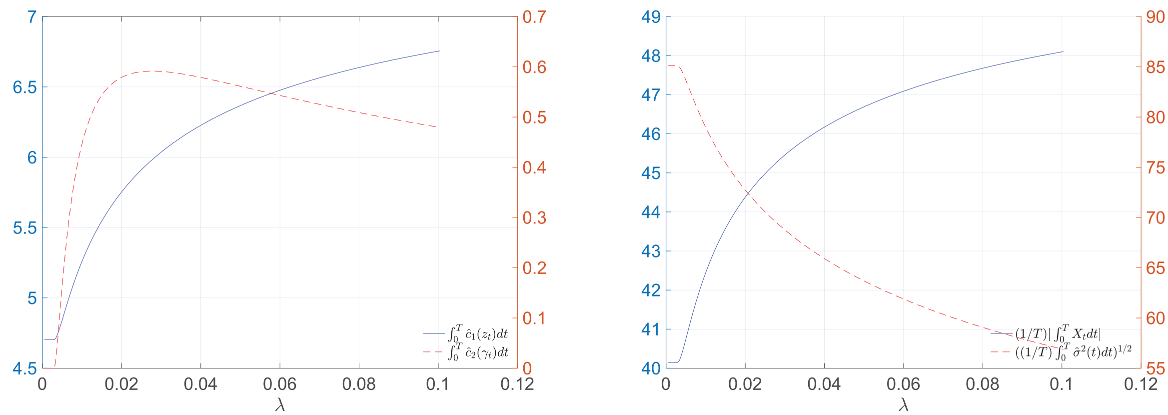
$$\frac{1}{T}\left|\mathbb{E}\left[\int_0^T X_t \mathrm{d}t\right]\right| = \frac{1}{3}\Lambda\bar{\mu}|\delta|T^2.$$

Recalling that $T = 5.5$, we obtain the value $\bar{\mu} = 9.3\,10^{-5}$. With this value of the parameter μ, the reduction of 40 W is obtained at the expense of a cost for the consumer, given by

$$\int_0^T \widehat{c}_1(z_t)\mathrm{d}t = \frac{1}{6}\bar{\mu}\Lambda^2\delta^2T^3 = 4.7 \text{ pence}.$$

A.6.8. Costs of Effort on Consumption Volatility λ. We have no way to calibrate on data a possible value for this parameter. Thus, we vary the parameter λ and observe the total cost of volatility reduction. We compare this cost with the total cost of average consumption reduction. Figure A.1 (left panel) shows the numerical estimation of the total cost of volatility reduction and the total cost of average consumption reduction as functions of λ with all other parameters fixed at the values above. For low values of λ, the corresponding consumer's effort is too costly, and thus the

Figure A.1. (Color online) (Left) Total costs of effort for average consumption reduction (left axis) and for volatility reduction (right axis), in pence, and (right) total volatility and average consumption reduction in watts.



resulting cost is zero (no effort). Then, as λ increases, the consumer starts marking efforts to be more responsive. When λ becomes large, the total cost of volatility reduction starts to decrease. We note that in this setting, the cost of volatility reduction is one order of magnitude lower than the cost for the average consumption reduction. Because we are interested in the question of what would happen if the consumer accepted to sign contracts indexed on his responsiveness, we take as a reference value for λ the value that corresponds to the maximum of total cost of efforts. This choice corresponds to a worst-case scenario for the consumer in terms of costs. It does not correspond to a maximum volatility reduction as Figure A.1 (right panel) demonstrates. We find a value of $\lambda = 2.8\,10^{-2}$. We summarise in Table 1 the reference case for the calibration of our model.

Endnotes

[1] Our work falls in the line of the works by Hölmstrom and Milgrom [21], Grossman and Hart [17], Hölmstrom and Milgrom [22], and Sannikov [33]. For an economic introduction to incentives theory, we refer to Laffont and Martimort [29]. The closest work on volatility control in the principal–agent framework is by Cvitanić et al. [12, 13].

[2] For technical reasons, we need to consider weak solutions of the stochastic differential equations. However, for expositional purposes, we deliberately ignore this technical aspect in this section to focus on the main message of the present paper.

[3] More general results are reported in the appendix, Sections A.1, A.2, and A.3, for a general increasing value function *f*.

[4] Results are provided in the appendix, Sections A.1, A.2, and A.3, for a general nondecreasing generation cost function.

[5] See the paradigmatic case of the Enerwise Company, which was fined a $780.000 penalty by the Federal Energy Regulatory Commission (143 FERC 61,218) as of June 7, 2013, for manipulation of a demand response program in the case of the management of the Baltimore stadium (see Brown and Sappington [7], Chao and De Pillis [10], Crampes and Léautier [11], Hogan [19, 20]).

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